Vom Big Bang bis heute mit Gravitation: Model for the Dynamics of Space[1]: Improved Derivation of Quantized Potential E_D, H.-O. Carmesin 2017

The purpose of this additional release[1] is to present an improved and direct derivation for the quantized potential term E_D [2].

According to the Friedmann Lemaître equation the density $\tilde{\rho}_D$ varies relatively slowly compared to the formation of the wave function. Correspondingly we apply an adiabatic separation of the slow and the fast dynamics. So the density $\tilde{\rho}_D$ is regarded as constant during the formation of the wave function of the probing mass m.

In order to derive the resulting probing mass m, we consider m as a function of its radius b (see fig. 1).

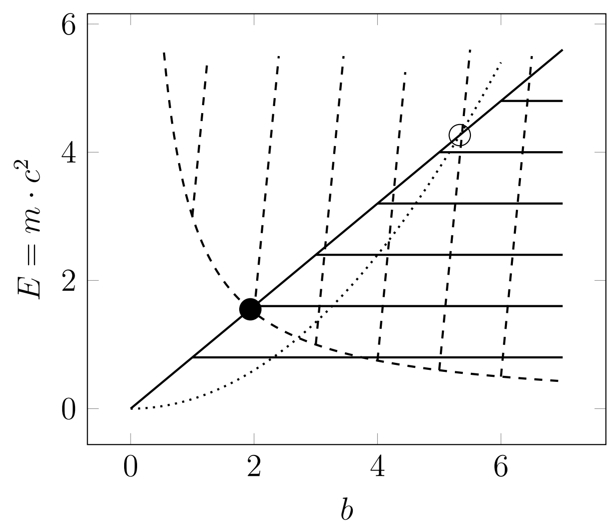

Figure 1: The concept of the Planck length is illustrated by a qualitative figure: The abscissa presents the length b that should be measured. The ordinate shows the energy E of the corresponding object. The dashed-line area marks situations that allow a measurement according to the uncertainty caused by quantum effects. The solid-line area corresponds to situations that allow a measurement according to the uncertainty caused by gravitational effects. Planck's length L_{PD} and mass M_{PD} correspond to the intersection point of the two borderlines, see full circle.

The probing mass m is a power function of its radius b, since $m = \rho_D \cdot V_D \cdot b^D$ (see dotted line in fig. 1). So the probing mass and its radius $(m|b)$ are located on the dotted line in fig. (1). The precise location is determined by the varaition principle as follows.

For the case of the local system with $p_a = 0$ and the matter era the potential E_D may be expressed as follows (see eq. (2.106)):

$$E_D = \frac{(\Delta p_a)^2}{2m^2 \cdot c^2} - \frac{G_D \cdot M}{c^2 \cdot (D-2) \cdot a^{D-2}} + \frac{G_D \cdot M}{2c^2 \cdot a^D} \cdot (\Delta a)^2$$

[1]H.-O. Carmesin (2017): Vom Big Bang bis heute mit Gravitation: Model for the Dynamics of Space. Berlin: Verlag Dr. Köster. The equation numbers used here refer to that book.

[2]Notation: Potential term $E_D = \frac{\langle E \rangle}{m \cdot c^2}$ or $E_D = U$. D-dimensional Planck length L_{PD}, Planck mass M_{PD} and volume of unit sphere V_D. D-dimensional density of mass m with radius b: $\rho_D = \frac{m}{V_D \cdot b^D}$. D-dimensional Planck density $\bar{\rho}_{PD} = \frac{M_{PD}}{V_D \cdot L_{PD}^D}$. Scaled density $\tilde{\rho}_D = \frac{\rho_D}{\bar{\rho}_{PD}}$. Momentum p_a of m corresponding to the scale factor a.

In the radiation era the mass M is proportional to $1/a$. We apply the convention $M(L_{PD}) = M_{PD}$ (see eq. (2.139)). So we obtain $M = M_{PD} \cdot L_{PD}/a$. With a derivation analogous to that presented by the equations (2.100) to (2.106) we obtain the corresponding term for the potential E_D in the radiation era:

$$E_D = \frac{(\Delta p_a)^2}{2m^2 \cdot c^2} - \frac{G_D \cdot M_{PD} \cdot L_{PD}}{c^2 \cdot (D-2) \cdot a^{D-1}} + \frac{G_D \cdot M_{PD} \cdot L_{PD} \cdot (D-1)}{2c^2 \cdot (D-2) \cdot a^{D+1}} \cdot (\Delta a)^2$$

This term decreases monotonously with increasing m. As a consequence the mass m takes the maximum possible value (see open circle in fig (1)), since the expectation value is determined by application of the variational principle. Thus the emerging radius b is equal to the Schwarzschild radius R_{SD} at the considered density. This is experessed as follows (see eqs. (2.111) to (2.119)):

$$b = L_{PD} \cdot \frac{1}{\sqrt{2 \cdot \tilde{\rho}_D}}$$

Accordingly the emerging probing mass m is expressed as follows (see eqs. (2.111) to (2.119)):

$$m = M_{PD} \cdot 2^{-D/2} \cdot \tilde{\rho}_D^{(2-D)/2}$$

Accordingly we eliminate probing mass term in the potential E_D in the above equation. So we express the potential E_D as follows:

$$E_D = 2^{D-1} \cdot \tilde{\rho}_D^{D-2} \frac{(\Delta p_a)^2}{M_{PD}^2 \cdot c^2} - \frac{G_D \cdot M_{PD} \cdot L_{PD}}{c^2 \cdot (D-2) \cdot a^{D-1}} + \frac{G_D \cdot M_{PD} \cdot L_{PD} \cdot (D-1)}{2c^2 \cdot (D-2) \cdot a^{D+1}} \cdot (\Delta a)^2$$

Next we eliminate Δp_a by application of the uncertainty relation for the case of Gaussian wave packets $\Delta p_a = \frac{D \cdot \hbar}{2\Delta a}$ (see eq. (2.120)) and obtain:

$$E_D = 2^{D-3} \cdot \tilde{\rho}_D^{D-2} \frac{D^2 \cdot \hbar^2}{M_{PD}^2 \cdot c^2 \cdot (\Delta a)^2} - \frac{G_D \cdot M_{PD} \cdot L_{PD}}{c^2 \cdot (D-2) \cdot a^{D-1}} + \frac{G_D \cdot M_{PD} \cdot L_{PD} \cdot (D-1)}{2c^2 \cdot (D-2) \cdot a^{D+1}} \cdot (\Delta a)^2$$

Next we eliminate the Planck mass M_{PD} by appropriate terms as follows: In the first fraction we apply the definition of the Planck mass $M_{PD} = \frac{\hbar}{L_{PD} \cdot c}$ (see eq. (2.138)). In the other fractions we identify the density terms $\rho_D = \frac{M_{PD} \cdot L_{PD}}{V_D \cdot a^{D+1}}$ and $\tilde{\rho}_D^{1/(D+1)} = \frac{L_{PD}}{a}$ (see eq. (2.139)) and we apply the term for the gravitational constant $\frac{G_D}{D-2} = \frac{c^2}{V_D \cdot L_{PD}^2 \cdot \bar{\rho}_{PD}}$ (see eq. (2.116)). Moreover we abbreviate the quantum fluctuations by $\hat{\alpha} = \frac{(\Delta a)^2}{D \cdot L_{PD}^2}$ (see eq. (2.110)). So we express the potential as follows:

$$E_D = D \cdot 2^{D-3} \cdot \tilde{\rho}_D^{D-2} \cdot \frac{1}{\hat{\alpha}} - \tilde{\rho}_D^{\frac{D-1}{D+1}} + \tilde{\rho}_D \cdot \frac{D \cdot (D-1)}{2} \cdot \hat{\alpha}$$

According to the variational principle this potential is minimzed by variation of $\hat{\alpha}$. As a result we obtain the following term for the potential[3]

$$E_D = -\tilde{\rho}_D^{\frac{D-1}{D+1}} + \sqrt{2^{D-2} \cdot (D-1) \cdot D^2} \cdot \tilde{\rho}_D^{\frac{D-1}{2}}$$

Conclusion: Here we derived the potential term E_D in a direct, graphical and concise manner. Several results can be derived[1] from E_D such as dimensional phase transitions, emergent structure formation and the explanation of the era of cosmic inflation by quantum gravity including excellent quantitative explanations of observations.

[3]The term in equation (2.143) exhibits a slight difference. This changes the numerical results slightly but it does not vary the conclusions of the present model. Detailed numerical results that additionally include various phase transition paths and the times required for the formation of the wave functions will be presented soon.