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How Volume Portions Form and Found Light, Gravity and Quanta

- the dynamics of volume in nature is derived from evident properties -

- the volume dynamics is a measurable foundation of present - day physics -

- the volume dynamics solves essential fundamental problems -

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Hans-Otto Carmesin

November 3, 2024

Back Cover

Volume in nature is a foundation of present-day physics. The volume dynamics, VD, is implied by evident properties: invariance of the speed of light and zero rest mass of volume

The VD includes:

- the existence and reality of volume portions, VPs
- change tensors of VPs, including electromagnetic waves
- the differential equation describing change tensors, form of wave packets, formation and propagation of VPs
- real quantities are identified via the Hacking criterion

The VD implies and explains present-day physics, e. g.

- gravity and transmission of gravitational interaction
- curvature of spacetime
- general relativity as a semiclassical and averaging approximation
- universal formation of quanta, a generalized Schrödinger equation, the postulates of quantum physics
- the Schrödinger equation as a nonrelativistic approximation
- an improved and generalized version of quantum field theory, including nonlocality and causality, without infinity, as expected, as real volume inside the light horizon cannot cause a real infinity

The VD implies, explains and clarifies properties of space, e. g.

- VD implies density of volume in our heterogeneous universe
- VD provides the time evolution of the Hubble constant (front cover), this solves the Hubble tension
- real VPs represent the wave function and its generalization
- VD clarifies real zero-point-oscillations, real Casimir force and real VPs and solves the cosmological constant problem
- causality with nonlocality is clarified

d

- formation of matter has been derived
- the era of cosmic *inflation* has been derived and clarified
- the elementary charge, the electroweak couplings and many elementary particles have been derived and explained
- the cosmological parameters have been derived fundamentally

Thus, the VD clarifies, generalizes and unifies present-day physics.

All results are in precise accordance with observation. Thereby, no fit has been executed, and no hypothesis has been introduced.

We derive all findings in a systematic, clear and smooth manner. We summarize our results by definitions, propositions and theorems. We are classes from grade 10 or higher, courses, research clubs, enthusiasts, observers, experimentalists, mathematicians, scientists, researchers . . .

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Contents

Part I

On Present-Day Theories and Methods

Chapter 1

Introduction

We humans find many objects in nature, for instance, stones, planets, molecules, stars, atoms, galaxies, nuclei, quasars (Fig. 1.1), electrons, galaxy clusters, quarks, super clusters of galaxies. But the largest natural phenomenon is the huge volume, in which these objects are located.

In fact, **volume in nature** is not only large. Moreover, an energy density of outer space has been observed, it is unexplained in general relativity, GR, and it represents 67 % of all energy in the universe, see e. g. Perlmutter et al. (1998), Riess et al. (2000), Smoot (2007), Bennett et al. (2013), Planck-Collaboration (2020).

Indeed, that volume is not static. In contrast, globally, the size and amount of volume in nature has been increasing permanently, since the Big Bang, see e. g. Einstein (1917), Slipher (1917) or Friedmann (1922), Wirtz (1922), Lemaître (1927) and-Hubble (1929), Helmcke et al. (2018). Moreover, locally, in the vicinity of a mass M, the volume in nature exhibits a curvature of space and time, see Figs. (1.1, 1.2) and e. g. Einstein (1915), Dyson et al. (1920), Hobson et al. (2006), Will (2014).

However, the usual present - day descriptions of the dynamics of volume consider essentially one entity of space that is essentially at rest, as that one entity does not move at a measurable velocity relative to a mass, see Fig. (1.2) or e. g. Hobson et al.

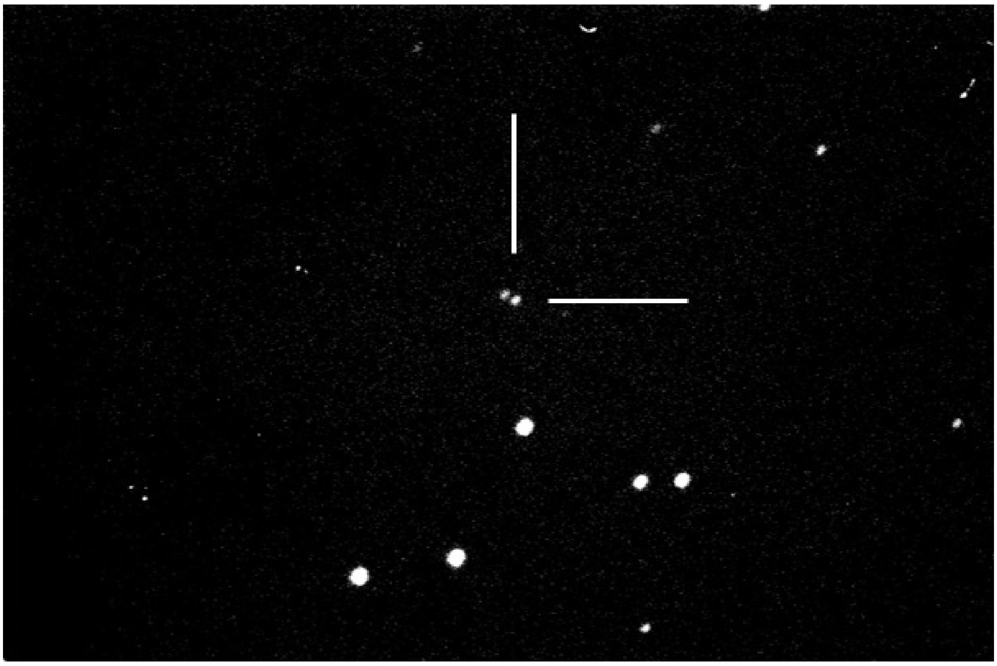

Figure 1.1: In the twin quasar (Q0957 + 561) we see two image points (Walsh et al. (1979)). Photo: Observatory Athenaeum Stade.

• The brightness of these two image points fluctuates in a correlated manner as a function of time. So the light comes from only one light source, a quasar. The two image points of the quasar are caused by a gravitational lens. That lens is explained by curvature of spacetime. That curvature will be explained by additional volume (THM 10).

• That quasar is at a light - travel distance d_{LT} of $9 \cdot 10^9$ light years. This indicates the huge amount of volume in nature.

• Light emitted by the quasar at a wavelength λ_{em} is observed at an increased wavelength $\lambda_{obs} = 2.41 \cdot \lambda_{em}$. This observation is summarized by the redshift $z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = 1.41$. This redshift implies a huge rate $d'_{LT} = \frac{\Delta d_{LT}}{\Delta \tau}$ of increase of that distance d_{LT} per time τ, no matter whether the redshift is interpreted by the optical Doppler effect or by a cosmological redshift, see e. g. Stephani (1980), Hobson et al. (2006).

• Wirtz (1922) discovered that all galaxies at very large distances d_{LT} exhibit such a huge rate d'_{LT} of increase of distance. Thus, all these galaxies permanently increase their distance to Earth. This has been a first indication of the expansion of our universe since the Big Bang.

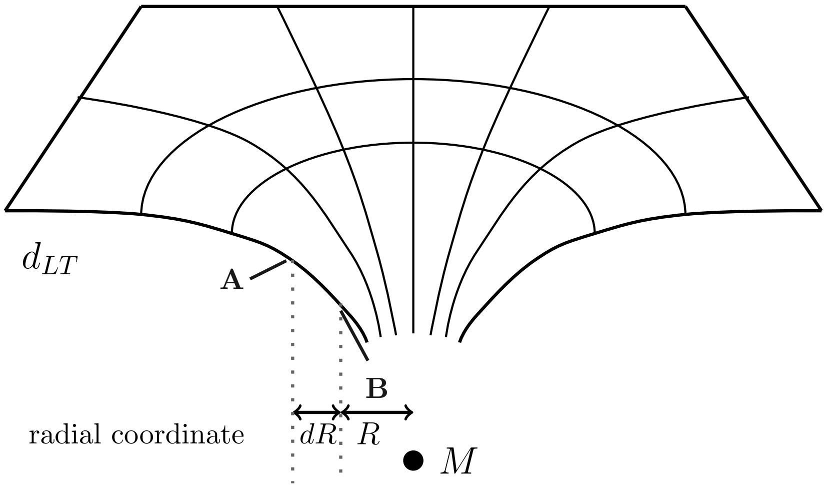

Figure 1.2: In its vicinity, a mass M causes a change of a distance dR to a light - travel distance dL. Such a change can be described with help of an element g_{RR} of a metric tensor: $dL = dR/\sqrt{g_{RR}}$. This increase of dL causes a curvature, see Fig. (1.1). The curvature can be illustrated by this figure with help of a map.

(2006), Planck-Collaboration (2020), Workman et al. (2022). But we show that volume in nature consists of many parts that propagate at very high velocity, see e. g. Carmesin (2024d). In this book, we derive the meaningful, insightful, momentous and exciting dynamics of volume in nature, the **volume dynamics, VD**.

We show that the VD implies the present - day essential theories of theoretical physics, **gravity**, see e. g. Newton (1687), Hobson et al. (2006), general relativity, GR, see e. g. Einstein (1915), Hobson et al. (2006) and quantum physics, QP, see Fig. (1.3) and e. g. Planck (1900), Heisenberg (1925), Schrödinger (1926d), Hilbert et al. (1928), Ballentine (1998), as well as electrodynamics, see Maxwell (1865), THM (7) and Carmesin (2021f). Moreover, the VD solves problems of fundamental physics beyond gravity, GR, QP and electrodynamics. And the VD provides predictions.

Thereby, we derive the VD from evident and measurable

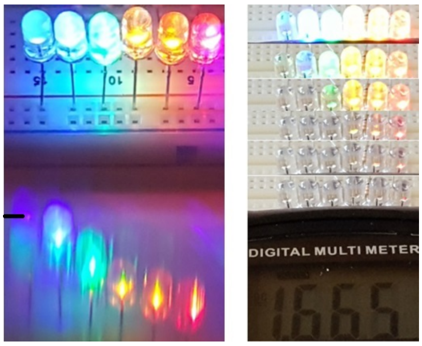

Figure 1.3: A light emitting diode, LED, has a specific colour.
• The left photo is taken with a diffraction grating in front of the camera. As a consequence, each LED exhibits two images in the photo, the upper image shows the zeroth order, the lower image shows the first order. The distance of such a pair of images provides the circular frequency ω of the emitted light, see e. g. Carmesin (2020d); Carmesin et al. (2020).
• The right photo shows the voltage $U_{required}$ that is required for the emission of light. With it, an electron in the LED has the required energy $E_{required}$. The set of LEDs shows that the required energy is a function of the circular frequency: $E_{required} = \hbar \cdot \omega$.
• Each LED can also absorb light at its specific circular frequency ω and provide a voltage $U_{provided}$ corresponding to the energy $E_{provided} = e \cdot U_{provided}$, that the light provides to each electron that moves from the valence band to the conduction band of the LED. The set of LEDs shows that the provided energy is a function of the circular frequency: $E_{provided} = \hbar \cdot \omega$.
• Both experiments indicate the following: A possible minimal energy portion of light $E_{min}(\omega)$ requires $E_{required} = \hbar \cdot \omega$, consequently $E_{min}(\omega) \leq E_{required} = \hbar \cdot \omega$. And $E_{min}(\omega)$ provides $E_{provided} = \hbar \cdot \omega$, consequently $E_{min}(\omega) \geq E_{provided} = \hbar \cdot \omega$. As a consequence, $E_{min}(\omega) = E_{provided} = \hbar \cdot \omega$: This equality describes the quanta of light, see THM (18).
• We will show that the increase of dL in Fig. (1.2) causes volume portions, VPs. These cause quanta, gravity, curvature & light. Thus, the VPs are the foundation of quanta, gravity, curvature, light and of many other objects in nature.
• The experiment is an example for the transformation of electric energy to light, or vice versa. This is essential for photovoltaics and reduction of climate change, see e. g. Carmesin (2009); Carmesin et al. (2012, 2015, 2020).

properties of volume, see section (4.2.2). In particular, we execute no fit, and we propose no hypothesis. Thereby, results achieved with the VD are in precise accordance with observation, see section (18.5). This is interpreted as a clear evidence for the VD.

Altogether, the VD is a measurable foundation of present - day physics, including gravity, GR, QP and electromagnetism, whereby the VD solves the essential problems of present - day physics. Furthermore, the VD provides a founded clarification of present - day physics, see e. g. THMs (9, 10, 18, 25, 31). Moreover, the VD provides generalizations to present - day physics, see e. g. THMs (26, 37, 20) or section (7.4) or Carmesin (2021a,f, 2022e)). Accordingly, the VD is an exciting, valuable and revealing theory with high evidence.

Concepts on volume and time proposed in science: Newton (1687) proposed a static concept of space and time. But space exhibits a dynamics, see e. g. Hubble (1929), Friedmann (1922), Lemaître (1927), Planck-Collaboration (2020).

Maxwell (1865) proposed a light aether, which can be excited by electromagnetic fields. This can be interpreted as a particular electromagnetic volume dynamics. However, it would cause a problem: The measurement of an absolute velocity relative to that aether would become possible - but such measurements failed to provide the expected velocities, see e. g. Michelson and Morley (1887), Will (2014), Carmesin (2002).

Einstein (1905b) solved the problem of the absolute velocity with his **s**pecial **r**elativity, SR. Moreover, Einstein (1915) discovered that the metric of space has a dynamics of the metric. He described it with general relativity, GR. However, that dynamics of the metric is incomplete, see e. g. Carmesin (2017c, 2018e,a, 2019b, 2021b), (Carmesin, 2021a, 2023g, Fig. 2.4).

Higgs (1964) proposed that space undergoes a transformation to matter in the process of a phase transition. Empirical

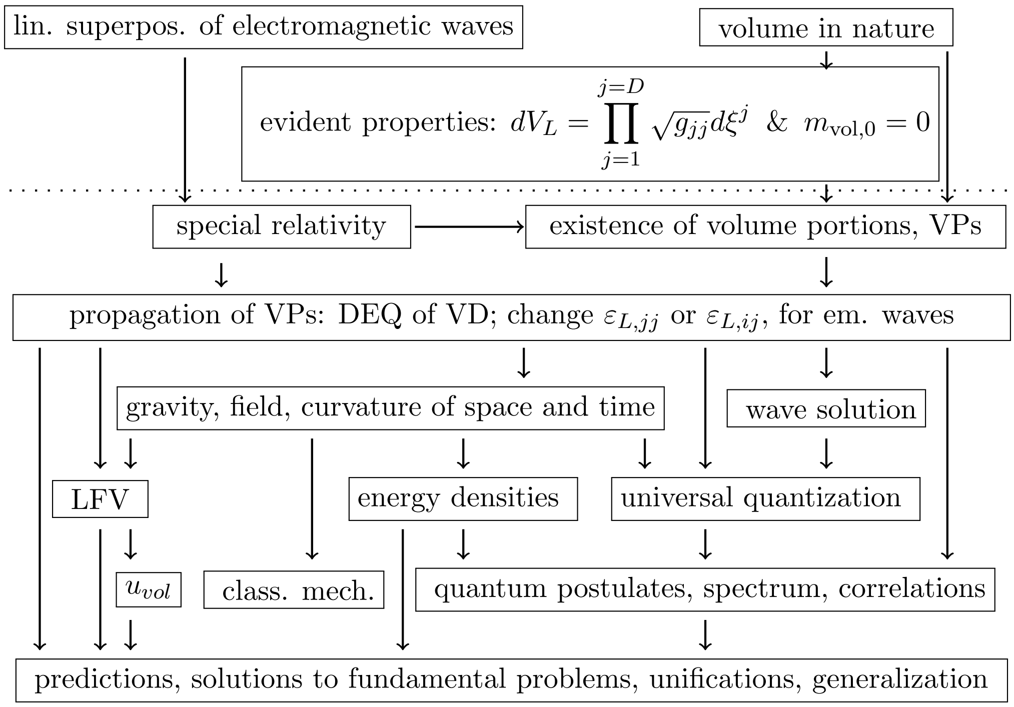

Figure 1.4: Paths of derivation: Evident properties imply volume portions, VPs. These propagate, form (via LFV, locally formed volume), change (for instance in electromagnetic, em., waves) and transform. Moreover, the volume portions, VPs, provide gravity, curvature and quanta.

evidence for it has been provided later, see e. g. Aad et al. (2012), Chatrchyan et al. (2012). However, that phase transition has been described by a postulated, phenomenological and relatively unspecific Φ^2 - Φ^4 - model only.

The volume dynamics presented in this book extends and explains the dynamics of metric, and it explicates and extends the phase transition from volume to matter, see e. g. Carmesin (2021a,f, 2022e, 2023g, 2024d) and Fig. (1.4).

Chapter 2

Linear superposition implies invariant c

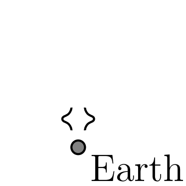

Figure 2.1: Binary star, see e. g. Karttunen et al. (2007): two stars rotate around their center of mass. For instance, when the stars have the same distance to Earth, they emit one light signal each. These signals arrive at Earth simultaneously, though the emitting stars move in opposite directions. Such observations confirm that light propagates in empty space at a constant velocity relative to an observer, irrespective of the velocity v of the light emitting source relative to the observer. For observations see e. g. de Sitter (1913), Carmesin (2006).

2.1 Electromagnetic waves of moving sources

The invariance of c has been introduced with help of observation, see Michelson and Morley (1887). In this section, we derive that invariance as a direct consequence of the fact that the phase velocity is the same for each circular frequency ω. This is a property of the Maxwell equation for empty space. It is insight- & useful, that our proof does not use any additional particular structure, such as the Lorentz transformation.

Theorem 1 Linear superposition implies invariance:

(1) As a consequence of linear superposition, the two signals $s_j(t, x)$ can be expressed by its Fourier components $s_{j,\omega}(t, x)$.

(2) As a consequence of linear superposition, each component $s_{j,\omega}(t, x)$ is a solution of a linear differential equation, DEQ. For each ω, the DEQ provides a wave vector $k(\omega)$.

In empty space, the ratio $\frac{\omega}{k}$ is equal to the velocity c of light in empty space, and it is not a function of ω. This fact is used, and it is well known, see e. g. Landau and Lifschitz (1971).

(3) As each component $s_{j,\omega}(t, x)$ propagates at c, each signal $s_j(t, x)$ propagates at c. Additionally, the signal $s_j(t, x)$ do not broaden, see e. g. Scheck (2013). Altogether, both signals propagate in a stable manner and at the same velocity c. This velocity is independent of the velocity of the sources of the signals, for instance the two stars in Fig. (2.1).

(4) In addition, sufficiently small values of the electromagnetic fields s_1 and s_2 of two electromagnetic wave signals exhibit the property of linear superposition.

Proof:

Parts (1), (2) and (3): These parts include their proofs.

Part (4): Each effect Y of the fields s_1 and s_2 of two electromagnetic wave signals can be expressed in the form of a power series. At sufficiently small fields, the linear order of the power series is sufficient for each desired accuracy (the linear order is marked by a dot at the equality sign):

$$Y(s_1, s_2) \doteq Y_0 + Y_1 \cdot s_1 + Y_1 \cdot s_2 \tag{2.1}$$

Hereby, we used the fact that the first order coefficient Y_1 is the same for s_1 and s_2, as these fields s_1 and s_2 represent the same type of physical entity. That coefficient is factorized:

$$Y(s_1, s_2) \doteq Y_0 + Y_1 \cdot (s_1 + s_2) \tag{2.2}$$

This equation represents the property of linear superposition of the fields s_1 and s_2 of electromagnetic wave signals s_1 and s_2. This completes the proof.

Interpretation: The invariance of c is very important: It implies **s**pecial **r**elativity, SR, see section (2.2). Accordingly, it is instructive to understand the source of that invariance. Our above derivation shows that the source of the invariance of c is the linear superposition of electromagnetic fields s_1 and s_2 of two electromagnetic wave signals, combined with the linear DEQ providing the dispersion relation $k(\omega) = \omega/c$.

A possible source of linear superposition is linear superposition of sufficiently small fields s_1 and s_2 of two electromagnetic signals in a power series. Hereby, radio signals are sufficiently small for this purpose of linear superposition.

It is exciting to analyze, how linear superposition works in the case of minimal energy objects. Indeed, this analysis will imply universal quantization, see THM (18).

2.2 Essential aspects of special relativity, SR

Einstein (1905b) realized that the invariance of the velocity of light has many implications. These are summarized in the field of **s**pecial **r**elativity, SR (Hobson et al., 2006, chapter 1) or

Carmesin (2020d). In this book, the following results of SR become essential:

2.2.1 Energy momentum relation

In its own frame, an object has an energy E_0, a mass m_0, an own length dL_{own} and an own time dt_{own}, see e. g. (Landau and Lifschitz, 1971, chapter I). In particular, this includes the possibility $E_0 = 0$ and $m_0 = 0$. Relative to a laboratory frame or lab frame, the own frame of the object has a velocity $\vec{v}$. In particular, this includes the case of a motion along a straight line and with a velocity v. In the lab frame, the energy is named E, the momentum is called p, the length is marked by dL and the time is denoted by dt. Thereby, the following energy momentum relation has been derived, see e. g. (Einstein, 1907, p. 384), (Landau and Lifschitz, 1971, Eq. 9.6), Carmesin (2020d):

$$E^2 = m_0^2 \cdot c^4 + p^2 \cdot c^2 \tag{2.3}$$

$$p^2 = \frac{m_0^2 v^2}{1 - v^2/c^2}, \quad \text{for} \quad m_0 > 0 \tag{2.4}$$

It is equivalent to the energy velocity relation of SR, see (Landau and Lifschitz, 1971, Eq. 9.4):

$$\frac{E_0^2}{E^2} = 1 - \frac{v^2}{c^2} \quad \text{with} \quad E_0 = m_0 c^2 \tag{2.5}$$

2.2.2 Time dilation

Two events are separated by a time difference dt and a spatial difference dL. This pair is marked by (dt, dL). A system 2 that moves at a velocity v with respect to a lab frame (see Fig. 2.2) is considered. Thereby, a time interval dt_{own} between two events A and B in the own frame of system 2 and the time interval dt between these two events A and B in the lab

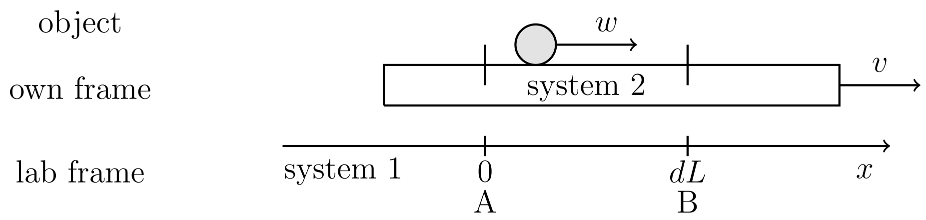

Figure 2.2: An object moves with a velocity w in a system 2, which moves with a velocity v relative to a system 1 (own frame). At what velocity u does the object move relative to system 1 (lab frame)?

frame are compared. For it, the following time dilation has been derived, see e. g. Einstein (1905b), Landau and Lifschitz (1971):

$$dt_{own} = dt \cdot \sqrt{1 - v^2/c^2} \tag{2.6}$$

In general, time intervals are different in different systems. Accordingly, Einstein (1907) analyzed the order of events in time. This is presented in the following.

2.2.3 Causality

In nature, the preparation of a state or cause occurs earlier than an observation of its effect, see e. g. D'Ariano (2018). This relation is called causality.

2.2.4 On Einstein's causality violation

(Einstein, 1907, p. 381-382) showed that a velocity $w > c$ can imply a negative time dt. For instance, the values $w = 2c$, $v = -0.8c$ and $dL > 0$ imply: As a consequence, in order to travel a distance dL from a point A to a point B, relative to the first system, the object requires the following time dt:

$$dt = dL \cdot \frac{1 - 1.6}{1.2} = dL \cdot \frac{-1}{2} < 0 \tag{2.7}$$

Moreover, (Einstein, 1907, p. 381-382) proposed that such a negative time and velocity $w > c$ would imply a causality violation and would not take place in nature:

$$\text{action at } c < w \text{ implies causality violation} \tag{2.8}$$

(Einstein, 1948, p. 321-322) named an unmediated action at $c < w$ nonlocal:

$$\text{nonlocal action} \leftrightarrow \text{unmediated action at } c < w \tag{2.9}$$

Einstein's relation of locality and causality is discussed next:

2.2.5 On Einstein's causality violation and locality

If the difference between unmediated action and action would be ignored, then the transitivity of implication could be applied to the above two relations (2.8, 2.9), so that nonlocal action would imply causality violation:

$$\text{nonlocal action would imply causality violation} \tag{2.10}$$

Correspondingly, in some theories, local action is effectively regarded as causality or micro causality, see e. g. Landau and Lifschitz (1971), Scheck (2013), Schwartz (2014).

Einstein's principle of locality is described in more detail in the next section:

2.2.6 On Einstein's principle of locality

(Einstein, 1948, p. 321-322) supposed a locality principle:

'Für die relative Unabhängigkeit räumlich distanter Dinge (A und B) ist die Idee charakteristisch: äußere Beeinflussung von A hat keinen unmittelbaren Einfluß auf B. Dies ist als Prinzip der Nahwirkung bekannt.'

'For the relative independence of spatially distant things (A and B), the following idea is characteristic: An external influence upon A has no unmediated influence upon B. This is known as locality principle.'

Thereby, two things with a spatial distance dL and a temporal distance dt and with negative $ds^2 : c^2dt^2 - dL^2$ are spatially distant or spacelike, see e. g. (Hobson et al., 2006, p. 7):

$$c^2dt^2 - dL^2 < 0 \quad \text{or} \quad c \cdot dt < dL$$
$$\text{or} \quad c < \frac{dL}{dt} := w_{eff} \text{ is spacelike} \tag{2.11}$$

Definition 1 Effective speed

If an effect and its cause have a spatial difference dL and a temporal difference dt, then we call the ratio $\frac{dL}{dt}$ the effective speed w_{eff}, at which the cause transmits the effect:

$$\frac{dL}{dt} := w_{eff} \tag{2.12}$$

Einstein (1948) excludes each unmediated action at a spacelike distance.

The principle of locality states that there is no unmediated action at $c < w_{eff}$:

$$\text{local action} \leftrightarrow \text{no unmediated action at } c < w_{eff} \tag{2.13}$$

Conversely:

$$\text{nonlocal action} \leftrightarrow \text{unmediated action at } c < w_{eff} \tag{2.14}$$

Altogether, Einstein's locality principle proposes that each unmediated action fulfills $w_{eff} \leq c$.

2.2.7 An example for $w_{eff} > c$

We analyze an example: The formation of intergalactic space can cause $\frac{\Delta L}{\Delta \tau} := w_{eff} > c$:

The light horizon R_{lh} is the present - day distance to the most distant light sources observable at Earth. At a time t, and during a time increment dt, the light signal propagates a light - travel distance $d_{LT} = c \cdot dt$. During the remaining light - travel time $t_0 - t$, that increment d_{LT} increases by the scale radius $r(t_0)$ divided by $r(t)$ to the comoving increment:

$$d_{comoving} = d_{LT} \cdot \frac{r(t_0)}{r(t)} \tag{2.15}$$

The light horizon is the integral of these comoving increments, see (Carmesin, 2019b, section 2.4):

$$R_{lh} = \int_0^{t_0} d_{comoving} = \int_0^{t_0} c \cdot dt \cdot \frac{r(t_0)}{r(t)} \tag{2.16}$$

Evaluation of the integral provides the value, see Carmesin (2021a):

$$R_{lh} \approx 4.1 \cdot 10^{26}\,\mathrm{m} \approx 43.3 \cdot 10^9\,\mathrm{ly} \tag{2.17}$$

Hereby, ly represents the unit light year, and y represents the unit year, see Workman et al. (2022). Thereby, the age t_0 of the universe is as follows, see Carmesin (2021a):

$$t_0 \approx 13.8 \cdot 10^9\,\mathrm{y} \tag{2.18}$$

As a consequence, the effective velocity w_{eff} is the ratio of the distance r_{lh} between cause (at the source) and effect (at Earth) divided by the time t_0 between cause and effect:

$$w_{eff} = \frac{43.3 \cdot 10^9\,\mathrm{ly}}{13.8 \cdot 10^9\,\mathrm{y}} = 3.14 \cdot c \tag{2.19}$$

This superluminal effective velocity $w_{eff} = 3.14 \cdot c$ is no contradiction to Einstein's locality principle, since that propagation of light is not unmediated, as a consequence of the increasing amount of intergalactic space.

2.2.8 Observed nonlocal action

However, nonlocal action has been observed, see e. g. Aspect et al. (1982) and Jaques et al. (2008). As a consequence, there are two possibilities:

(1) Either the observed nonlocal action is mediated.

(2) Or Einstein's locality principle is violated.

As volume in nature consists of many portions with zero rest mass (THM 3), each action can be regarded as a mediated action. This is analyzed further in chapter (13). Thereby, we will show that the volume portions make possible the above interpretation (1).

2.2.9 People are used to nonlocality with causality

Meanwhile, nonlocality has been observed in many experiments, see e. g. Aspect et al. (1982), Jaques et al. (2008). Thereby, no causality violation has been observed. Accordingly people are used to the fact that QP is typically nonlocal, whereby no causality violation occurs, see e. g. Levi (2015).

However, it is essential to understand this fact of observation. Indeed, we will show in THM (20) that Einstein's causality violation is not fully founded. And in fact, we will show in THM (37) that the volume portions can provide nonlocal action.

2.2.10 Three categories of objects

According to the great importance of the velocity c of light in empty space, (Einstein, 1907, p. 381-382), the objects in nature can be separated into three categories $(<, =, >)$, with respect to the effective velocity $w_{eff} = \frac{\Delta L}{\Delta \tau}$, including the velocity v as a special case:

(1) Objects at $w_{eff} < c$: Examples are objects with nonzero own mass $m_0 > 0$, such as electrons.

(2) Objects at $w_{eff} = c$: Examples are electromagnetic waves.

(3) Objects at $w_{eff} > c$: Examples are derived with help of the dynamics of volume in nature. Such objects are essential for the explanation of nonlocality in nature, see chapter (13).

These categories are elaborated further when they are needed in section (9.3).

2.2.11 Problems and solutions

Einstein (1905b) proposed SR, including time dilation. Quite rapidly, Einstein (1907) realized that the time dilation requires an analysis of causality. Thereby, he considered three essential categories, objects with $v < c$, objects with $v = c$ and objects with $v > c$. While the first and second category are treated in a successful manner, the third category has been treated in a problematic manner:

Einstein (1907) tried to exclude objects with $v > c$ with help of two proposals: causality violation and a principle of locality. However, both proposals are hardly founded in a clear manner. Moreover, nonlocal effects have been measured in quanta, e. g. Aspect et al. (1982). Furthermore, the lack of nonlocality in Einsteins's relativity prevented the necessary unification of GR and QP, see e. g. Einstein et al. (1935), Einstein (1948).

The volume dynamics, VD provides a natural solution: The VD provides quanta (chapter 11), gravity (chapter 17) and nonlocal harmonic waves (chapters 8, 12) - this triple provides nonlocality (chapter 13) and unifies GR and QP (part II).

In particular, the VD provides classical mechanics: In a first route, the VD implies QP, which implies classical mechanics in a semiclassical limit. In a second route, the VD implies curvature, which implies classical mechanics (for it, a non relativistic limit is applied), see section (19.4) and Fig. (1.4).

Chapter 3

Concepts of volume in nature

3.1 The present-day dynamical metric

In present-day physics, the main[1] basic theories are the theory of **g**eneral **r**elativity, GR, and **q**uantum **p**hysics, QP. In both theories, space or spacetime are conceptualized in terms of a flat (in QP) or dynamical (in GR) metric, so that only a metric concept of the volume is used, see e. g. (Hobson et al., 2006, S. 2.14).

3.1.1 Incompleteness of the dynamical metric

An energy density of *outer space*, $u_{\text{outer space}}$, has been measured, see Perlmutter et al. (1998), Riess et al. (2000), Smoot (2007). Thereby, an energy density is the ratio of an energy per volume,

$$u_{\text{outer space}} = \frac{\delta E_{\text{outer space}}}{\delta V_{\text{outer space}}}. \tag{3.1}$$

Consequently, outer space has a metric property, $\delta V_{\text{outer space}}$, as well as a physical property $\delta E_{\text{outer space}}$. Thereby, the energy $\delta E_{\text{outer space}}$ of outer space is essential, as it amounts to 68 % of all energy in the universe, see Bennett et al. (2013),

[1]The standard model of elementary particles, SMEP, is the thrid main theory. It treats particular particles and fundamental interactions, see e. g. Workman et al. (2022). More generally, GR, QP and partially the SMEP are implied by the VD, see for instance THM (7), Carmesin (2021a,f, 2022e).

Planck-Collaboration (2020). As a consequence, the present-day dynamical metric is **incomplete**. In order to overcome this incompleteness, we investigate the properties of volume in nature.

3.1.2 Dynamical metric in present-day theories

Indeed, the incomplete dynamical metric is used as a basis in both present-day basic theories of physics: GR and QP.

For instance, in GR, space and time are conceptualized as a combined four-dimensional spacetime that can be curved, see Einstein (1915), Hilbert (1915), Landau and Lifschitz (1971), Hobson et al. (2006). That curvature is described by a metric tensor g_{ij}. With it, the volume is described with help of the determinant $|g_{ij}|$ of that tensor g_{ij}, see (Hobson et al., 2006, Eq. 2.22). Consequently, the dynamical metric concept is underlying GR.

In QP, space or spacetime are conceptualized and used as the frame in which wave functions can propagate according to a dynamical equation. Hereby, the metric tensor of SR is used, see e. g. Landau and Lifschitz (1982), Feynman (1985), Sakurai and Napolitano (1994), Ballentine (1998), Workman et al. (2022). The basic dynamical equation is the Schrödinger equation, SEQ, see Schrödinger (1926b), Schrödinger (1926c), Schrödinger (1926d). Thereby, other dynamical equations of QP (including quantum field theory, QFT) can be developed from the SEQ or according to the SEQ, see Dirac (1928), Feynman (1965), Feynman (1985), Sakurai and Napolitano (1994), Ballentine (1998). Additionally, a stochastic structure and a Hilbert space are included in QP with help of postulates, see Hilbert et al. (1928), Sakurai and Napolitano (1994), Ballentine (1998), Kumar (2018). Thus, also QP and QFT use a metric-volume-concept.

3.1.3 Restriction to dynamical metric hardly founded

However, there is no empirical foundation of the widely used restriction to a dynamical metric, that does not consider VPs moving independently of each other. Though the metric tensor has been used in order to explain various empirical findings, see e. g. Will (2014), there is no empirical foundation of the idea that a dynamical metric is sufficient. This is the case, even though many scientists used a dynamical metric, see Newton (1687), Maxwell (1865), Einstein (1905b), Einstein (1915), Hilbert et al. (1928), Hobson et al. (2006), Sakurai and Napolitano (1994).

In contrast, the observation of the energy density of outer space (Eq. 3.1) directly shows the incompleteness of the dynamical metric. Moreover, the two present-day basic theories GR and QP appear incompatible, see e. g. Einstein et al. (1935), Einstein (1948), this additionally indicates an incompleteness of GR and/or QP.

3.1.4 The new concept implies GR and QP

In this book, we present a very successful, useful, informative and founded concept beyond the dynamical metric: the new, fundamental and derived concept of *volume portions*.

We will show in this book, that the present-day theories GR and QP are derived with help of the new concept as special cases. Moreover, we will show that many fundamental problems of present-day physics are solved by the new concept and theory.

3.2 New concept of volume-portions, VPs

In this section, we present examples that provide clear evidence for a concept of **volume portions**. These examples are interconnecting different fields in nature, such as global (S. 3.2.1) and local (S. 3.2.2) dynamics in GR as well as quantum-portions in QP (S. 3.2.3) and zero-point oscillations in a generalized and modified QFT (S. 3.2.4).

3.2.1 Volume portions form since the Big Bang

The global expansion of space since the Big Bang has been observed, see Hubble (1929), Penzias and Wilson (1965), Perlmutter et al. (1998), Planck-Collaboration (2020). In GR, this is typically described by a uniform scaling, Hobson et al. (2006). Hereby, in a time interval δt, each volume dV increases by a factor k^3, with $k > 1$. Consequently, the additional *volume portion* in three-dimensional space

$$\delta V = dV \cdot k^3 - dV \tag{3.2}$$

is added to the volume dV. As a consequence, a one-portion-concept is **incomplete**, since there is a permanent formation of the *volume portions* in Eq. (3.2).

3.2.2 Volume portions occur locally

In GR, volume portions can occur even locally. For instance, a radial increase of length in the vicinity of a mass M has been observed, Dyson et al. (1920), Pound and Rebka (1960), Will (2014). Thereby, in spherical polar coordinates (R, ϑ, φ), the mass M increases a volume $dV_R = dR \cdot Rd\vartheta \cdot R\sin\vartheta d\varphi$ in flat space to the volume $dV_L = dL \cdot Rd\vartheta \cdot R\sin\vartheta d\varphi$, with $dL = \frac{dR}{\sqrt{1-\frac{R_S}{R}}}$, and with the Schwarzschild radius $R_S = \frac{2GM}{c^2}$, see Schwarzschild (1916).

Consequently, the additional volume portion

$$\boxed{\delta V = dV_L - dV_R} \tag{3.3}$$

is added to the volume dV_R. As a consequence, a one-portion-concept is **incomplete**, since there are the observable *volume portions, VPs* in Eq. (3.3). Consequently, in the expansion of space since the Big Bang, typically, the number or amount of VPs increases.

3.2.3 Quantum-portions have been observed

In QP, *quantum-portions* are observed. For instance, quantum-portions of momentum

$$\vec{p} = \hbar\vec{k} \tag{3.4}$$

of electrons have been measured, see Davisson and Germer (1927). Hereby, $\vec{k}$ is the *wave-vector* of the portion. For example, quantum-portions of energy

$$E = \hbar\omega \tag{3.5}$$

of light have been observed, see Einstein (1905a). Hereby, ω is the *circular frequency* of the portion. Thereby, these quantum-portions are described with help of *wave-functions* Ψ, see for instance Schrödinger (1926a), Schrödinger (1926b), Schrödinger (1926c), Schrödinger (1926d) and of *quantum-postulates*, see Hilbert et al. (1928), Sakurai and Napolitano (1994), Ballentine (1998). However, the physical explanation of these wave-functions and quantum-postulates has not been provided, see Heisenberg (1958). As a consequence, the *present-day quantum-concept* is **incomplete**, since the observable *quantum-portions* in Eqs. (3.4, 3.5) are described by physically unexplained wave-functions and quantum-postulates.

3.2.4 Observed zero - point oscillations, ZPOs

In QP and QFT, *zero-point oscillations, ZPOs*, have been observed. For instance, at a ZPO with a circular frequency ω, the *zero-point energy, ZPE*

$$ZPE = \frac{1}{2}\hbar\omega \tag{3.6}$$

has been measured in crystals, see for instance Fornasini and Grisenti (2015).

However, similar ZPOs of the electromagnetic field have been introduced, see (Ballentine, 1998, C. 19). In empty space, these ZPOs provide an energy density that exceeds the observed energy density of *outer space* by a factor of roughly 10^{123}, see (Nobbenius, 2006, section 1.1) or Cugnon (2012). This problem has been called *cosmological constant problem.*

As a consequence, the *present-day quantum-concept* is **incomplete**, since observable ZPOs in Eq. (3.6) imply an energy density at empty space that exceeds the observed density of outer space by an enormous factor.

3.3 Incompleteness, problems and solutions

In this book, the theory of *volume dynamics, VD*, is developed. The VD is helpful and revealing, as it overcomes the above four instances of incompleteness. Moreover, we show that the VD is very fundamental, as we derive it from very basic and evident insights in the following.

For instance, GR is incomplete with respect to VPs. As a consequence, in present - day physics, there remain unresolved problems, e. g. the energy density of volume is not derived, the generalized and modified QFT of VPs is not derived (chapter 12), and the Hubble tension is not solved.

Moreover, QP is incomplete with respect to VPs. Thus, there remain unresolved problems, e. g. the cosmological constant problem is not solved, the quantum postulates remain unexplained, the necessary unification with gravity and general relativity is not provided, and the meaning of the wave function remains a mystery.

The VD overcomes the above instance of incompleteness, and it solves the above mentioned problems, as well as further problems of physics.

Chapter 4

Epistemology and methods

4.1 Epistemology

In this book, we analyze fundamental properties of nature. In order to distinguish our results from opinion, we discuss the used method, its validity and its scope in advance:

(Kircher et al., 2001, section 4.1.2) describe the **hypothetic deductive method**. In the epistemological literature, this method is also called hypothetico-deductive testing, see e. g. (Niiniluoto et al., 2004, p. 214). The method consists of three steps:

(1) In the hypothetic step, a thesis or hypothesis is suggested for testing.

(2) In the deductive step, implications are derived.

(3) The implications are compared with observation. Hereby, in principle, a falsification should be possible.

Ad (1): In the present book, we use evident properties of volume in nature, see section (4.2.2). Consequently, we use a hypothesis that is already tested in an evident manner. As a consequence, the derived VD is hardly hypothetical. This is regarded as a great advantage of the VD. Moreover, as the results are not based on any particular hypothesis, postulate, differential equation, action or field equation, the result is not restricted by such particular suppositions - so the scope of the achieved results is very general.

Ad (2): In the present book, we derive implications of these evident properties in section (4.2.2). Thereby, especially essential results are presented in the form of theorems. Hereby, the methods of the derivations are standard mathematical methods, see section (4.3). As a consequence, if a derived result would be falsified by observation, then either one of the evident properties would be incorrect or not applicable, or one of the standard mathematical methods would be incorrect or not applicable. Both cases are especially unlikely. Thus, this method provides an especially high level of validity. This is regarded as a great advantage of the derived results about the VD.

Ad (3): In the present book, we derive gravity, GR and QP from the VD. These essential present - day theories have already been compared with observation. These observations apply to the derived VD as well. Moreover, fundamental problems are solved. Hereby, the comparison with observation is presented in the respective chapters. Additionally, the results are summarized and reflected in the discussion.

The hypothetic deductive method is regarded as a very essential and widely accepted epistemological standard in science, see e. g. Popper (1935, 1974, 2002), Niiniluoto et al. (2004).

4.2 Evident properties of volume in nature

4.2.1 Observation of volume in nature

Volume in nature occurs in a relatively pure form in the volume of outer space or intergalactic space, $V_{\text{outer space}}$, (section 3.1.1).

Volume of outer space, $V_{\text{outer space}}$, represents a useful approximate representation of volume in nature:

(1) The volume of outer space has a very low density of atoms

or molecules. Thus, it has very small disturbances by atoms or molecules. Hence, it represents volume in nature in a relatively pure form.

(2) The volume of outer space or intergalactic space can be investigated empirically, see e. g. Karttunen et al. (2007), (Choudhuri, 2010, sections 1.5 and 11.8).

(3) The principle of translation invariance implies that undisturbed volume of the intergalactic space is the same as the undisturbed and empty volume at any other place, for instance, at Earth.

Accordingly, the volume of intergalactic or outer space is a very pure representative of volume in nature. In contrast, light or radiation or matter or mass can disturb or modify empty volume or pure volume in nature: The volume dynamics, VD, describes such changes of (pure) volume in nature - for instance with help of tensors or change tensors in DEF (5).

In the following, in most cases, we will not distinguish between volume in nature and the relatively pure representative of volume of outer or intergalactic space, shortly:

$$V_{\text{outer space}} \approx V_{\text{intergalactic space}} \approx V_{empty} \approx V \tag{4.1}$$

4.2.2 Evident properties

(1) Volume and time are fundamental.

These are basic, as objects exist in volume in nature and evolve as a function of time. Moreover, volume can undergo a change or a phase transition or transformation to matter, see e. g. Carmesin (2021a). Higgs (1964) proposed transitions from vacuum to matter, while Aad et al. (2012) and Chatrchyan et al. (2012) discovered the resulting particle.

(2) Volume in nature has the volumetric property.

For instance, in flat space, an incremental volume dV of a cuboid is the product of the three edges dx_j:

$$dV = dx_1 \cdot dx_2 \cdot dx_3 \tag{4.2}$$

More generally, in curved space, the edges $dx_{L,j}$ of a cuboid volume dV_L are usually measured with help of the light - travel distance d_{LT}, see e. g. Condon and Mathews (2018). The volume is the product of the edge lengths.

$$dV_L = dx_{L,1} \cdot dx_{L,2} \cdot dx_{L,3} \tag{4.3}$$

Thereby, in the present - day metric - tensor concept of space, the curvature is described by the elements of a metric tensor g_{ij}, for an original see e. g. Riemann (1868), for a systematic explanation based on the elementary concept of Gaussian curvature see Lee (1997), for a relatively geometric explanation see Hobson et al. (2006), for a relatively algebraic and generalizable explanation see Landau and Lifschitz (1971). Hereby, each edge length $dx_{L,j}$ is the product of a parallel parameter increment $d\xi_j$ and the root $\sqrt{|g_{jj}|}$:

$$dx_{L,j} = d\xi_j \cdot \sqrt{|g_{jj}|} \tag{4.4}$$

Altogether, the usual notation is as follows:

$$dV_L = d\xi_1\sqrt{|g_{11}|} \cdot d\xi_2\sqrt{|g_{22}|} \cdot d\xi_3\sqrt{|g_{33}|} \tag{4.5}$$

dV_L can also be expressed with the product notation:

$$dV_L = \prod_j^3 d\xi_j \cdot \sqrt{|g_{jj}|} \tag{4.6}$$

(3) Volume in nature has the rest mass m_0 equal to zero. In addition to the present - day evidence, this is confirmed by observation, see e. g. (Workman et al., 2022, p. 1142).

In fact, nobody has ever observed a nonzero rest mass of the volume in nature, se e. g. Workman et al. (2022).

$$m_{vol,0} = 0 \tag{4.7}$$

(4) At a global level, (empty or pure) volume in nature is isotropic. This is an idealization and at least a good approximation, see e. g. Smoot (2007), Planck-Collaboration (2020), Carmesin (2023g).

$$\text{(empty) volume is globally isotropic} \tag{4.8}$$

(5) Translation invariance holds in empty space.

(6) Moreover, electromagnetic waves are used as a probe for the investigation of volume in nature[1]: Electromagnetic waves exhibit the property of linear superposition. With it, the invariance of the velocity of light in empty space with respect to the velocity of the source has been derived, see THM (1). With it, laws of nature can be derived in two frames moving relative to each other: Resulting differences are derived and summarized in the field of **s**pecial **r**elativity, SR, see e. g. Einstein (1905b), Carmesin (2020d,e). Thereby, distances are measured with help of the light - travel distance d_{LT}, see e. g. Condon and Mathews (2018), Carmesin (2021d, 2023g). As a consequence, space and time merge to spacetime, see e. g. Einstein (1905b), Carmesin (2023g). In the following, space or spacetime are analyzed, corresponding to the respective situation.

(7) Moreover, the principle of energy conservation can be used in an energetically closed system that exhibits translation invariance in time (it is stationary), see e. g. Mayer (1842), Noether (1918). The stationarity is based on two facts: The laws of nature are stationary. And there is no time dependent force acting from outside our universe upon our universe.

As a consequence, a lack of stationarity can occur in a part of our universe, since other parts can exhibit time dependent forces upon that part.

[1]In THM 3, the relation to the Michelson and Morley (1887) experiment will become clear.

As an advanced organizer, note that volume in nature exhibits energy conservation in many cases, see e. g. Carmesin (2020d, 2021a), see for instance THMs (11, 16).

4.3 Methods used in derivations

In our derivations, we use standard mathematical methods only. These are analysis, see e. g. Adams and Essex (2010), linear algebra, see e. g. Lay et al. (2016), stochastics, see e. g. Olofsson and Andersson (2012), geometry, see e. g. Hart (1912), differential geometry, see e. g. Lee (1997), differential equations, see e. g. Pollard and Tenenbaum (1963), differential forms, see e. g. Flanders (1989), and functional analysis, see e. g. Teschl (2014).

Part II

Volume Dynamics, VD

Chapter 5

Observation and existence

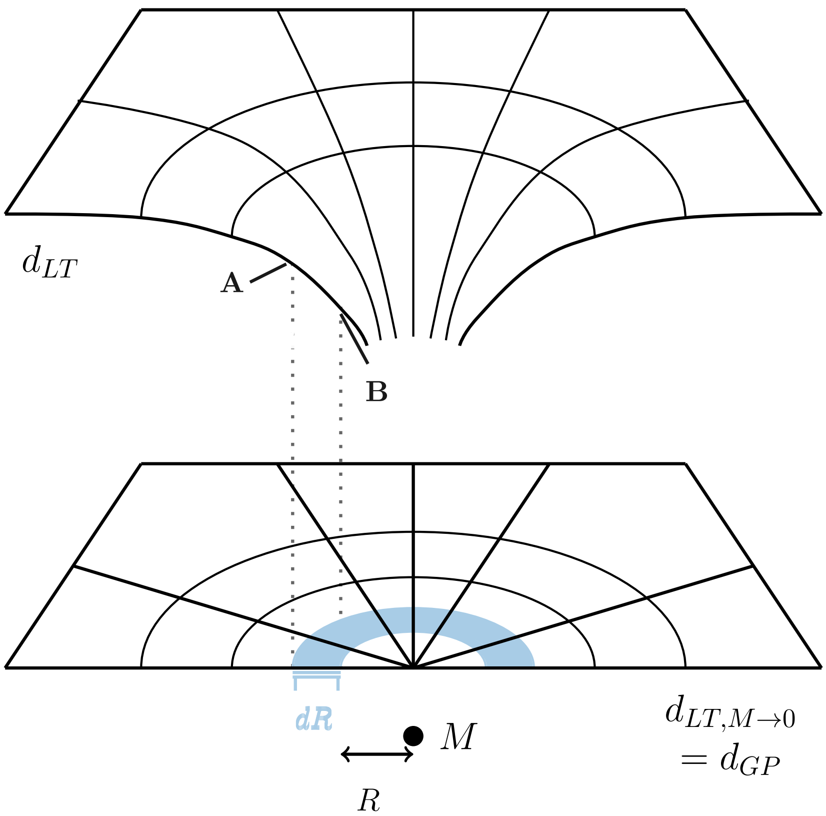

Figure 5.1: Two maps: A mass M causes nonzero curvature in its vicinity, illustrated by the upper map. In the zero mass limit, that curvature vanishes, illustrated by the lower map. A measurement procedure for both distances, dR and dL, is utile and instructive. It is developed next.

5.1 Observation of a reference volume

In the vicinity of a mass M, it is useful and informative to measure the light - travel distance d_{LT} of curved space, and to simultaneously measure the gravitational parallax distance d_{GP}, that space would have at $M = 0$, see Fig. (5.1). That is elaborated in this section.

Idea of the d_{GP}:

Firstly, a spaceship with sensors measuring quantities q_j can be used in a closed feedback loop, in order to achieve $q_j = 0$, for many quantities. With it, distances with respect to M can be measured without any disturbance by q_j. In particular, the distances with respect to M can be measured in the limit M to zero.

Secondly, distances based on a parallax can be measured with help of light, see Karttunen et al. (2007). Similarly, distances based on a parallax can be measured with help gravity, it is the helpful and illuminating gravitational parallax distance d_{GP}, see Figs. (5.2, 5.3).

Definition 2 Gravitational parallax distance to a mass

The gravitational parallax distance, d_{GP}, between an observer and a mass or dynamical mass M is defined by the following measurement procedure, see Fig. (5.3):

(1) The observer places two hand leads at points D_1 and D_2 in Fig. (5.3). Hereby, the d_{LT} between D_1 and D_2 is named b. Thereby, the hand leads are placed so that D_1, D_2 and M are the corners of an isosceles triangle. For it, the hand leads are placed so that the observer measures the same angle of gravitational parallax p_{grav} at both hand leads. The center of the baseline D_1D_2 is named A. Hereby, the observer has zero rotational velocity, $\vec{\omega} = 0$, and a constant gravitational angle of parallax

p_{grav}. *This can be achieved by the following measurement device and procedure, for instance:*

(2a) The observer (see Fig. (5.3)) uses a spacecraft. With it, the observer fulfills two conditions.

(2a1) The gravitational parallax distance d_{GP} *does not change as a function of time. For it, the observer can use a closed loop control that keeps* p_{grav} *at a constant value.*

(2a2) The angular velocity $\vec{\omega}$ *is zero. For it, the observer can use a gyroscope and a closed loop control that keeps* $\vec{\omega}$ *at the value zero.*

(2b) As a consequence, the state of the observer can be described in spherical polar coordinates (R, ϑ, φ)*, with the mass* M *at the origin, whereby the observer has constant polar coordinates* (R, ϑ, φ)*. Hereby, the radial coordinate is called gravitational parallax distance between* M *and the observer,* $R = d_{GP}$.

(2c) Consequently, the observer has a fixed position relative to the mass M.

(3) The observer calculates the gravitational parallax distance d_{GP} *according to the following equations:*

$$\text{triangle } SD_2A, \text{ is rectangular, with } \tan(p_{grav}) = \frac{b/2}{d_{GP}} \quad (5.1)$$

$$\text{thus,} \quad d_{GP} = \frac{b/2}{\tan(p_{grav})} \quad (5.2)$$

(4a) In a system with an isolated field generating mass M *at the origin, see Fig. (5.2), and with an acceleration* $\vec{a}_M$*, and with the unit vector* $\vec{e}_x$ *in the x-direction, and with two observers at the locations* $\vec{r}_1 = 10^4\,\mathrm{km} \cdot \vec{e}_x$ *and* $\vec{r}_2 = -10^4\,\mathrm{km} \cdot \vec{e}_x$*, the acceleration sensors of the observers show the following results:*

$$\vec{a}_1 = \vec{a}_M - \vec{G}_1^* \text{ and } \vec{a}_2 = \vec{a}_M + \vec{G}_1^*, \quad (5.3)$$

$$\text{thus,} \ \frac{\vec{a}_2 - \vec{a}_1}{2} = \vec{G}_1^*. \quad (5.4)$$

Hereby, $\vec{G}_1^*$ *is the gravitational field at* $\vec{r}_1$*.*

(4b) (Tipler and Llewellyn, 2008, p. 99) and (Karttunen et al., 2007, p. 365) mark the gravitational field by g*. However, (Karttunen et al., 2007, p. 212, 240) mark the gravitational acceleration by* g *as well. In order to distinguish both, we mark the gravitational field by* $\vec{G}^*$*.*

Thereby, the superscript in $\vec{G}^*$ *is just a marker. It should be distinguished from the superscript that marks the complex conjugate, for instance* $(3 + 5 \cdot i)^* = 3 - 5 \cdot i$ *or* $(e^{i\alpha})^* = e^{-i\alpha}$*.*

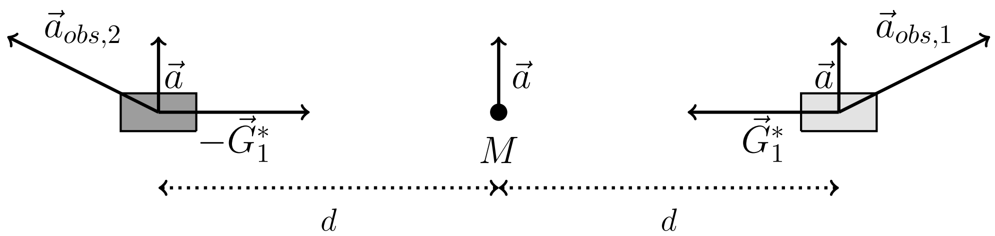

Figure 5.2: Measurement of the gravitational field $\vec{G}_1^*$ at an acceleration sensor (light gray) providing the result $\vec{a}_{obs,1}$. Another acceleration sensor (dark gray) provides the result $\vec{a}_{obs,2}$. Both sensors have the same and constant distance d to the field generating mass M and exhibit no rotation $\vec{\omega}_1 = 0 = \vec{\omega}_2$. For it, each acceleration sensor is at a spacecraft using closed loop control based on a gyroscope and laser measurement device. The system has an unknown acceleration $\vec{a}$. As a consequence, the correct field $\vec{G}_1^* = \frac{\vec{a}_{obs,2} - \vec{a}_{obs,1}}{2}$ is achieved.

More generally, there can be several masses or dynamical masses in the vicinity of the observer. In that case, the observer can measure the gravitational parallax distance to an effective mass M_{eff} as follows in DEF (3), see Fig. (5.3):

Definition 3 Gravitational parallax distance to an effective mass

The gravitational parallax distance between an observer and an effective mass or effective dynamical mass M_{eff} *is defined by the following measurement procedure, see Fig. (5.3):*

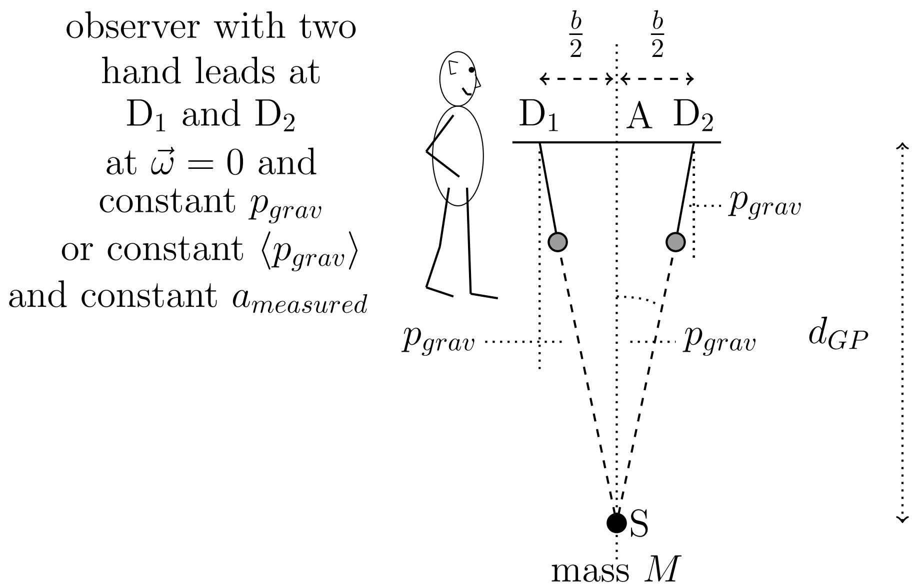

Figure 5.3: Measurement of the angle of gravitational parallax p_{grav} and of the corresponding gravitational parallax distance d_{GP} between an observer and a mass M. Hereby, D_1, D_2 and S form an isosceles triangle D_1D_2S, with the baseline D_1D_2. It has a center A.

(1) The observer places two hand leads at points in the same manner as in part (1) in Def. (2).

(2) Similarly as in part (1) Def. (2), the observer takes care for zero rotational velocity, $\vec{\omega} = 0$.

(3) Similarly as in part (1) Def. (2), the observer takes care for a constant gravitational angle of parallax p_{grav}. Consequently, there is no kinematic acceleration, since a kinematic acceleration would vary the position and p_{grav} as a function of time.

(4a) Additionally, the observer measures with an acceleration sensor. Hereby, the measured acceleration $\vec{a}_{measured}$ is constant. Since otherwise, the gravitational angle of parallax p_{grav} would vary as a function of time. Accordingly, the acceleration sensor indicates a gravitational field $\vec{G}^*$.

(4b) The absolute value of that gravitational field $\vec{G}^*$ is max-

imized in another closed loop feedback. At the maximal value $|\vec{G}^*_{max}|$*, the two hand leads are directed towards* M_{eff} *in an optimal manner. Accordingly, the gravitational angle of parallax* p_{grav} *is measured at that configuration.*

(5) The observer calculates the gravitational parallax distance according to Eq. (5.2).

(6) The concept of the effective mass is a generic term or superordinate concept including one mass as a special case.

Theorem 2 Law of the measurable gravitational parallax distance

In the vicinity of a mass M *or an effective mass* M_{eff}*, the following holds:*

(1) The gravitational parallax distance d_{GP} *between* M *or* M_{eff} *and the observer can be measured.*

(2) The gravitational parallax distance d_{GP} *does not depend on the value of* M *or* M_{eff}*. Thus,* d_{GP} *is the same distance as the light-travel distance* d_{LT} *that would occur at zero* M *or zero* M_{eff}*. Thus, the gravitational parallax distance* d_{GP} *is the same distance as the corresponding distance in flat space.*

(3) For a volume that is determined by events or landmarks, see Fig. (5.1), the same events or landmarks determine the corresponding volume in flat space. The amount of that volume can be measured with help of the gravitational parallax distance d_{GP}*.*

Proof:

Part (1): The gravitational parallax distance between M or M_{eff} and the observer can be measured by using the definitions 1 and 2.

Part (2): The gravitational parallax distance d_{GP} does not depend on the value of M or M_{eff}, as the angle of parallax does not depend on the value of M or M_{eff}.

Part (3): The volume is determined by events or landmarks in curved and in flat space, see for instance the volume of the gray region in Fig. (5.1). Thus, the volume in flat space can be measured with help of the light-travel distance d_{LT} corresponding to flat space. This is equal to the gravitational parallax distance d_{GP}. Thus, the amount of the corresponding volume in flat space can be measured with help of d_{GP}. q. e. d.

5.2 Existence of volume portions in nature

Volume in nature consists of many rapidly moving portions. This momentous insight is derived in this section.

Definition 4 Localizable volume portions

Each observer has a light horizon R_{lh}. A volume portion δV is called localizable, if the following conditions are fulfilled:

(1) Each localizable VP can be described with help of a function $f(\tau, \vec{L})$ of space $\vec{L}$ and time τ.

(2) A localizable volume portion, VP, has the potential to describe local phenomena, and it has the potential to describe non-local phenomena.

(3) A VP can be observed within the observer's light horizon R_{lh}.

(4) In D-dimensional space, for some VPs, the function $f(\tau, \vec{L})$ can describe an energy density $u(\tau, \vec{L})$ and a center of energy $\vec{L}_{E,c}(\tau)$:

$$\vec{L}_{E,c}(\tau) = \frac{\int \vec{L} \cdot u(\tau, \vec{L}) d^D L}{\int u(\tau, \vec{L}) d^D L} \tag{5.5}$$

(5) The following possibilities are included:

For instance, the function $f(\tau, \vec{L})$ can be a harmonic wave or a Gaussian wave packet.

Moreover, the function $f(\tau, \vec{L})$ can describe deterministic as well as stochastic properties of the VP or object.

Additionally, the function $f(\tau, \vec{L})$ can describe the object as a whole or a change of the object or VP.

In addition, the function $f(\tau, \vec{L})$ can be a scalar, vector or tensor. Thereby, a possible polarization is marked by a subscript p.

Furthermore, the function $f(\tau, \vec{L})$ can describe continuous behavior or discontinuous events, such as discontinuous phase transitions, see e. g. Carmesin (2021d,a, 2023g).

Proposition 1 Properties at effective velocities

A VP can transform to a mass, see Carmesin (2021a,f). As a consequence, the three categories in section (2.2.10) are included as follows:

(1) In empty space, the VP moves at $v < c$ or $w_{eff} < c$: In that case, the VP has transformed to a mass, as $w_{eff} < c$ implies $m_0 > 0$.

(2) In empty space, the VP propagates at $v = c$ or or $w_{eff} = c$: As a consequence, the VP has no rest mass. Moreover, the VP has a center of energy, as otherwise, it could travel at $w_{eff} > c$, see part (3).

(3) In empty space, the VP provides effects at $w_{eff} > c$: As a consequence, the VP has no center of energy.

Otherwise, the proof of THM (3) could be applied, so that v would be equal to c. However, the VP has $w_{eff} > c$, so that the VP has no a center of energy.

Proof: It is included in the theorem.

Theorem 3 Law of the existence of volume portions in nature

(1) Volume in nature exhibits localizable volume portions, VP, see Fig. (5.1).

(2) A localizable VP in nature with a center of energy propagates at the velocity of light c.

(3) Volume in nature consists of many VPs.

Proof:

Part (1): It is sufficient to derive one localizable VP in nature. For it, a portion of additional volume in the vicinity of a mass in Fig. (5.1) is derived: The center of the shell is M. The radius is R. In the zero mass limit, the thickness is dR. Thus, the volume is as follows:

$$dV_R = 4\pi R^2 \cdot dR \tag{5.6}$$

In general, the thickness is dL. Hence, the volume is as follows: Thus, the volume is as follows:

$$dV_L = 4\pi R^2 \cdot dL \tag{5.7}$$

As $dL > dR$, the shell has the following positive additional volume localized at M:

$$\delta V = dV_L - dV_R = 4\pi R^2 \cdot (dL - dR) > 0 \tag{5.8}$$

Part (2): The volume in nature has no rest mass.

In order to show $v = c$ for the case of a VP, we apply the energy velocity relation of SR, see property (6) in section (4.2.2), Eq. (2.5) or (Landau and Lifschitz, 1971, Eq. 9.4):

$$\frac{E_0^2}{E^2} = 1 - \frac{v^2}{c^2} \text{ with } E_0 = m_0 c^2 \tag{5.9}$$

In the above energy relation, $m_{vol,0} = 0$ (see Eq. 4.7) is used. At $m_{vol,0} = 0$, the energy momentum relation in Eq. (2.3) implies

$E = p \cdot c$. With it and $m_{vol,0} = 0$, Eq. (5.9) implies that the velocity v is equal to c:

$$\frac{E_0^2}{E^2} = \frac{E_0^2}{p^2 c^2} = 0 = 1 - \frac{v^2}{c^2} \quad \text{or} \quad v = c \tag{5.10}$$

The proof of part (2) is completed.

(3a) Volume in nature can be observed within the particle horizon or light horizon R_{lh}, see e. g. (Workman et al., 2022, section 22.2). Thereby, the velocities $\vec{v}_j$ of the volume portions within R_{lh} average to zero,

$$\langle \vec{v}_j \rangle = 0 \tag{5.11}$$

as volume in nature is globally isotropic, see Eq. (4.8).

(3b) Volume in nature consists of more than one volume portion, as the average of their velocity vectors is zero, see Eq. (5.11):

$$\text{different VPs exist with} \quad \vec{v}_j \neq \vec{v}_j \tag{5.12}$$

Consequently, there exist several different VPs in nature. This completes the proof.

Chapter 6

Representation and propagation of VPs

6.1 Representation

It is very valuable to represent portions of volume in nature, as well as the possible changes of these portions. With it, the enlightening and momentous volume dynamic, VD, can be derived.

6.1.1 Additional volume

Theorem 4 Law of representation of additional volume

At each point in space in a D-dimensional space, and for each system of local orthogonal incremental coordinates $d\xi^j$, these coordinates can be used as follows for a representation of the diagonal elements of the metric tensor g_{jj} and of the additional volume that is caused by the curvature of spacetime:

(1) Scaled coordinates can be introduced, see Fig. (6.1):

$$dx_{L,j} := \sqrt{|g_{jj}|} \cdot d\xi^j \tag{6.1}$$

(2) The volume increment $dV_L = \prod_j^D \sqrt{|g_{jj}|} \cdot d\xi^j$ is a product

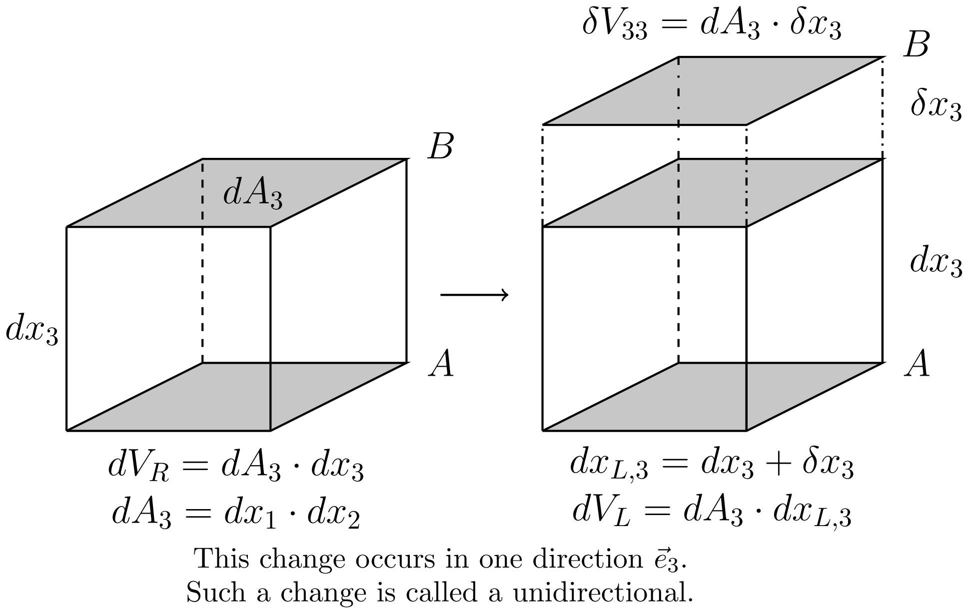

Figure 6.1: Additional volume in the x_3-direction: A cube with lower and upper surface dA_3 is enlarged, whereby the upper surface is a portion or increment δx_3 higher than at the left.

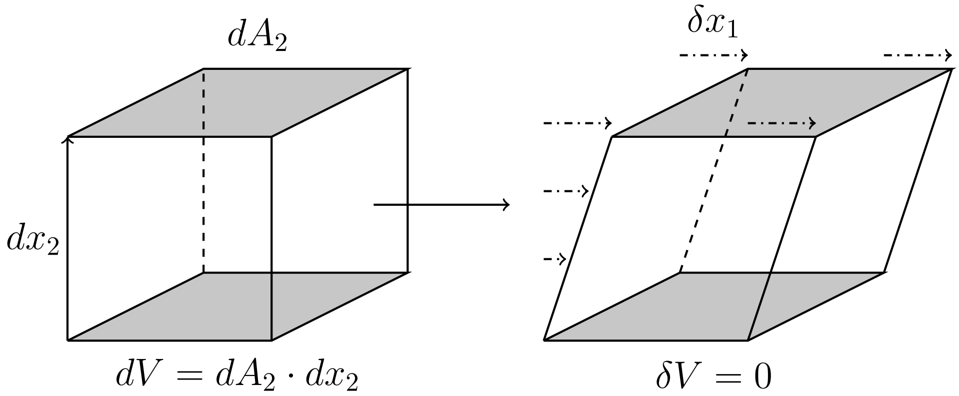

Figure 6.2: Shear: A cube with a cross section dA_2. At each height δx_2, the cross section is shifted according to an element $\varepsilon_{L,1,2} = (\varepsilon_{L,1,2}+\varepsilon_{L,2,1})/2$ of the change tensor by $\delta x_1 = \varepsilon_{L,1,2} \cdot \delta x_2$.

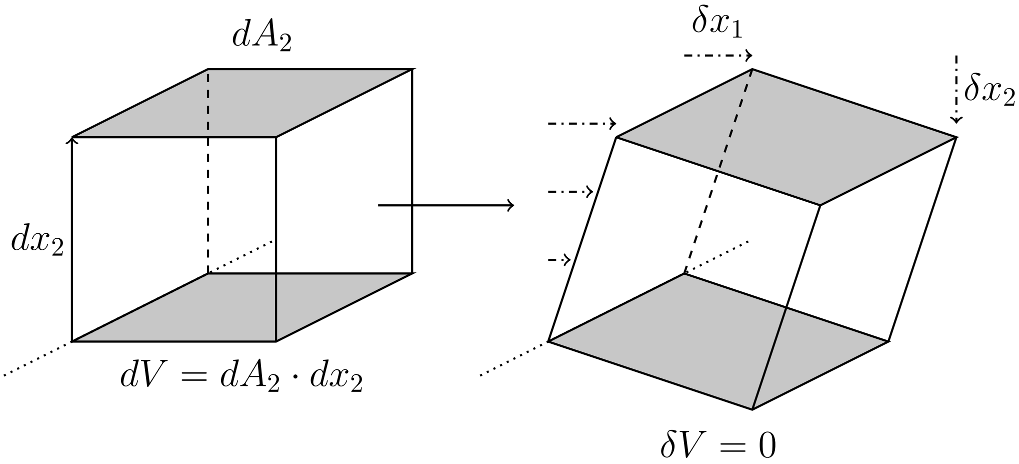

Figure 6.3: Rotation: A cube with a cross section dA_2 is rotated around the axis $\vec{e}_3$ (dotted) by the angle $d\varphi \approx -18.43^o$.

of coordinates:

$$dV_L = \prod_j^D dx_{L,j} \tag{6.2}$$

(3) In the vicinity of a mass M or effective mass M_{eff}, the following quantities can be measured:

(3a) The same volume in nature can be described simultaneously and based on measurements by a map describing a curved space and by a map describing a flat space. These measurable descriptions are possible, as the light-travel distance d_{LT} and the gravitational parallax distance d_{GP} can be measured simultaneously, see THM (2). For an illustration see Fig. (5.1).

As a consequence, the volume in nature can be described by corresponding distances and increments that occur in both spaces:

Two objects or landmarks A and B (like cities Atlanta and Bremen) are represented in a flat space map based on d_{GP} and in a curved space map based on d_{LT}, see Figs. (5.1, 6.1). The landmarks A and B are connected by an increment $d\xi_{j,flat}$ in the flat space map and by $d\xi_j$ in the curved space map.

If and only if, iff, the two increments $d\xi_{j,flat}$ and $d\xi_j$ have equal endpoints A and B, then $d\xi_{j,flat}$ and $d\xi_j$ correspond to each other:

$$\text{Iff } d\xi_j \& \, d\xi_{j,flat} \text{ have equal endpoints, } d\xi_j \text{ corresponds to } d\xi_{j,flat}. \tag{6.3}$$

Corresponding scaled coordinates can be introduced,

$$dx_{j,flat} := \sqrt{|g_{jj,flat}|} \cdot d\xi_{j,flat} = dx_j \tag{6.4}$$

with the corresponding metric tensor

$$g_{ij,flat} = d\vec{\xi}_{i,flat} \cdot d\vec{\xi}_{j,flat} \tag{6.5}$$

and the corresponding volume increment

$$dV_R = \prod_j^D dx_j \tag{6.6}$$

(3b) The additional increment that is caused by unidirectional change in a direction $d\xi^j$ can be measured, see Fig. (6.1):

$$\delta\xi_j := dx_{L,j} - dx_j \tag{6.7}$$

Similarly, the additional volume that is caused by the curvature of spacetime can be measured:

$$\delta V := dV_L - dV_R \tag{6.8}$$

Moreover, the relative additional volume that is caused by the curvature of spacetime can be measured:

$$\varepsilon_L := \frac{\delta V}{dV_L} = 1 - \frac{dV_R}{dV_L} \tag{6.9}$$

(3c)Additional volume that is caused by unidirectional change in a direction $d\xi^j$ can be measured:

$$\delta V_{jj} := dV_L - dV_R \tag{6.10}$$

Moreover, the relative additional volume that is caused by uni-directional change in a direction $d\xi^j$ can be measured:

$$\varepsilon_{L,jj} := \frac{\delta V_{jj}}{dV_L} = \frac{\delta x_j}{dx_{L,j}} \tag{6.11}$$

(3d) In linear order (marked by a dot at the equality sign) in additional increments $\frac{\delta x_j}{dx_{L,j}}$, the relative volumes are related as follows:

$$\varepsilon_L \doteq \sum_j^D \varepsilon_{L,jj} \tag{6.12}$$

Proof:

Part (1): At each point, the following constructions and measurements are possible: A local orthogonal coordinate system can be constructed, see (Lay et al., 2016, section 6.4). The local coordinates $d\xi^j$ can be measured with help of the light-travel distance, see Hobson et al. (2006), Carmesin (2023g). With it, the basis vectors $\vec{d\xi}_i$ can be determined. Using these, the metric tensor can be evaluated, see (Hobson et al., 2006, sections 4.1-4.4):

$$g_{ij} = \vec{d\xi}_i \cdot \vec{d\xi}_j \tag{6.13}$$

Application of these results provides the scaled coordinates in Eq. (6.1). This completes the proof of part (1).

Part (2): At each point, a volume increment dV_L is spanned by the basis vectors $\vec{d\xi}_i$ of a possible local orthogonal coordinate system. Its volume dV_L is as follows, see (Hobson et al., 2006, section 2.10):

$$dV_L = \prod_j^D \sqrt{|g_{jj}|} \cdot d\xi_j \tag{6.14}$$

Scaled coordinates (Eq. 6.1) used in Eq. (6.2) provide:

$$dV_L = \prod_j^D dx_{L,j} \quad \text{q.e.d.} \tag{6.15}$$

Part (3a): At each point in the vicinity of M or M_{eff} the following can be constructed or measured: A local orthogonal system of coordinates $d\xi_j$ can be constructed, which includes the radial direction from M or M_{eff} to the point. In that system, each coordinate $d\xi_j$ can be measured with help of the light-travel distance, see Condon and Mathews (2018), Carmesin (2023g). Moreover, the gravitational parallax distance d_{GP} can be measured, see THM (2). With it, the radial coordinate $d\xi_L$ can be supplemented by the corresponding value of flat space. The other coordinates $d\xi_j$ are the same in curved and flat space. Altogether, for each coordinate $d\xi_j$ in curved space, the corresponding coordinate $d\xi_{flat,j}$ in flat space can be measured.

With it, the corresponding scaled coordinates can be introduced,

$$dx_{j,flat} := \sqrt{|g_{jj,flat}|} \cdot d\xi_{j,flat} \tag{6.16}$$

with the corresponding metric tensor

$$g_{ij,flat} = d\vec{\xi}_{i,flat} \cdot d\vec{\xi}_{j,flat} \tag{6.17}$$

and the corresponding volume increment

$$dV_R = \prod_j^D dx_{j,flat} \tag{6.18}$$

This completes the proof of part (3a).

Part (3b): The additional increment in Eq. (6.7), the additional volume in Eq. (6.8) and the relative additional volume in Eq. (6.9) represent definitions of measurable quantities. We apply the additional volume in Eq. (6.8) to the relative additional volume in Eq. (6.9):

$$\varepsilon_L = \frac{\delta V}{dV_L} = \frac{dV_L - dV_R}{dV_L} = 1 - \frac{dV_R}{dV_L} \tag{6.19}$$

This completes the proof of part (3b).

Part (3c): The additional volume that is caused by unidirectional change in Eq. (6.10) and the respective relative additional volume in Eq. (6.11) represent definitions of measurable quantities, see THM (2). This completes the proof of part (3c).

Part (3d): We apply the product representations of the volume increments (Eqs. 6.1 and 6.2) to the relative additional volume in Eq. (6.9):

$$\varepsilon_L = 1 - \frac{dV_R}{dV_L} = 1 - \prod_j^D \frac{dx_j}{dx_{L,j}} \tag{6.20}$$

We represent the numerator with help of the additional increment in Eq. (6.7), $dx_j = dx_{L,j} - \delta x_j$:

$$\varepsilon_L = 1 - \prod_j^D \frac{dx_{L,j} - \delta x_j}{dx_{L,j}} = 1 - \prod_j^D \left(1 - \frac{\delta x_j}{dx_{L,j}}\right) \tag{6.21}$$

In the above Eq., the fraction $\frac{\delta x_j}{dx_{L,j}}$ is expanded by the surface A_j of the incremental volume dV_L orthogonal to the direction x_j, see Fig. (6.1):

$$\varepsilon_L = 1 - \prod_j^D \left(1 - \frac{\delta x_j \cdot A_j}{dx_{L,j} \cdot A_j}\right) \tag{6.22}$$

In the above Eq., the product $\delta x_j \cdot A_j$ is identified with the additional volume δV_{jj} caused by unidirectional change in direction $\vec{e}_j$. Similarly, the product $dx_{L,j} \cdot A_j$ is identified with the incremental volume dV_L. With it, we derive:

$$\varepsilon_L = 1 - \prod_j^D \left(1 - \frac{\delta V_{jj}}{dV_L}\right) \tag{6.23}$$

In the above Eq., we use terms up to the linear order in the relative differences $\delta V_{jj}/dV_L$. This is also the first order in the fractions $\frac{\delta x_j}{dx_{L,j}}$, as the A_j can be canceled out. This approximation becomes exact in the limit δV_{jj} to zero. This limit is realized in the limit of large distance of the observer to M or M_{eff}. Correspondingly, the approximation is called far distance approximation FDA. The first order is marked by a dot at the equality sign. Thus, we derive:

$$\varepsilon_L \doteq 1 - \left(1 - \sum_j^D \frac{\delta V_{jj}}{dV_L}\right) = \sum_j^D \frac{\delta V_{jj}}{dV_L} \tag{6.24}$$

In the above Eq., the fraction is identified with the relative additional volume caused by unidirectional change in direction $\vec{e}_j$, see part (3c):

$$\varepsilon_L \doteq \sum_j^D \varepsilon_{L,jj} \tag{6.25}$$

This completes the proof of part (3d).

6.1.2 Change tensors

In the case of additional volume (section 6.1.1), we analyzed a change of a reference cube with an initial volume dV_R and a final volume dV_L. In particular, a change δx_j in a direction $\vec{e}_j$ is proportional to the edge $dx_{L,j}$ of the box (Eq. 6.11), whereby the proportionality factor is $\varepsilon_{L,jj}$.

$$\varepsilon_{L,jj} = \frac{\delta x_j}{dx_{L,j}} \tag{6.26}$$

More generally, we analyze non-diagonal elements in addition:

$$\varepsilon_{L,ij} = \frac{\delta x_i}{dx_{L,j}} \tag{6.27}$$

Similar tensors are used in the theory of deformations in elastic media, see (Landau and Lifschitz, 1975, paragraph 1) or Sommerfeld (1978), and in the theory of fluids, see (Landau and

Lifschitz, 1987, paragraph 14). Note that there are differences. For instance, the denominator $dx_{L,j}$ used here represents the edge including the change, as the change is present in reality - and as this provides an exact unification.

We solve the above Eq. for the change, and we express the edge by its endpoints:

$$\delta x_i = \varepsilon_{L,ij} \cdot dx_{L,j} = \varepsilon_{L,ij} \cdot (x_{L,j} - x_{L,j,0}) \tag{6.28}$$

We apply the derivative with respect to $x_{L,j}$:

$$\frac{\partial \delta x_i}{\partial x_{L,j}} = \varepsilon_{L,ij} \tag{6.29}$$

With it, we define essential change tensors:

Definition 5 Change tensors

In D-dimensional space, a change $\delta x_i = dx_{L,i} - dx_i$ of a reference cube with initial edges dx_j and changed edges $dx_{L,j}$ can be described with the following change tensors, see Fig. (6.1):

(1) A single change δx_i that varies proportional to $dx_{L,j}$ is described by the following tensor element:

$$\frac{\partial \delta x_i}{\partial x_{L,j}} = \varepsilon_{L,ij} = \frac{\partial}{\partial x_{L,j}} (dx_{L,i} - dx_i) \tag{6.30}$$

(2) Shear in the directions $\vec{e}_i$ and $\vec{e}_j$ of the cube is described by the following symmetric non-diagonal change tensor, see Fig. (6.2):

$$\varepsilon_{L,ij} = \frac{1}{2} \cdot \left(\frac{\partial \delta x_i}{\partial x_{L,j}} + \frac{\partial \delta x_j}{\partial x_{L,i}} \right) \tag{6.31}$$

(3) A rotation in a plane spanned by directions $\vec{e}_i$ and $\vec{e}_j$ is described by the following antisymmetric non-diagonal change tensor, see Fig. (6.3):

$$\varepsilon_{L,ij} = \frac{1}{2} \cdot \left(\frac{\partial \delta x_i}{\partial x_{L,j}} - \frac{\partial \delta x_j}{\partial x_{L,i}} \right) \tag{6.32}$$

Proposition 2 Law of representation of and change tensors, including volume changes by additional volume

In the vicinity of each mass M or effective mass M_{eff} and in D - dimensional space, the following changes of volume portions can be **measured***: (1) additional volume, (2) shear, (3) rotation, (4) translation and (5) linear combinations thereof. These changes are as follows:*

(1) The diagonal tensor elements of change represent the change of a volumetric property:

$$\varepsilon_{L,jj} = \frac{\delta x_i}{dx_{L,j}} = \frac{\sqrt{|g_{jj}|} - 1}{\sqrt{|g_{jj}|}} = 1 - \frac{1}{\sqrt{|g_{jj}|}} \tag{6.33}$$

$\varepsilon_{L,jj}$ represents a unidirectional change in the direction j, see Fig. (6.1).

(2) Shear in the directions $\vec{e}_i$ and $\vec{e}_j$ according to

$$\varepsilon_{L,ij} = \frac{1}{2} \cdot \left(\frac{\partial \delta x_i}{\partial x_{L,j}} + \frac{\partial \delta x_j}{\partial x_{L,i}} \right) \quad \text{with} \quad i \neq j \tag{6.34}$$

can become effective. Such shear is also described by the metric tensor, and it is included in the theory of general relativity.

(3) As volume in nature consists of different volume elements (see the law of the existence of volume portions in nature), a rotation in a plane spanned by directions $\vec{e}_i$ and $\vec{e}_j$ according to

$$\varepsilon_{L,ij} = \frac{1}{2} \cdot \left(\frac{\partial \delta x_i}{\partial x_{L,j}} - \frac{\partial \delta x_j}{\partial x_{L,i}} \right) \quad \text{with} \quad i \neq j \tag{6.35}$$

can become effective. Such rotation is not described by the metric tensor, as the metric tensor is symmetric, by DEF, see e. g. Hobson et al. (2006). Correspondingly, such a rotation is not included in the theory of general relativity. This is a fundamental lack of generality of the present-day general relativity theory.

(4) A translation of a volume portion localized at a position $\vec{L}$ is represented as follows:

$$\delta\vec{L} = \frac{\partial\vec{L}}{\partial\tau} \cdot \delta\tau \tag{6.36}$$

The corresponding coordinates of translation are as follows:

$$\delta L_j = \frac{\partial L_j}{\partial\tau} \cdot \delta\tau \tag{6.37}$$

Corresponding changes of the relative additional volume are described by the law of propagation of relative additional volume.

(5) Linear combinations of the changes in (1) to (4) can be measured.

(5.1) In particular, the following isotropic change can be measured:

$$\varepsilon_{L,iso} = \sum_j^D \varepsilon_{L,jj} \text{ with } \varepsilon_{L,jj} = \varepsilon_{L,ii} \tag{6.38}$$

(5.2) Moreover, the following change in a gravitational wave can be measured:

$$\varepsilon_{L,ij} = \hat{\varepsilon} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{6.39}$$

Hereby, a gravitational wave propagates in a direction $\vec{e}_k$, and it exhibits changes in the directions orthogonal to $\vec{e}_k$, the transverse directions. These can be described by the above tensor of relative additional volume in the plane orthogonal to $\vec{e}_k$, and there is a second polarisation as follows, see e. g. Abbott (2016)) or Carmesin (2017b):

$$\varepsilon_{L,ij} = \hat{\varepsilon} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{6.40}$$

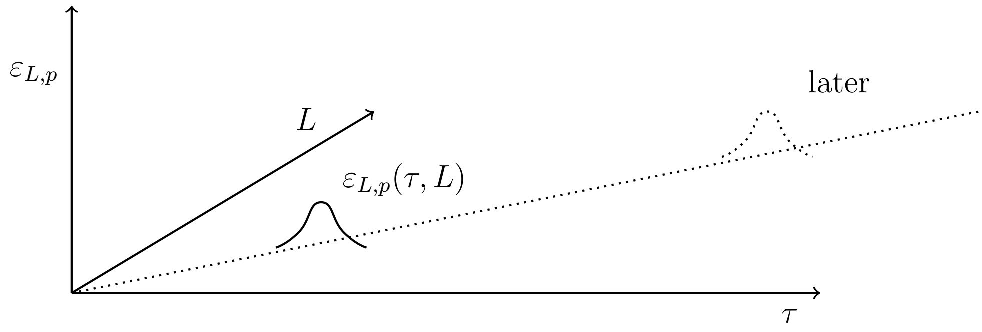

Figure 6.4: Sketch of a localizable portion of relative additional volume $\varepsilon_{L,p}$ that propagates in space. So $\varepsilon_{L,p}$ is a function of time τ and $\vec{L}$, symbolized in a reduced manner by L.

Proof: These change tensors can be measured, as the gravitational parallax distance d_{GP} or dx_j and the light - travel distance d_{LT} or $dx_{L,j}$ can be measured in the vicinity of a mass or effective mass M_{eff}, see THM (2). Each tensor element is a function of such increments, see Eq. (6.30). Of course, in many cases, there are even more effective, precise, robust, specific and sensitive procedures or instruments of measurement.

6.2 Propagation

The THM (3) shows that VPs exist and move: Next, we analyze that process: We consider generally formed and forming VPs that propagate within the volume that has already formed since the Big Bang.

6.2.1 DEQ of VD

Theorem 5 Law of propagation of relative additional volume

Localizable VPs propagate as follows.

(1) During an increment of time $d\tau$, the local maximum of the relative additional volume $\varepsilon_{L,p}$ changes its position by a spatial

increment $d\vec{L}$ as follows, see Fig. (6.4):

$$d\vec{L} := \frac{\partial \vec{L}}{\partial \tau} \cdot d\tau \text{ with } \frac{\partial \vec{L}}{\partial \tau} = c \cdot \vec{e}_v \tag{6.41}$$

(2) If the relative additional volume or change, described by a change tensor $\varepsilon_{L,p}$ of rank[1] *two or larger, is analyzed as a function of time τ and location $\vec{L}$, see Fig. (6.4), then it fulfills the following differential equation, DEQ:*

$$\boxed{\frac{\partial}{\partial \tau}\varepsilon_{L,p} = -v \cdot \vec{e}_v \cdot \frac{\partial}{\partial \vec{L}}\varepsilon_{L,p} \text{ with } v = c} \tag{6.42}$$

This DEQ is the **DEQ of VD**. *It holds at each location in empty space, according to the principle of translation invariance.*

(3) In principle, there is no difference between a portion of additional volume δV and a localizable VP. Consequently, Eqs. (6.41 and 6.42) hold for each localizable VP.

(4) Each localizable volume portion propagates according to the following Lorentz invariant DEQ:

$$\boxed{\dot{\varepsilon}_{L,p}^2 - c^2 \cdot \left(\frac{\partial}{\partial \vec{L}}\varepsilon_{L,p}\right)^2 = 0} \tag{6.43}$$

Hereby, the abbreviation $\dot{\varepsilon}_{L,p} := \frac{\partial}{\partial \tau}\varepsilon_{L,p}$ has been used. In the Lorentz invariance, the time component and the spatial components are determined by the respective derivatives, these are explicated as follows:

$$c^2 \left(\frac{\partial}{\partial \tau}\varepsilon_{L,p}\right)^2 - \left(\frac{\partial}{\partial L_1}\varepsilon_{L,p}\right)^2 - \left(\frac{\partial}{\partial L_2}\varepsilon_{L,p}\right)^2 - \left(\frac{\partial}{\partial L_3}\varepsilon_{L,p}\right)^2 = 0 \tag{6.44}$$

[1] In D-dimensional space, a tensor of rank n has D^n components, see e. g. (Moore, 2013, p. 83).

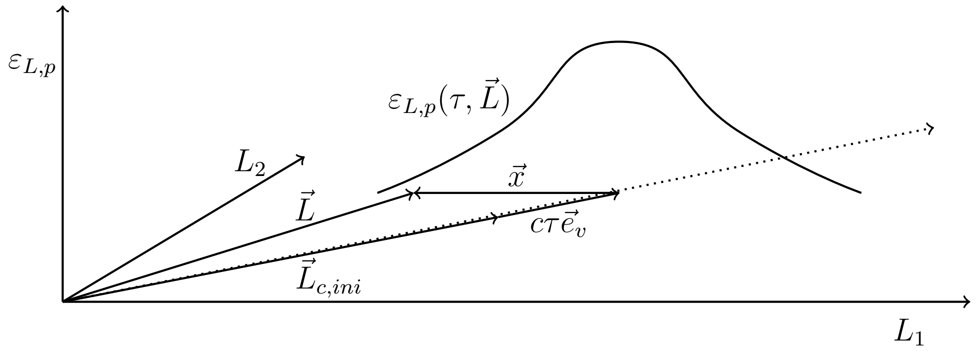

Figure 6.5: Sketch of a localizable portion of relative additional volume $\varepsilon_{L,p}$. The center is at $\vec{L}_c = \vec{L}_{c,ini} + c \cdot \tau \cdot \vec{e}_v$. A location is represented by $\vec{x}$ relative to the center and by $\vec{L} = \vec{L}_c + \vec{x}$ relative to the origin. In empty space, according to translation invariance, the DEQ of VD also holds for each such off-center $\vec{L}$.

Proof: Part (1): A localizable VP propagates at $v = c$. With it, the chain rule implies that the increment $d\vec{L}$ is described by Eq. (6.41).

Part (2): When a localizable portion of relative additional volume $\varepsilon_{L,p}$ propagates, $\varepsilon_{L,p}$ is a function of position $\vec{L}$ and time τ, whereby $\vec{L}$ is a function of τ according to Eq. (6.41):

$$\varepsilon_{L,p} = \varepsilon_{L,p}(\tau, \vec{L}) \tag{6.45}$$

As a consequence of Eq. (6.41), $\vec{L}$ is the following function of τ and of an initial value $\vec{L}(\tau_0)$:

$$\vec{L}(\tau) = \vec{L}(\tau_0) + \int_{\tau_0}^{\tau} \frac{\partial \vec{L}}{\partial \tau_1} d\tau_1 \tag{6.46}$$

Consequently, $\varepsilon_{L,p}$ is a function of τ as follows:

$$\varepsilon_{L,p} = \varepsilon_{L,p}(\tau, \vec{L}(\tau)) \tag{6.47}$$

At each time, each local maximum is determined by the value zero of its total time derivative:

$$d\varepsilon_{L,p}(\tau, \vec{L}(\tau)) = 0 \tag{6.48}$$

That derivative is evaluated with help of the chain rule:

$$\frac{\partial}{\partial \tau}\varepsilon_{L,p}(\tau, \vec{L})d\tau + \frac{\partial \vec{L}}{\partial \tau}\frac{\partial}{\partial \vec{L}}\varepsilon_{L,p}(\tau, \vec{L})d\tau = 0 \tag{6.49}$$

The above Eq. is divided by $d\tau$. Moreover, Eq. (6.41) is applied. Thus, the propagation is as follows:

$$\frac{\partial}{\partial \tau}\varepsilon_{L,p}(\tau, \vec{L}) + c \cdot \vec{e}_v \cdot \frac{\partial}{\partial \vec{L}}\varepsilon_{L,p}(\tau, \vec{L}) = 0 \tag{6.50}$$

In the above Eq., the right summand is subtracted. So Eq. (6.42) is derived.

Part (3) is self-evident.

Part (4): In Eq. (6.42), the term $\frac{\partial}{\partial \tau}\varepsilon_{L,p}(\tau, \vec{L})d\tau$ is subtracted. Then the square is applied to the resulting Eq.:

$$\left(\frac{\partial}{\partial \tau}\varepsilon_{L,p}(\tau, \vec{L})\right)^2 = c^2 \cdot \left(\frac{\partial}{\partial \vec{L}}\varepsilon_{L,p}(\tau, \vec{L})\right)^2 \tag{6.51}$$

In the above Eq., the right hand side is subtracted. So Eq. (6.43) is derived. This completes the proof of the theorem.

6.2.2 Stationary solution of the DEQ of VD

Stationary solutions are informative, as they show how localizable objects can form in a durable manner, see Fig. (6.6).

Theorem 6 Law of the stationary solution of the DEQ of the VD

(1.1) A Gaussian wave packet $\varepsilon_{L,jj}$, which extends in a one-dimensional subspace with a coordinate x_j, which has a characteristic wave number $k_{0,j}$, which is described as follows, see Figs. (6.5,6.6):

$$\varepsilon_{L,jj}(\tau, L_j) = t_n \cdot \exp\left(i \cdot k_{0,j} \cdot x_j - \frac{x_j^2}{4\sigma^2}\right), \tag{6.52}$$

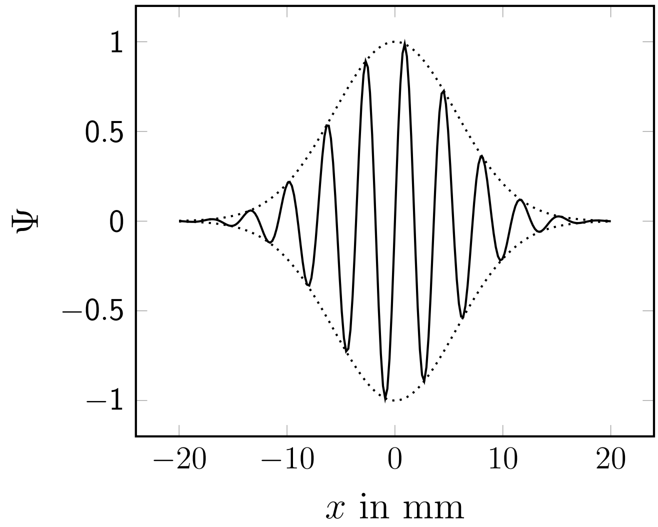

Figure 6.6: Wave packet of relative additional volume $\varepsilon_{L,p}$ or of unidirectional relative additional volume $\varepsilon_{L,p}$ or of a wave function Ψ at $t = 0$: $\Psi(t,x) = \sin(k \cdot x - \omega \cdot t) \cdot \exp(\frac{-x^2}{4 \cdot \sigma^2})$ (solid line). Envelope (dotted). $k = 100/\text{mm}$ and $\sigma = 4\text{mm}$.

$$\text{with} \;\; t_n = \frac{1}{(2\pi\sigma^2)^{1/4}}, \;\; \text{with} \tag{6.53}$$

a center $L_{c,j}$, off-center differences x_j, and coordinates L_j

$$x_j = L_j - L_{c,j}, \tag{6.54}$$

with a center propagating in the considered one - dimensional subspace (with $\omega = k_{0,j} \cdot c$ and $k_j = k$) via

$$L_{c,j} = L_{c,j,ini} + c \cdot \tau, \;\; \text{so that} \tag{6.55}$$

$$ik_{0,j}x_j = ik_{0,j}(L_j - L_{c,j,ini} - c\tau) = ik_{0,j}(L_j - L_{c,j,ini}) - i\omega \cdot \tau, \tag{6.56}$$

has the following properties:

(1.2) The Gaussian wave packet solves the DEQ of the VD (6.42).

(2) The Gaussian wave packet has the norm one:

$$\int_{-\infty}^{\infty} \varepsilon_{L,jj} \cdot \varepsilon^*_{L,jj} dx_j = 1 \tag{6.57}$$

(3) The Gaussian wave packet $\varepsilon_{L,jj}$ does not broaden as a function of time. As a consequence, the Gaussian wave packet is a stationary solution of the DEQ of VD.

(4) In the limit of infinite σ, the Gaussian wave packet $\varepsilon_{L,jj}$ forms a harmonic wave. This result is a mathematical idealization, as the observable causal structure of each wave packet is limited by the light horizon R_{lh}.

(5) **Generalization:** *More generally, for each polarization p, there are stationary solutions $\varepsilon_{L,p}$ of the DEQ of VD in THM (5) in the form of such Gaussian wave packets or of Gaussian wave packets in a D-dimensional space $\varepsilon_{L,p}(\tau, \vec{x})$.*

Proof: Part (1): The derivative with respect to time is as follows:

$$\frac{\partial}{\partial \tau}\varepsilon_{L,jj}(\tau, L_j) = \frac{\partial x_j}{\partial \tau} \cdot \frac{\partial}{\partial x_j}\varepsilon_{L,jj} = -c\vec{e}_v \cdot \frac{\partial}{\partial x_j}\varepsilon_{L,jj}, \qquad (6.58)$$

$$\text{with } \frac{\partial}{\partial x_j}\varepsilon_{L,jj} = \left(i \cdot k_{0,j} - \frac{x_j}{2\sigma^2}\right) \cdot \varepsilon_{L,jj} = \frac{\partial}{\partial L_j}\varepsilon_{L,jj} \qquad (6.59)$$

These derivatives imply the DEQ of VD in the one-dimensional subspace:

$$\frac{\partial}{\partial \tau}\varepsilon_{L,jj}(\tau, L_j) = -c\vec{e}_v \cdot \frac{\partial}{\partial L_j}\varepsilon_{L,jj} \qquad (6.60)$$

q. e. d.

Part (2): The norm is evaluated:

$$\int_{-\infty}^{\infty} \varepsilon_{L,jj} \cdot \varepsilon^*_{L,jj} dx_j = \int_{-\infty}^{\infty} t_n^2 \cdot \exp\left(-\frac{x_j^2}{2\sigma^2}\right) dx_j, \text{ thus,} \qquad (6.61)$$

$$\int_{-\infty}^{\infty} \varepsilon_{L,jj} \cdot \varepsilon^*_{L,jj} dx_j = t_n^2 \cdot (2\pi\sigma^2)^{1/2} = 1, \text{ q.e.d.} \qquad (6.62)$$

Part (3): As the solution is completely verified by inserting, it is stationary, in particular.

More generally, a Gaussian wave packet does not broaden, if the group velocity $v_g = \frac{\partial \omega}{\partial k}$ is a constant function. Hereby, $v_g = \frac{\partial \omega}{\partial k}$ represents the leading order term of a power series of a Fourier transform, see (Scheck, 2013, section 1.3).

In the present fully relativistic case, $\omega = c \cdot k$. Consequently, $v_g = \frac{\partial \omega}{\partial k} = c$.

In contrast, in a non-relativistic case with $E = \frac{p^2}{2m} = \hbar\omega = \frac{k^2 \cdot \hbar^2}{2m}$, the group velocity $v_g = \frac{\partial \omega}{\partial k} = \frac{k \cdot \hbar}{m} = \frac{p}{m} = v$ is a linear function of k. Thus, the Gaussian broadens in space as a function of time.

Part (4): In the limit σ to infinity, the wave function in Eqs. (6.52, 6.53, 6.54, 6.55, 6.56) becomes the following harmonic wave:

$$\varepsilon_{L,jj}(\tau, L_j) = t_n \cdot \exp\left(ik_{0,j}(L_j - L_{c,j,ini}) - i\omega\tau\right), \text{ q.e.d.} \tag{6.63}$$

Part (5): The proof is performed in essentially the same manner as above. This completes the proof.

6.2.3 VD exactly implies electromagnetic waves

Maxwell (1865) analyzed electromagnetic elongations of a supposed aether, whereby he derived the electromagnetic waves in the form of a wave equation and its solution.

However, the idea of the aether was too static, so that it has been withdrawn, see e. g. Michelson and Morley (1887), Einstein (1905b), and is falsified, see Will (2014).

In this section, we derive the same wave equation in Eq. (6.64) for antisymmetric change $\varepsilon_{L,ij}$ of volume portions. As a consequence, the solutions Eq. (6.64) are identical to the solutions of the wave equation in Maxwell's theory and in the theory of classical electrodynamics, see e. g. Jackson (1975), Landau and Lifschitz (1963). Consequently, the antisymmetric changes of VPs are identified with the electromagnetic waves.

This result is insightful, as it shows how electromagnetic waves are exactly included in the VD. This result is very valuable, as it shows how the electromagnetic waves are incremental changes of the volume portions, whereby the VPs are not static, such as the aether proposed by Maxwell (1865). Moreover, Carmesin (2021f) extended that result by showing how the VD implies the elementary charge, the electroweak couplings, classical electrodynamics and the theory of electroweak interactions, see Carmesin (2022e).

Theorem 7 Derived DEQ of electromagnetic waves

The Lorentz invariant DEQ of VD in Eq. (6.43) implies electromagnetic waves as follows:

(1) The Lorentz invariant DEQ of VD in Eq. (6.43) **implies the wave equation***:*

$$\Delta\varepsilon_{L,p} - \frac{1}{c^2} \cdot \left(\frac{\partial}{\partial\tau}\right)^2 \varepsilon_{L,p} = 0 \tag{6.64}$$

Hereby, Δ marks the Laplace operator, see for instance Jackson (1975)):

$$\Delta = \left(\frac{\partial}{\partial x}\right)^2 + \left(\frac{\partial}{\partial y}\right)^2 + \left(\frac{\partial}{\partial z}\right)^2 \tag{6.65}$$

(2) In general, the relative additional volume $\varepsilon_{L,p}$ can represent each possible polarization. Physically, this can be achieved by a phase transition, similarly as in the Higgs (1964) mechanism. In this case, $\varepsilon_{L,p}$ can take the form of a vector. The corresponding solution $\varepsilon_{L,p}$ of the DEQ (6.64) represents the vector potential $\vec{A}(\tau, \vec{L})$ that describes electromagnetic waves.

$$\Delta\vec{A} - \frac{1}{c^2} \cdot \left(\frac{\partial}{\partial\tau}\right)^2 \vec{A} = 0 \tag{6.66}$$

Hereby, the vector potential $\vec{A}(\tau, \vec{L})$ is an axial vector or pseudo *vector, which is an antisymmetric tensor as follows:*

$$\varepsilon_{L,ij} = \hat{\varepsilon} \cdot \begin{pmatrix} 0 & -A_3 & A_2 \\ A_3 & 0 & -A_1 \\ -A_2 & A_1 & 0 \end{pmatrix} \tag{6.67}$$

The electric potential ϕ is caused by the elementary charge, see e. g. Landau and Lifschitz (1971). It is derived on the basis of the VD in Carmesin (2021f, 2022e). Altogether, the Lorentz invariant DEQ of VD in Eq. (6.43) implies the description of electromagnetic waves in terms of the wave equation and the vector potential.

Proof: Part (1): The Lorentz invariant DEQ of VD in Eq. (6.43) is used, and the subtrahend is added:

$$\left(\frac{\partial}{\partial \tau}\varepsilon_{L,p}\right)^2 = c^2 \cdot \left(\frac{\partial}{\partial \vec{L}}\varepsilon_{L,p}\right)^2 \tag{6.68}$$

The direction vector $\vec{e}_v$ of the gradient is factorized. Additionally, $\vec{e}_v^{\,2} = 1$ is used:

$$\left(\vec{e}_v \cdot \frac{\partial}{\partial \tau}\varepsilon_{L,p}\right)^2 = c^2 \cdot \left(\vec{e}_v \cdot \left|\frac{\partial}{\partial \vec{L}}\varepsilon_{L,p}\right|\right)^2 \tag{6.69}$$

Application of the root implies the following DEQ:

$$\vec{e}_v \cdot \frac{\partial}{\partial \tau}\varepsilon_{L,p} = c \cdot \vec{e}_v \cdot \left|\frac{\partial}{\partial \vec{L}}\varepsilon_{L,p}\right| = c \cdot \frac{\partial}{\partial \vec{L}}\varepsilon_{L,p} \tag{6.70}$$

The derivative $\frac{\partial}{\partial \tau}$ is applied and substituted by $\frac{\partial \vec{L}}{\partial \tau} \cdot \frac{\partial}{\partial \vec{L}}$:

$$\vec{e}_v \cdot \left(\frac{\partial}{\partial \tau}\right)^2 \varepsilon_{L,p} = c \cdot \frac{\partial \vec{L}}{\partial \tau} \cdot \left(\frac{\partial}{\partial \vec{L}}\right)^2 \varepsilon_{L,p} \tag{6.71}$$

The equality of derivatives $\frac{\partial \vec{L}}{\partial \tau} = c \cdot \vec{e}_v$ in Eq. (6.41) is applied:

$$\vec{e}_v \cdot \left(\frac{\partial}{\partial \tau}\right)^2 \varepsilon_{L,p} = c^2 \cdot \vec{e}_v \cdot \left(\frac{\partial}{\partial \vec{L}}\right)^2 \varepsilon_{L,p} \tag{6.72}$$

The direction vector $\vec{e}_v$ is multiplied, and $\vec{e}_v^{\,2} = 1$ is used:

$$\left(\frac{\partial}{\partial\tau}\right)^2 \varepsilon_{L,p} = c^2 \cdot \left(\frac{\partial}{\partial\vec{L}}\right)^2 \varepsilon_{L,p} \tag{6.73}$$

The square of the gradients is identified with the Laplace operator Δ, see e. g. Jackson (1975), Landau and Lifschitz (1971):

$$\left(\frac{\partial}{\partial\vec{L}}\right)^2 = \Delta \tag{6.74}$$

With it, the above DEQ has the form:

$$\Delta\varepsilon_{L,p} - \frac{1}{c^2} \cdot \left(\frac{\partial}{\partial\tau}\right)^2 \varepsilon_{L,p} = 0 \tag{6.75}$$

q. e. d.

Part (2): Electromagnetic waves can be described by the vector potential $\vec{A}(\tau, \vec{L})$ and by the following DEQ, it is called the **wave equation**, (Landau and Lifschitz, 1971, Eq. 46.7):

$$\Delta\vec{A} - \frac{1}{c^2} \cdot \left(\frac{\partial}{\partial\tau}\right)^2 \vec{A} = 0 \tag{6.76}$$

In general, the relative additional volume $\varepsilon_{L,p}$ can exhibit each possible polarization. Physically, this can be achieved by a phase transition, similarly as in the Higgs (1964) mechanism, for instance. In this case, the tensor $\varepsilon_{L,p}$ in Eq. (6.75) can take the form of a vector. The corresponding solution $\varepsilon_{L,p}$ of the DEQ (6.64 or 6.75) represents the vector potential $\vec{A}(\tau, \vec{L})$ that describes electromagnetic waves. This result is additionally confirmed by (Landau and Lifschitz, 1971, Eqs. 46.10 and 46.11). q. e. d.

6.3 Periodicity φ_{per} of a rank 2 tensor

A rotation by an angle ϕ can be described by the following matrix:

$$D(\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \tag{6.77}$$

Proposition 3 Periodicity of a rank 2 tensor

(1) If a tensor $\varepsilon_{L,ij}$ of rank two is rotated by an angle φ, then the tensor $\varepsilon_{L,ij}(\varphi)$ is the following function of φ:

$$\varepsilon_{L,ij}(\varphi) = D(\varphi) \cdot (\varepsilon_{L,ij}) D^{-1}(\varphi) \tag{6.78}$$

Hereby, the bracket notation $(\varepsilon_{L,ij}(\varphi = 0))$ describes the whole tensor or vector (dx_j):

$$(\varepsilon_{L,ij}) = \begin{pmatrix} \varepsilon_{L,11} & \varepsilon_{L,12} \\ \varepsilon_{L,21} & \varepsilon_{L,22} \end{pmatrix} \text{ and } (dx_j) = \begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix} \tag{6.79}$$

(2) The tensor $\varepsilon_{L,ij}(\varphi)$ has the periodicity π:

$$\varepsilon_{L,ij}(\varphi_{per}) = \varepsilon_{L,ij} \text{ for } \varphi_{per} = \pi \tag{6.80}$$

Proof: Part (1): The tensor changes increments dx_j.

$$\delta x_i = \sum_j \varepsilon_{L,ij} \cdot dx_j \text{ or } (\delta x_i) = (\varepsilon_{L,ij}) \cdot (dx_j) \tag{6.81}$$

A rotation $D(\varphi)$ is applied:

$$D(\varphi) \cdot (\delta x_i) = D(\varphi) \cdot (\varepsilon_{L,ij}) \cdot (dx_j) \tag{6.82}$$

The identity $D^{-1}(\varphi) \cdot D(\varphi)$ is inserted:

$$D(\varphi) \cdot (\delta x_i) = [D(\varphi) \cdot (\varepsilon_{L,ij}) D^{-1}(\varphi)] \cdot \{D(\varphi) \cdot (dx_j)\} \tag{6.83}$$

The curly bracket is identified with the rotated increment vector (dx_j), and the rectangular bracket is identified with the rotated tensor $(\varepsilon_{L,ij}(\varphi))$ in Eq. (6.78).

Part(2): At $\varphi = \pi$, the rotation matrix in Eq. (6.84) is:

$$D(\pi) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \tag{6.84}$$

Consequently, the rotated tensor is $(\varepsilon_{L,ij}(\pi)) = (\varepsilon_{L,ij})$, q. e. d.

Chapter 7

VD implies gravity and curvature

7.1 Exact gravity

The DEQ of VD includes a useful, momentous and instructive interaction. Moreover, this interaction is described by a potential and by a field in an exact manner. In contrast, Newton's gravitational interaction is not exact, as it can be derived as an approximation (see section 19.4) of the gravitational interaction described by curved space or by the position factor ε_E in THMs (9 and 10). But the interaction inherent to the DEQ of VD is exact, in flat and curved space. That interaction is identified in this section. In the following section, the interaction is related to gravity and to curvature in space time.

Theorem 8 Law of the constant energy density of volume in an empty universe

(1) Einstein (1917) introduced the cosmological constant Λ *by the criterion, that* Λ *is the same for each cosmological redshift* z*. As a consequence, the corresponding energy density* $u_\Lambda = \frac{c^4 \cdot \Lambda}{8\pi}$*, (Hobson et al., 2006, p. 389), is the same for each cosmological redshift* z*.*

(2) In general, u_Λ *can be a function of space and time, see DEF (4). Part (1) implies that* u_Λ *is not a function of the cosmolog-*

ical redshift, which corresponds to a cosmological calendar date or time. Consequently, u_Λ is a function of space $\vec{L}$, in general:

$$u_\Lambda = u_\Lambda(\vec{L}) \tag{7.1}$$

Thus, $u_\Lambda(\vec{L})$ can be heterogeneous.

(3) In contrast, an empty universe is filled by volume only, without content. Such a universe exhibits translation invariance, see section (4.2.2, part (4)). As a consequence, $u_\Lambda(\vec{L})$ becomes homogeneous in an empty universe:

$$\frac{\partial}{\partial \vec{L}} u_{\Lambda,empty}(\vec{L}) = 0 \tag{7.2}$$

Since there is only volume in an empty universe, that energy density $u_{\Lambda,empty}(\vec{L})$ is identified with the energy density of volume u_{vol}:

$$u_{\Lambda,empty}(\vec{L}) = u_{vol} \tag{7.3}$$

(4) Altogether, the energy density of volume u_{vol} is the same for each cosmological redshift z, for each time t and for each location $\vec{L}$.

Proof: The proof is included in the THM.

Theorem 9 Law of the derived exact generalized gravitational interaction

In the vicinity of a mass M, see Fig. (5.1), the unidirectional radial relative additional volume $\varepsilon_{L,rr}$ exhibits the following properties:

(Hereby, we use spherical polar coordinates (r, ϑ, φ) with M at the origin, $d_{GP} = dR$ and $d_{LT} = dL$.)

(1) The relative additional volume $\varepsilon_{L,rr}$ propagates according to Eq. (6.42). That Eq. is multiplied by c:

$$c \cdot \frac{\partial}{\partial \tau} \varepsilon_{L,rr} = \vec{e}_v \cdot \frac{\partial}{\partial \vec{L}} \left(-c^2 \cdot \varepsilon_{L,rr} \right) \tag{7.4}$$

The bracket in the above DEQ has the form of a generalized potential Φ_{gen}:

$$\boxed{\Phi_{gen} = -c^2 \cdot \varepsilon_{L,rr}} \tag{7.5}$$

(Hereby, the potential is generalized, as it describes volume. Moreover, volume can generate matter in a phase transition, see e. g. Higgs (1964), Aad et al. (2012), Chatrchyan et al. (2012).

The negative gradient of that generalized potential is the generalized field $\vec{G}_{gen}$, see Fig. (7.1):

$$\vec{G}_{gen} = -\frac{\partial}{\partial \vec{L}} \left(-c^2 \cdot \varepsilon_{L,rr}\right) = -\frac{\partial}{\partial \vec{L}} \Phi_{gen} \tag{7.6}$$

The DEQ of VD (6.42 or 7.4) in THM (5) takes the form of the following rate gravity relation:

$$c \cdot \frac{\partial}{\partial \tau} \varepsilon_{L,rr} = \vec{e}_v \cdot \frac{\partial}{\partial \vec{L}} \Phi_{gen} = \vec{e}_v \vec{G}_{gen} \tag{7.7}$$

(2) That rate gravity relation can be expressed with help of the following rate gravity scalar RGS_{gen}:

$$RGS_{gen} := \left(c \cdot \frac{\partial}{\partial \tau} \varepsilon_{L,rr}\right)^2 - \vec{G}^2_{gen}, \quad \text{thus} \tag{7.8}$$

$$RGS_{gen} = \left(c \cdot \frac{\partial}{\partial \tau} \varepsilon_{L,rr}\right)^2 - \sum_j^D G^2_{gen,j} \quad \text{and} \tag{7.9}$$

$$RGS_{gen} = \left(c \cdot \frac{\partial}{\partial \tau} \varepsilon_{L,rr}\right)^2 - \left(c \frac{\partial}{\partial \vec{L}} \Phi_{gen}\right)^2 \quad \text{and} \tag{7.10}$$

$$RGS_{gen} = 0 \tag{7.11}$$

(3) The generalized field is proportional to $\frac{1}{R^{D-1}}$:

$$\boxed{|\vec{G}_{gen}| \propto \frac{1}{R^{D-1}} \quad \text{for } D \geq 3} \tag{7.12}$$

(4) **Generalization:** *The relations in parts (1) to (3) also hold for an effective mass* M_{eff}.

Proof: Part (1): These DEQs (7.4 and 7.7) are direct consequences of the DEQ of VD (6.42).

Part (2): The DEQ (7.7) is multiplied by $\vec{e}_L$ (or by $\vec{e}_v$), the resulting Eq. is squared and then the left hand side of the Eq. is subtracted. Consequently, the DEQ (7.9) is derived.

In that DEQ, the left hand side is identified with a scalar. It is valuable, as it is a coordinate independent quantity. That scalar is called rate gravity scalar RGS_{gen}. Hereby, $\left(c \cdot \frac{\partial}{\partial \tau} \varepsilon_{L,rr}\right)^2$ is the time like component with a positive sign, and $\sum_j^D G_{gen,j}^2$ is the space like component with a negative sign.

This sign convention has been used by Einstein (1915), and it is quite common, see also Landau and Lifschitz (1971) or Hobson et al. (2006), some authors use the opposite sign convention, see e. g. Moore (2013).

Part (3): (7.7) is multiplied by the unit vector $\vec{e}_v$:

$$\vec{e}_v c \cdot \frac{\partial}{\partial \tau} \varepsilon_{L,rr} = -\vec{G}_{gen} \tag{7.13}$$

The resulting Eq. is integrated via the separation of variables and from a time 0 to a time $\delta\tau$, during which a relative additional volume $\varepsilon_{L,rr}$ forms:

$$\vec{e}_v c \cdot \int_0^{\varepsilon_{L,rr}} d\varepsilon_{L,rr,1} = -\int_0^{\delta\tau} \vec{G}_{gen} d\tau \tag{7.14}$$

In the present incremental analysis, the time $\delta\tau$ can be chosen sufficiently small, so that the field or its averaged value is relatively constant during the interval of integration. With it, both integrals are evaluated:

$$\vec{e}_v c \cdot \varepsilon_{L,rr}(\delta\tau) = -\delta\tau \cdot \vec{G}_{gen} \tag{7.15}$$

In the above Eq., the DEF $\varepsilon_{L,rr} = \frac{\delta V_{L,rr}}{dV_L}$ is used:

$$\vec{e}_v c \cdot \frac{\delta V_{L,rr}}{dV_L} = -\delta\tau \cdot \vec{G}_{gen} \tag{7.16}$$

The volume portion $\delta V_{L,rr}$ propagates at $v = c$. Consequently, the VP has a nonzero energy δE and a nonzero energy density u_{vol}. Thence, $\delta V_{L,rr}$ is equal to the ratio of its energy δE and the energy density of volume u_{vol}. Note that the value of u_{vol} is not needed in this derivation, and note that the value of u_{vol} has been derived from the VD, see e. g. Carmesin (2021d, 2023g). Moreover, the energy δE is equal to the product of c and the corresponding momentum δp:

$$\delta V_{L,rr} = \frac{\delta E}{u_{vol}} = \frac{c \cdot \delta p}{u_{vol}} \tag{7.17}$$

The volume of a hyperball with radius 1, and in a dimension D is named V_D, and it can be determined as follows:

$$V_D = \frac{\pi^{D/2}}{\Gamma(1 + D/2)} \text{ with} \tag{7.18}$$

$$\Gamma(x + 1) = \Gamma(x) \cdot x \text{ with} \tag{7.19}$$

$$\Gamma(1) = 1 \text{ and } \Gamma(1/2) = \sqrt{\pi} \tag{7.20}$$

As a consequence, the surface area of a hyperball in a dimension D, named A_D, can be determined as follows:

$$A_D = V_D \cdot D \tag{7.21}$$

We analyze the additional volume that occurs in a shell with a radius R and a thickness dL. The volume dV_L of that shell is as follows:

$$dV_L = A_D \cdot R^{D-1} \cdot dL \tag{7.22}$$

As the volume propagates the thickness dL outwards, it requires the time $d\tau = dL/c$. With it, the volume of the shell is as follows:

$$dV_L = A_D \cdot R^{D-1} \cdot c \cdot d\tau \tag{7.23}$$

With it and Eq. (7.17), the field in Eq. (7.16) is as follows:

$$-\vec{e}_v \cdot \frac{c}{u_{vol} \cdot A_D \cdot d\tau} \cdot \frac{\delta p}{\delta \tau} \cdot \frac{1}{R^{D-1}} = \vec{G}_{gen} \tag{7.24}$$

In the above Eq., the first fraction is invariant, as $d\tau$ can be chosen invariant. A_D is a mathematical constant. Moreover, c and u_{vol} are physical invariants, see THM (8).

The second fraction is the momentum current of $\varepsilon_{L,rr}$. It is invariant at each radial coordinate R: The reason is that the volume is not a source of momentum. Thus, at each increment of time $\delta\tau$, the same momentum δp flows through each surface area $A_D R^{D-1}$ of a hyperball with its center at the considered mass M. Thus, the generalized field is proportional to $\frac{1}{R^{D-1}}$.

Part (4): In the case of an effective mass M_{eff}, in general, there is no rotational symmetry with respect to the center M_{eff}. However, an observer who measures the gravitational parallax distance d_{GP}, the generalized field $\vec{G}_{gen}$ and the effective mass M_{eff}, and who derives $\varepsilon_{L,rr}$ with the help of two measurements at distances R and $R + \Delta R$, obtains the same relations among these quantities as an observer in the vicinity of a mass M would obtain, see THM (2). As a consequence, based on these observations, such an observer in the vicinity of M_{eff} obtains the relations in parts (1) to (3). This completes the proof.

Interpretation: It is helpful and revealing, that the interaction inherent to the DEQ of VD exists in each dimension $D \geq 3$, and that its field is proportional to $\frac{1}{R^{D-1}}$.

7.2 Curvature and gravity

In the vicinity of a mass or effective mass, the DEQ of VD provides the curvature of space and time as well as the exact gravitational interaction. This shows how the VD provides essential results of gravity and GR.

For it, the ratio $\varepsilon_E(R)$ of the increments in the radial $j = r$ direction, $dx_{j=r} = dR$ in flat space and the corresponding increment $dx_{L,j=r} = dL$ in curved space is useful:

$$\varepsilon_E(R) := \frac{dx_{j=r}}{dx_{L,j=r}} = \frac{dR}{dL} = \frac{\sqrt{|g_{rr,flat}|}}{\sqrt{|g_{rr}|}} \tag{7.25}$$

Theorem 10 Law of the derived curvature, exact interaction and RGS

In the vicinity of a mass or dynamics mass M, the curvature and exact gravity are described by the following items:

(1) volume curvature relation,

(2) form of the field,

(3) form of the curvature,

(4) universal constant,

(5) Schwarzschild radius,

(6) curvature equation,

(7) exact field equation,

(8) generalizations.

(1) **Volume curvature relation:**

The position factor fulfills the following Eq.:

$$\boxed{\varepsilon_E(R) = 1 - \varepsilon_{L,rr}} \tag{7.26}$$

(2) **Form of the field:**

The generalized field is proportional to the mass M and to $\frac{1}{R^{D-1}}$, see Fig. (7.1):

$$|\vec{G}_{gen}| \propto \frac{M}{R^{D-1}} \tag{7.27}$$

Hereby, the proportionality factor is identified as a **positive universal constant of nature** G_{gen}*, as it is the same for* *each mass M and dimension $D \geq 3$. Accordingly, it must be obtained from observation:*

$$|\vec{G}_{gen}| = G_{gen} \cdot \frac{M}{R^{D-1}} \tag{7.28}$$

(3) **Form of the curvature:**

ε_E is a function of R, and $\varepsilon_E(R)$ fulfills the following DEQ:

$$\frac{G_{gen} \cdot M}{c^2 R^{D-1}} = \varepsilon_E \cdot \frac{\partial \varepsilon_E}{\partial R} \tag{7.29}$$

That DEQ and the relation $\lim_{R\to\infty} \varepsilon_E = 1$ imply:

$$\varepsilon_E = \sqrt{1 - \frac{2G_{gen} \cdot M}{c^2} \cdot \frac{1}{R^{D-2} \cdot (D-2)}} \tag{7.30}$$

(4) **Universal constant:**

Observation shows that the constant G_{gen} is the universal constant of gravitation, for $D \geq 3$. In three-dimensional space, it has the following value, see e. g. (Workman et al., 2022, table 2.1):

$$G_{gen}(D=3) = G, \quad \text{with} \tag{7.31}$$

$$G = 6.674\,30(15) \cdot 10^{-11}\, \frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2} \tag{7.32}$$

Hereby, G is the universal gravitational constant. It is also called Newton's constant of gravitation.

As the constant $G_{gen}(D)$ is related to the gravitational constant G, the constant $G_{gen}(D)$ is also named G_D. At each dimension $D \geq 3$, the gravitational constant is as follows, see e. g. (Carmesin, 2019c, section 2.6):

$$G_{gen}(D \geq 3) = G \cdot L_P^{D-3} \cdot (D-2) = G_D \tag{7.33}$$

Hereby, L_P is the Planck length:

$$L_P = \sqrt{\frac{\hbar \cdot G}{c^3}} = 1.616 \cdot 10^{-35}\,\mathrm{m} \tag{7.34}$$

(5) **Schwarzschild radius:**

At $D = 3$, the Schwarzschild radius $R_S = 2GM/c^2$ implies:

$$\varepsilon_E = \sqrt{1 - \frac{R_S}{R}} \text{ at } D = 3 \tag{7.35}$$

At a dimension $D \geq 3$, the Schwarzschild radius is as follows, see e. g. (Carmesin, 2019c, section 2.6):

$$R_{SD} = (R_S \cdot L_P^{D-3})^{1/(D-2)} \tag{7.36}$$

(6) **Curvature equation:**

Consequently, the position factor is as follows:

$$\boxed{\varepsilon_E = \sqrt{1 - \left(\frac{R_{SD}}{R}\right)^{D-2}} \text{ at } D \geq 3} \tag{7.37}$$

(7) **Exact field equation:**

As a consequence, in three-dimensional space, the generalized field is equal to the gravitational field $\vec{G}^*$, see Fig. (7.1) and DEF (2):

$$\vec{G}_{gen} = -\frac{G \cdot M}{R^2}\vec{e}_L = \vec{G}^*, \text{ at } D = 3 \tag{7.38}$$

Hereby, $\vec{e}_L = \vec{e}_v$. Consequently, at a dimension $D \geq 3$, the gravitational field is as follows:

$$\vec{G}_{gen} = -\frac{G_D \cdot M}{R^{D-1}}\vec{e}_v \text{ at } D \geq 3 \tag{7.39}$$

$$\text{Thus, } \vec{G}_{gen} = -\frac{G \cdot M}{R^2}\vec{e}_v \cdot (D-2) \cdot \left(\frac{L_P}{R}\right)^{D-3} \text{ at } D \geq 3 \tag{7.40}$$

(8) **Generalization:** *The relations in parts (1) to (7) also hold for an effective mass M_{eff}.*

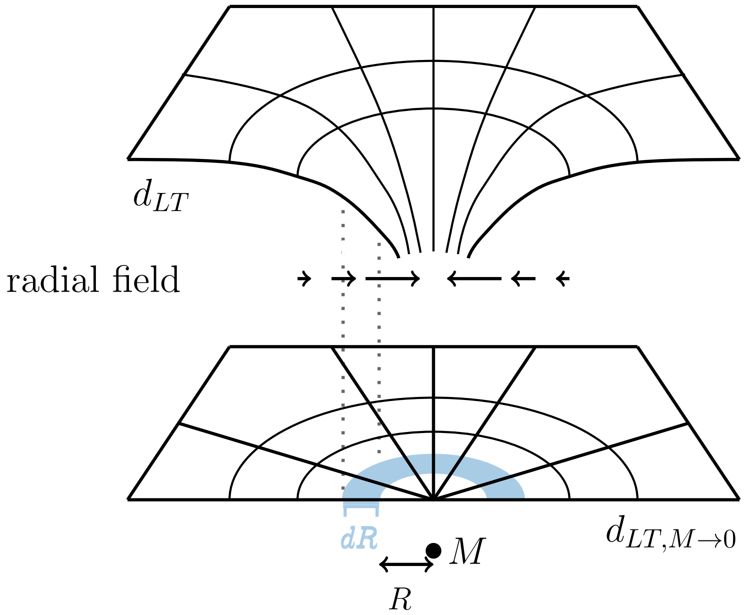

Figure 7.1: Vectors of the exact gravitational field $\vec{G}_{gen} = -\frac{G_{gen}\cdot M}{R^{D-1}}\cdot\vec{e}_L = -\partial_{\vec{L}}\Phi_{gen}(R)$, with $\Phi_{gen} = -c^2\varepsilon_{L,rr}$ and $\varepsilon_{L,rr}(R) = 1 - \varepsilon_E(R)$.

(9) **Curvature of spacetime:** *In the vicinity of M or M_{eff}, and in polar coordinates, the position factor provides the tensor elements of spacetime:*

$$g_{rr}(R) = \frac{1}{\varepsilon_E^2(R)} \quad \text{and} \quad g_{tt}(R) = \varepsilon_E^2(R), \quad \text{for } D \geq 3 \tag{7.41}$$

The remaining tensor elements are the same as in flat space.

Proof:

Part (1): Eq. (6.28) is used:

$$\delta x_i = \varepsilon_{L,ij} \cdot dx_{L,j} = dx_{L,i} - dx_i \tag{7.42}$$

Thus,

$$\varepsilon_{L,jj} = 1 - \frac{dx_j}{dx_{L,j}} \tag{7.43}$$

In the radial direction $i = r = j$, the above relation is as follows:

$$\varepsilon_{L,rr} = 1 - \frac{dR}{dL} \tag{7.44}$$

With it, the DEF of ε_E in Eq. (7.25) yields:

$$\varepsilon_{L,rr}(R) = 1 - \varepsilon_E(R) \tag{7.45}$$

With it, the proof of part (1) is completed.

Part (2): The source of the generalized field $\vec{G}_{gen}(R)$ is the field generating mass M. Thereby, the principle of linear superposition holds, as the field is derived in an exact manner from the solutions of the linear DEQ of VD. Consequently, the field is proportional to the mass M. This completes the proof of part (2).

Part (3): The gradient $\frac{\partial}{\partial \vec{L}}$ is applied to Eq. (7.26):

$$\frac{\partial}{\partial \vec{L}}\varepsilon_E = -\frac{\partial}{\partial \vec{L}}\varepsilon_{L,rr} \tag{7.46}$$

As a consequence of the rotational symmetry with respect to M, the gradient $\frac{\partial}{\partial \vec{L}}$ applied to $\varepsilon_{L,rr}$ is equal to the derivative $\frac{\partial}{\partial \vec{L}}$ with respect to the d_{LT} - based length L in the radial direction multiplied by the negative direction vector $-\vec{e}_L$, as $\varepsilon_{L,rr}$ decreases as a function of L. Consequently,

$$\frac{\partial}{\partial \vec{L}}\varepsilon_E = \vec{e}_L \frac{\partial}{\partial L}\varepsilon_E. \tag{7.47}$$

Moreover, $-\varepsilon_{L,rr}c^2$ is the potential Φ_{gen}:

$$\frac{\partial}{\partial \vec{L}}\varepsilon_E = \vec{e}_L \frac{\partial}{\partial L}\varepsilon_E = c^{-2}\frac{\partial}{\partial \vec{L}}\Phi_{gen} \tag{7.48}$$

The negative gradient of the potential is the field:

$$\vec{e}_L \frac{\partial}{\partial L}\varepsilon_E = -c^{-2}\vec{G}_{gen} \tag{7.49}$$

Hereby, the field is directed antiparallel to the radial direction, as $\varepsilon_{L,rr}$ decreases as a function of L, so that ϕ_{gen} increases as a function of L:

$$\vec{e}_L \frac{\partial}{\partial L}\varepsilon_E = \vec{e}_L c^{-2}|\vec{G}_{gen}| \tag{7.50}$$

The above Eq. is multiplied by the direction vector:

$$\frac{\partial}{\partial L}\varepsilon_E = c^{-2}|\vec{G}_{gen}| \tag{7.51}$$

The chain rule is used:

$$\frac{\partial R}{\partial L}\frac{\partial}{\partial R}\varepsilon_E = c^{-2}|\vec{G}_{gen}| \tag{7.52}$$

Eqs. (7.25, 7.28) are applied:

$$\varepsilon_E\frac{\partial}{\partial R}\varepsilon_E = G_{gen}\cdot\frac{M}{c^2R^{D-1}} \tag{7.53}$$

Altogether, the DEQ in part (3) is derived.

That DEQ is integrated by parts:

$$\int \varepsilon_E d\varepsilon_E = \int G_{gen}\cdot\frac{M}{c^2R^{D-1}}dR \tag{7.54}$$

The above integrals are evaluated with a constant K of integration:

$$\frac{1}{2}\varepsilon_E^2 = K - \frac{G_{gen}\cdot M}{c^2R^{D-2}\cdot(D-2)} \tag{7.55}$$

That Eq. is solved for the position factor:

$$\varepsilon_E = \sqrt{2K - \frac{2G_{gen}\cdot M}{c^2R^{D-2}\cdot(D-2)}\frac{1}{R^{D-2}}} \tag{7.56}$$

The physical far distance limit $\lim_{R\to\infty}\varepsilon_E = 1$ implies $K = 1/2$:

$$\varepsilon_E = \sqrt{1 - \frac{2G_{gen}\cdot M}{c^2R^{D-2}\cdot(D-2)}\cdot\frac{1}{R^{D-2}}} \tag{7.57}$$

Altogether, the solution in part (3) is derived. This completes the proof of part (3).

Parts (4) to (7) include their derivations.

Part (8): In the case of an effective mass M_{eff}, in general, there is no rotational symmetry with respect to the center M_{eff}.

However, in spherical polar coordinates (r, ϑ, φ) with M_{eff} at the center in each direction (ϑ, φ), the same relations in parts (1) to (7) hold as in the system with rotational invariance with respect to M, see THM (2).

Part (9): A derivation is provided in Carmesin (2023g). This completes the proof.

7.3 Density of gravitational energy

Theorem 11 Energy density of the gravitational field

(1) The gravitational energy is inherent to modifications of space such as curvature or additionally formed volume or a gravitational field.

(2) In three-dimensional space, a gravitational field $\vec{G}_{gen}$ or $\vec{G}^*$ has the energy density $u_{gr.f.}$ as follows:

$$u_{gr.f.} = \rho_{gr.\ f.} \cdot c^2 = -\frac{\vec{G}^{*2}}{8\pi \cdot G} = -\frac{\vec{G}^2_{gen}}{8\pi \cdot G} \tag{7.58}$$

(3) In D - dimensional space, with $D \geq 3$, a gravitational field $\vec{G}_{gen}$ or $\vec{G}^*$ has the energy density $u_{gr.f.}$ as follows:

$$\boxed{u_{gr.f.} = \rho_{gr.\ f.} \cdot c^2 = -\frac{\vec{G}^{*2}}{2A_D \cdot G_D} = -\frac{\vec{G}^2_{gen}}{2A_D \cdot G_D}} \tag{7.59}$$

(4) In general, a field $\vec{F}$ has the 'large-distance-limit-zero', iff the field that is caused by a source tends to zero, when the distance to the source tends to infinity. Examples are the gravitational field $\vec{G}^*$ and the electric field $\vec{E}^*$.

(5) The energy density $u_{\vec{F}}$ of $\vec{F}$ is determined on the basis of the principle of energy conservation, see (Jackson, 1975, section 6.8, Eqs. 6.110, 6.111, 6.112).

(6) In general, a field $\vec{F}$ that has the 'large-distance-limit-zero' has the following 'sign property':

(6.1) When the two sources have infinite distance, then the field $\vec{F}$ is zero. Consequently, the energy density $u_{\vec{F}}$ is zero.

(6.2) If the field $\vec{F}$ is caused by two mutually attracting sources, then the sign of the energy density $u_{\vec{F}}$ is negative for the following reason:

In a process of approach of the two sources, the sources provide an energy ΔE as an output. Consequently, the energy of interaction decreases by ΔE. As a consequence of parts (5) and (6.1), the energy density $u_{\vec{F}}$ is negative.

(6.3) If the field $\vec{F}$ is caused by two mutually repelling sources, then the sign of the energy density $u_{\vec{F}}$ is positive for the following reason:

In a process of approach of the two sources, the sources require an energy ΔE as an input. Consequently, the energy of interaction increases by ΔE. As a consequence of parts (5) and (6.1), the energy density $u_{\vec{F}}$ is positive.

(6.4) The sign of the energy density $u_{\vec{F}}$ of a field $\vec{F}$ between two mutually interacting sources depends on the pair of these sources. In contrast, that sign property is neither considered nor fulfilled in present-day physics, see e. g. (Jackson, 1975, section 6.8 about a system of charged particles, Eqs. 6.110, 6.111, 6.112), (Landau and Lifschitz, 1971, Eq. 31.5 about a 'closed system consisting of the electromagnetic field and particles in it'), (Griffiths, 2013, a positive u in Eq. 8.5 describes the 'total energy stored in electromagnetic fields per unit volume').

(6.5) Moreover, that sign property is considered, but not fulfilled in (Maxwell, 1865, in item (72), a positive E in Eq. (I) describes the 'total energy existing in the field'). This is essentially a consequence of a polarization of a dielectric by an electric field, see (Maxwell, 1865, item (66)). Though such a dielectric can be considered as a particular case, (Maxwell, 1865,

items (71), (74), (82)) generalizes that energy density to a universal context. For instance, item (74) states:

'The energy in electromagnetic phenomena is mechanical energy. The only question is, Where does it reside? In the old theories, it resides in the electrified bodies, conducting circuits, and magnets, in the form of an unknown quality called potential energy, or the power of producing certain effects at a distance. On our theory it resides in the electromagnetic fields, in the space surrounding the electrified and magnetic bodies, as well as in those bodies themselves, and is in two different forms, which may be described without hypothesis as magnetic polarization and electric polarization, or, according to a very probable hypothesis, as a motion and the strain of one and the same medium'.

(Maxwell, 1865, item (82)) realized that the energy density $u_{gr.f.}$ *of the gravitational field is negative.*

(6.6) In the VD, there is a similarity to item (6.5), and there are differences to item (6.5):

(6.6.1) In the VD and in item (6.5), the electromagnetic energy resides in the space between matter.

(6.6.2) Essential differences are as follows:

In the VD, the volume portions have the correct and derived energy density, see THM (38).

The VD naturally explains gravity, quanta and electrodynamics in a fully relativistic manner, see THM (7) and Carmesin (2021a, 2022e).

In the VD, an electric field is typically caused by a virtual vacuum polarization or by a polarization due to pair production, as the universe is electrically neutral at a global scale, so that the energy density of the electric field is negative, see item (6.2). For the representation, see THM (7).

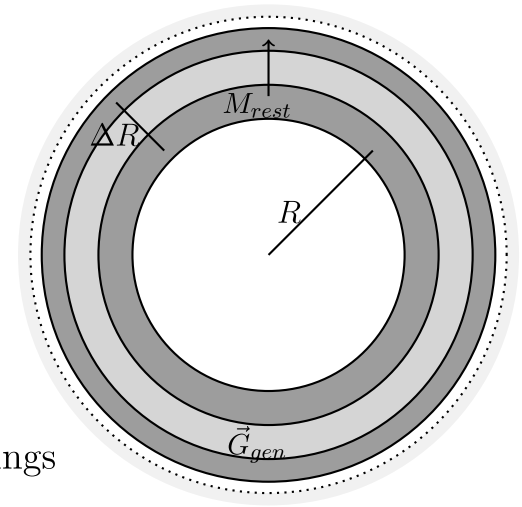

Figure 7.2: A mass M (dark gray) in a shell at a radius R is lifted to a radius $R + \Delta R$ as follows: Differential parts dM are lifted, while the rest M_{rest} is still at R. Thereby the field $\vec{G}_{gen}$ (medium gray) in the shell with radius R and thickness ΔR becomes zero, when the whole mass is at $R + \Delta R$ in Fig. (7.3).

In contrast, (Maxwell, 1865, item (66)) considers an electric field that causes polarization, so that existing charges move parallel to the electric force, so that the resulting energy density is positive. If Maxwell (1865) would have analyzed the field caused by opposite charges (as done in the VD), then he could have arrived at a negative energy density.

Proof:

Part (1): As the exact gravitational field as well as the curvature are derived from relative additional volume, the energy is inherent to the same modification of space that can be described by the exact gravitational field as well as the curvature as well as the relative additional volume.

Part (2): For the case of three-dimensional space, the proof is presented in (Carmesin, 2023g, section 6, THM 8) or also in (Carmesin, 2021d, section 5.6). Moreover, that case is included in part (3).

Part (3): The corresponding analysis is started for D - dimensional space with $D \geq 3$: We analyze the energy ΔE_M that is necessary in order to lift a mass M in a shell with a radius R to a shell with a radius $R + \Delta R$, see Fig. (7.2).

Thereby, the mass is lifted as follows: Differential parts dM are lifted, while the part M_{rest} is still at R. Moreover, the velocity of M remains approximately zero. So, a part dM is lifted at the following gravitational field of the part M_{rest}.

$$|\vec{G}_{gen,of\ M_{rest}}(R)| = \frac{G_D \cdot M_{rest}}{R^{D-1}} \tag{7.60}$$

So, the field $|\vec{G}_{gen,of\ M_{rest}}(R)|$ is proportional to the part M_{rest} (Fig. 7.3). When a mass dM is lifted, and when the mass M_{rest} is still at R, then dM experiences the following force: $|\vec{F}_G| = |\vec{G}_{gen,of\ M_{rest}}(R)| \cdot dM$. Thus, the following energy $dE = |\vec{F}_G| \cdot \Delta R$ is required:

$$dE = |\vec{G}_{gen,of\ M_{rest}}(R)| \cdot dM \cdot \Delta R = \frac{G_D \cdot M_{rest}}{R^{D-1}} \cdot dM_{lifted} \cdot \Delta R \tag{7.61}$$

We derive the full change of the gravitational energy ΔE_M by integrating the above Eq.:

$$\Delta E_M = \int_0^E dE' \tag{7.62}$$

We apply Eq. (7.61):

$$\Delta E_M = \int_0^M \frac{G_D \cdot M_{rest}}{R^{D-1}} dM_{lifted} \cdot \Delta R \tag{7.63}$$

We substitute the change dM_{lifted} of the lifted mass to the change dM_{rest} of the rest mass. As the rest mass M_{rest} is decreased by dM_{lifted}, we obtain $dM_{lifted} = -dM_{rest}$. At the lower bound of the integral, M_{rest} has the value M of the complete mass that is lifted. And at the upper bound of the integral,

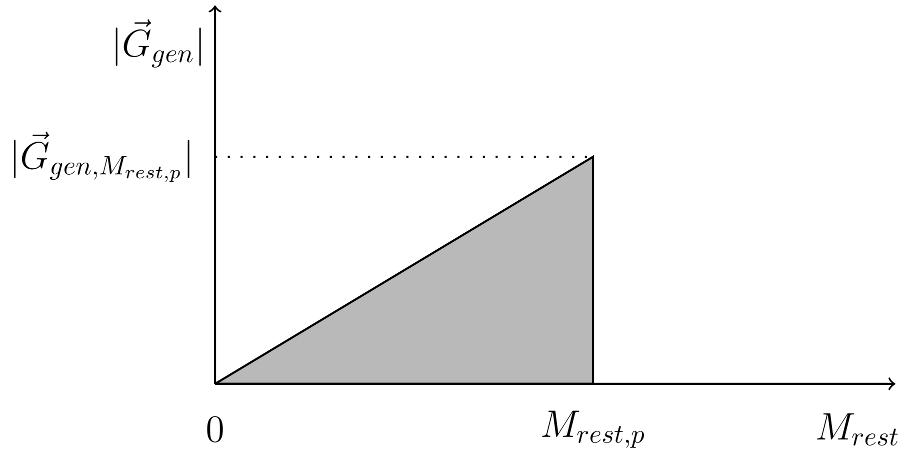

Figure 7.3: The field $|G_{gen}|$ is shown as a function of the mass M_{rest}, that is still at the shell with the radius R. A particular value $M_{rest,p}$ is marked.

M_{rest} has the value zero, as the whole mass has been lifted. Altogether, the substitution yields:

$$\Delta E_M = -\int_M^0 \frac{G_D \cdot M_{rest}}{R^{D-1}} dM_{rest} \cdot \Delta R \tag{7.64}$$

We evaluate the integral:

$$\Delta E_M = \frac{G_D \cdot M^2 \cdot \Delta R}{2R^{D-1}} \tag{7.65}$$

7.3.1 Free fall of M

In order to identify the energy density of the gravitational field, we analyze the inverse process:

Initially, the mass M is distributed isotropically at $R + \Delta R$. Then the mass M falls freely towards R. Thereby, the following holds:

Firstly, the potential energy E_{pot} is decreased by the value ΔE_M in Eq. (7.65):

$$\Delta E_{pot} = -\Delta E_M = -\frac{G_D \cdot M^2 \cdot \Delta R}{2R^{D-1}} \tag{7.66}$$

Secondly, the energy $E(M)$ of M does not change, according to the law of energy conservation.

Thirdly, the mass M does not change, as $M = E(M)/c^2$.

Fourthly, the internal energy $E_{internal}$ of M is equal to ΔE_M in Eq. (7.65), as the energy of M does not change:

$$\Delta E_{internal} = \Delta E_M = \frac{G_D \cdot M^2 \cdot \Delta R}{2R^{D-1}} \tag{7.67}$$

Fifthly, the potential energy E_{pot} is located outside M. At a radial coordinate larger than $R + \Delta R$, there is no change of the field, as M is not changed outside $R + \Delta R$. At a radial coordinate smaller than R, there is no change of the field, as there is no field. Thus, the change of the field occurred in the shell at radial coordinate between R and $R+\Delta R$. Moreover, in that shell, there emerged the gravitational field. Accordingly, we derive the energy density in that shell, and we identify that energy density with the energy density of the gravitational field $u_{gr.f.}$.

The above analysis holds for an observer, who interprets M as a sum of rest masses, e. g. of elementary particles[1].

Absolute value $|u_{gr.f.}|$ of the energy density $u_{gr.f.}$ of the field: The field $|G_{gen}|$ is in the shell with radius R and thickness ΔR (see Fig. 7.2). The corresponding volume is $\Delta V = A_D \cdot R^{D-1} \cdot \Delta R$. So, we derive the energy density by dividing the energy ΔE_M by the volume ΔV. So, we get:

$$|u_{gr.f.}| = \frac{\Delta E_M}{\Delta V} = \frac{G_D \cdot M^2 \cdot \Delta R}{2R^{D-1} \cdot A_D \cdot R^{D-1} \cdot \Delta R} \tag{7.68}$$

[1]If the observer assigns the change of energy via the position factor ε_E to the mass M, then the same energy can hardly be assigned a second time to the field.

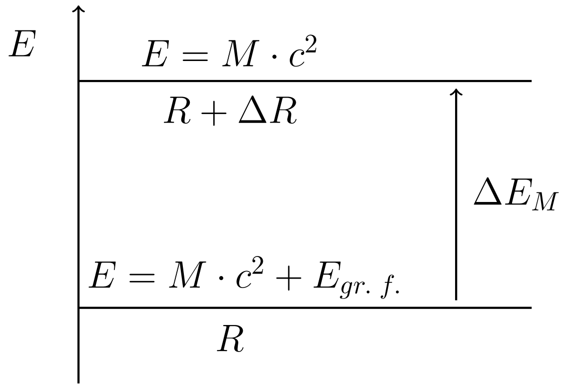

Figure 7.4: The energy of the mass is shown at the initial radius R and at the final radius $R + \Delta R$.

We simplify the above term, we expand by G, and we apply the field $|G_{gen}| = \frac{G_D \cdot M}{R^{D-1}}$. So, we derive:

$$\boxed{|u_{gr.f.}| = \frac{\vec{G}_{gen}^2}{2A_D \cdot G_D} = |\rho_{gr.\ f.}| \cdot c^2} \tag{7.69}$$

Hereby, $\rho_{gr.\ f.}$ is the mass density corresponding to the energy density of the gravitational field according to the equivalence of mass and energy.

7.3.2 Sign of $u_{gr.f.}$

As the potential energy has a negative sign, the energy density $u_{gr.\ f.}$ of the gravitational field has a negative sign as well. Note that the negative sign of the energy density $u_{gr.\ f.}$ is a direct consequence of the fact that gravity is attractive. Moreover, that negative sign does not cause any difficulty in the following, see also Carmesin (2021d). This completes the proof of parts (2) and (3).

Parts (4), (5) and (6): These parts include their derivation. This completes the proof.

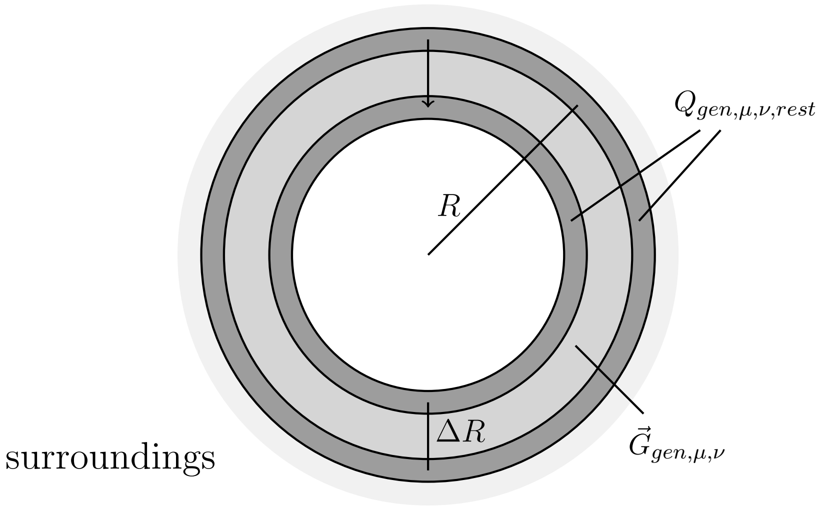

Figure 7.5: A generalized charge $Q_{gen,\mu,\nu}$ (dark gray) in a shell at a radius R is lowered (i. e. moved towards the center) to a radius $R-\Delta R$ as follows: Differential parts $dQ_{gen,\mu,\nu}$ are lowered, while the rest $Q_{gen,\mu,\nu,rest}$ is still at R. Thereby the generalized field $\vec{G}_{gen,\mu,\nu}$ (medium gray) in the shell with radius R and thickness ΔR increases from zero.

7.4 Generalized field

The above results about gravity and curvature are derived for the diagonal components of the change tensor. In this section, these results are generalized for the case of non - diagonal change tensors.

7.4.1 Generalized potential and field

In this section, generalized potentials, fields and charges are derived.

(1) VD provides a generalized potential: For each change tensor $\varepsilon_{\mu,\nu}$ in PROP (2), a generalized potential $\Phi_{gen,\mu,\nu}$ is identified as follows:

$$\Phi_{gen,\mu,\nu} = -c^2 \cdot \varepsilon_{\mu,\nu} \tag{7.70}$$

With it, the DEQ of VD in Eq. (6.42) implies the following DEQ for each localizable VP:

$$c \cdot \frac{\partial}{\partial \tau} \varepsilon_{L,\mu,\nu} = \vec{e}_v \cdot \frac{\partial}{\partial \vec{L}} \Phi_{gen,\mu,\nu} \tag{7.71}$$

(2) VD provides a generalized field: In general, the gradient of the potential in Eq. (7.71), multiplied by -1, provides the generalized field:

$$\vec{G}_{gen,\mu,\nu} = -\frac{\partial}{\partial \vec{L}} \Phi_{gen,\mu,\nu} \tag{7.72}$$

(3) VD provides a generalized charge: In a D - dimensional space, for each d_{GP} based distance R, the generalized field in Eq. (7.72) can be described with a generalized charge $Q_{gen,\mu,\nu}$ and with a generalized coupling constant $G_{D,\mu,\nu}$:

$$\vec{G}_{gen,\mu,\nu} = \frac{Q_{gen,\mu,\nu} \cdot G_{D,\mu,\nu}}{R^{D-1}} \tag{7.73}$$

This result is derived analogously to the proof of THM (9)[2].

(4) VD provides the energy density of the generalized field: Without loss of generality, the self interaction is analyzed with the setting in Fig. (7.5): A generalized charge is located homogeneously in a shell with radius R and thickness dR and moved towards the center as explained in Fig. (7.5). Without loss of generality, the portions of the charge repel each other, see THM (11 part 6). Consequently, the energy is increased by the following amount:

$$\Delta E = \frac{G_{D,\mu,\nu} \cdot Q^2_{gen,\mu,\nu} \cdot \Delta R}{2R^{D-1}} \tag{7.74}$$

[2]The charge and the coupling constant can be combined, as a convention, see for instance the electroweak theory, see Carmesin (2022e).

The above Eq. is transformed equivalently: The above fraction is expanded by the generalized coupling constant. Additionally, the volume of the shell $\Delta V = A_D \cdot R^{D-1} \Delta R$ is expanded.

$$\Delta E = \frac{G_{D,\mu,\nu}^2 \cdot Q_{gen,\mu,\nu}^2}{(R^{D-1})^2} \cdot \frac{\Delta R \cdot \Delta V}{2A_D \cdot G_{D,\mu,\nu} \cdot \Delta R} \tag{7.75}$$

In the above Eq., the first fraction is identified with the squared generalized field in Eq. (7.73):

$$\Delta E = \frac{\vec{G}_{gen,\mu,\nu}^2 \cdot \Delta V}{2A_D \cdot G_{D,\mu,\nu}} \tag{7.76}$$

The above Eq. is transformed equivalently: It is divided by $\Delta V \neq 0$. Moreover, the ratio $\frac{\Delta E}{\Delta V}$ is identified with the energy density of the field u_{field}:

$$\frac{\Delta E}{\Delta V} = u_{field,\mu,\nu} = \frac{\vec{G}_{gen,\mu,\nu}^2}{2A_D \cdot G_{D,\mu,\nu}} \tag{7.77}$$

Thereby, the energy density is positive, as the portions of equal charge repel each other, see THM (11 part 6).

(5) VD provides the density of the kinetic energy: In the DEQ of VD in Eq. (7.71), the field in Eq. (7.72) is inserted:

$$c \cdot \frac{\partial}{\partial \tau} \varepsilon_{L,\mu,\nu} = c\dot{\varepsilon}_{L,\mu,\nu} = -\vec{e}_v \cdot \vec{G}_{gen,\mu,\nu} \tag{7.78}$$

The above Eq. is transformed equivalently: The square is applied. Moreover, the resulting Eq. is multiplied by $\frac{1}{2A_D \cdot G_{D,\mu,\nu}}$:

$$\frac{c^2 \dot{\varepsilon}_{L,\mu,\nu}^2}{2A_D \cdot G_{D,\mu,\nu}} = \frac{\vec{G}_{gen,\mu,\nu}^2}{2A_D \cdot G_{D,\mu,\nu}} \tag{7.79}$$

In the above Eq., the right hand side is identified with the positive field energy density $-u_{field,\mu,\nu}$ in Eq. (7.77), and the left hand side is identified with the density of kinetic energy $u_{kin,\mu,\nu}$:

$$u_{kin,\mu,\nu} = \frac{c^2 \dot{\varepsilon}^2_{L,\mu,\nu}}{2A_D \cdot G_{D,\mu,\nu}} \tag{7.80}$$

$$u_{kin,\mu,\nu} = u_{field} \text{ for repelling charges} \tag{7.81}$$

(6) VD provides the density for attractive interaction: Globally, the universe is neutral with respect ot electric charges and with respect to charges of quantumchromodynamics, see e. g. Workman et al. (2022). As a consequence, most interactions are attractive. This case is treated next:

$$u_{kin,\mu,\nu} = -u_{field} \text{ for attracting charges} \tag{7.82}$$

The above Eq. is transformed equivalently: $u_{field,\mu,\nu}$ is added:

$$u_{ZPO,kin,\mu,\nu} + u_{field,\mu,\nu} = 0 \tag{7.83}$$

Moreover, the sum $u_{field,\mu,\nu} + u_{kin,\mu,\nu}$ is identified with the complete energy density:

$$u_{kin,\mu,\nu} + u_{field,\mu,\nu} = u_{\mu,\nu} = 0 \tag{7.84}$$

This relation of energy densities is derived for each localizable VP, in general.

Chapter 8

VD implies formation of volume in nature

8.1 Expansion of space

Idea: We derived exact local gravity and curvature in THMs (9, 10). In this chapter, we derive global expansion of space from that local gravity and curvature. Thus, we derive laws of macrocosm from laws of microcosm.

Similarly, the kinetic gas theory shows how the dynamics of local molecules provides the equation of state of the global ideal gas.

Accordingly, in this section, we unify the classical local dynamics of the position factor and the classical global expansion of space. For it, we derive the **Friedmann Lemaître equation, FLE**, from the position factor.

Einstein (1917) analyzed a possible expansion of the space. Slipher (1917) discovered the redshift of distant galaxies, Wirtz (1922) analyzed empirical evidence for the expansion of space, and Hubble (1929) obtained a convincing empirical basis for that expansion of space. That expansion is usually described by a **uniform scaling**.

Models of that expansion of space since the Big Bang typically apply the cosmological principle: isotropy and homogeneity of space, including its content, see e. g. Karttunen et al.

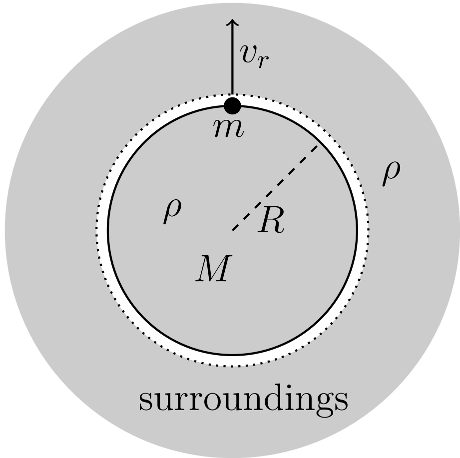

Figure 8.1: Ball with mass M and radius R embedded in a homogeneous surrounding and exhibited to a probe mass m with a radial velocity $v_r = \dot{R}$.

(1996). Accordingly, we model space by a homogeneous ball with a density ρ, see Fig. (8.1). So, we derive the DEQ for the time evolution of the radius of such a homogeneous ball.

8.2 Derivation of the FLE

In this section, we derive the Friedmann Lemaître equation, Friedmann (1922) and Lemaître (1927). The DEQ describes the expansion of space since the Big Bang. For it, we apply the position factor.

8.2.1 DEQ of uniform scaling: derivation

In this section, we analyze how the ball in Fig. (8.1) exhibits a uniform scaling as a function of time.

The surroundings of the ball in Fig. (8.1) do not generate a field $\vec{G}_{gen}$ inside the embedded sphere, as the surroundings are homogeneous. A homogeneous sphere with a mass M and radius R generates a field in its vicinity that is equal to the field generated by the mass M in the center of the ball, if there is an incremental empty probe slit at R as shown in Fig. (8.1).

Accordingly, the energy of a probe mass is described by the pair $(R|v_r)$ or $(R|\dot{R})$. Hereby, we consider the boundary condition $(\infty|0)$ at some time. This corresponds to a globally flat universe. In fact, it has been observed that our universe is globally flat, see Planck-Collaboration (2020). Moreover, it has been proven that the dynamics of our universe causes a globally flat universe, see Carmesin (2023h), Carmesin (2023g). Other conditions are analyzed in Carmesin (2020d).

As a consequence, at the probe mass at R, the curvature is described by the position factor, see THMs (9, 10), and the effect of the velocity v_r is described by the Lorentz factor:

$$E(R, v_r) = m_0 \cdot c^2 \cdot \gamma(v_r) \cdot \varepsilon_E(R) = E_0 \text{ alias } E_{ref} \quad (8.1)$$

Thereby, the factors are as follows:

$$\gamma(v_r) = \frac{1}{\sqrt{1 - v_r^2/c^2}} \;;\; \varepsilon_E(R) = \sqrt{1 - \frac{R_S}{R}} \text{ and } m_0 \cdot c^2 = E_0 \quad (8.2)$$

The Eq. (8.1) represents a DEQ, as it contains v_r, which in turn represents a derivative. This DEQ describes the dynamics of the probe mass. Next, we **transform** this DEQ, in order to obtain a transformed DEQ, still describing the dynamics of m and $R(t)$.

8.2.2 Structured energy function

In this section, we derive a **structured energy function**. This may be interpreted as a result of a mathematical transformation of the DEQ, or it may be interpreted physically in addition:

The structured energy function might be interpreted as a normalized **excess energy**, see Carmesin (2020d), as follows:

In SR, the difference of the square E^2 of the energy and of the square of the own energy $m_0^2 \cdot c^4 = E_0^2$ represents the square of the kinetic energy. By construction, it represents the square of the excess energy that the mass m has compared to its own mass or rest mass m_0.

According to the position factor, that excess energy contains the kinetic energy and, additionally, a gravitational energy in the field.

Correspondingly, we derive the excess energy as follows: We take the square of Eq. (8.1), and we subtract the squared own energy $m_0^2 c^4$ (so we obtain the square of the generalized excess energy):

$$E(R, v_r)^2 - m_0^2 c^4 = m_0^2 \cdot c^4 \cdot (\varepsilon_E(R)^2 \cdot \gamma(v_r)^2 - 1) \tag{8.3}$$

In order to transform the local physics of the probe mass to the global physics of the uniform scaling of space, we divide by γ^2 (Eq. 8.1):

$$\frac{E(R, v_r)^2 - m_0^2 c^4}{\gamma^2} = m_0^2 \cdot c^4 \cdot (\varepsilon_E(R)^2 - \gamma(v_r)^{-2}) \tag{8.4}$$

In order to simplify, we insert the factors $\varepsilon_E(R)$ and $\gamma(v_r)$:

$$\boxed{\frac{E(R, v_r)^2 - m_0^2 c^4}{\gamma^2(v_r)} = m_0^2 c^4 \cdot \left(\frac{v_r^2}{c^2} - \frac{R_S}{R}\right)} \tag{8.5}$$

Conventional form: In this paragraph, we derive a conventional energy function with a conventional kinetic and potential energy term. For it, we divide by $2m_0c^2$. So, we derive:

$$\frac{E(R, v_r)^2 - m_0^2 c^4}{2\gamma^2 m_0 c^2} = m_0 \cdot c^2 \cdot \left(\frac{v_r^2}{c^2} - \frac{R_S}{R}\right) \cdot \frac{1}{2} \tag{8.6}$$

We apply the Schwarzschild radius $R_S = \frac{2GM}{c^2}$: So, the result is a conventional structured energy function. We denote that energy function by a bar, $\bar{E}(R, v_r)$. Thus, we derive $\bar{E}(R, v_r)$:

$$\boxed{\frac{E(R, v_r)^2 - E_0^2}{2\gamma^2(v_r) E_0} =: \bar{E}(R, v_r) = \frac{m_0 \cdot v_r^2}{2} - \frac{G \cdot M \cdot m_0}{R}} \tag{8.7}$$

Form with the Hubble parameter: In this part, we transform the DEQ (8.5) further, so that we obtain a term for the **Hubble parameter**:

$$H = \frac{\dot{R}}{R} \text{ and } R \neq 0 \tag{8.8}$$

For it, we multiply Eq. (8.5) with $\frac{1}{m_0^2 \cdot c^4} \cdot \frac{c^2}{R^2}$, and we use the density $\rho = \frac{M}{R^3 \cdot 4\pi/3}$. So, we derive:

$$\frac{E(R,\dot{R})^2 - m_0^2 c^4}{m_0^2 \cdot c^4 \gamma^2(\dot{R})} \cdot \frac{c^2}{R^2} = \frac{\dot{R}^2}{R^2} - \frac{8\pi G \cdot \rho}{3} \tag{8.9}$$

We identify the scaled squared energy $-\frac{E(R,\dot{R})^2 - m_0^2 c^4}{m_0^2 \cdot c^4 \gamma^2(\dot{R})}$, or the scaled energy term $-\frac{2\bar{E}(R,\dot{R})}{m_0 \cdot c^2}$, with the **curvature parameter** k (Friedmann (1922), Lemaître (1927), Stephani (1980)), Carmesin (2021d), Carmesin (2021a)). We identify $\frac{\dot{R}^2}{R^2}$ with the squared Hubble parameter H^2, and we solve for H^2. So, we derive the **Friedmann Lemaître equation, FLE** (Friedmann (1922) and Lemaître (1927)), the DEQ for the homogeneous system:

$$\boxed{H^2 = \frac{8\pi G \cdot \rho}{3} - k \cdot \frac{c^2}{R^2}} \tag{8.10}$$

In the above Eq. the curvature parameter is $-\frac{2}{m_0 c^2}$ multiplied by the structured energy term:

$$k = -\frac{2}{m_0 c^2} \cdot \bar{E}(R,\dot{R}) = -\frac{2}{m_0 c^2} \cdot \frac{E(R,\dot{R})^2 - E_0^2}{2\gamma^2(\dot{R}) E_0} \tag{8.11}$$

Hereby, the energy $E(R,\dot{R})$ takes the value E_0 at R to infinity. Thus, for R to infinity, the structured energy function $\bar{E}(R,\dot{R})$ is zero. Hence, $\bar{E}(R,\dot{R})$ is zero during the whole time evolution of the system, as the law of energy conservation holds. Thence, the curvature parameter is zero, as a result of the dynamics, $k_{theo} = 0$.

That theoretical result $k_{theo} = 0$ is confirmed by observations (Collaboration (2020), Bennett et al. (2013)). As the curvature parameter k is zero, space is globally flat. This has been derived for most times or cosmological redshifts in general in Carmesin (2023g). We summarize our derivation:

Theorem 12 Local dynamics implies global dynamics

The FLE is derived from the position factor.

The expansion of the universe has the following properties, see (Carmesin, 2021d, THM 3).

(1) In classical GR, it is described by a **uniform scaling** *with a scale factor $R(t)$ Fig. (8.1).*

(2) In classical GR, the time evolution of the scale factor $R(t)$ is described by the FLE:

$$\boxed{H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G \cdot \rho}{3} - k \cdot \frac{c^2}{R^2}} \tag{8.12}$$

Hereby, the density in the universe is separated into three components, according to their dynamics with respect to the cosmological redshift z, see e. g. Hobson et al. (2006), Carmesin (2019b): The dynamical density of radiation ρ_r is proportional $(z+1)^4$. The density of matter ρ_m is proportional $(z+1)^3$. The dynamical density ρ_Λ corresponding to the cosmological constant Λ introduced by Einstein (1917) is proportional $(z+1)^0$.

$$\rho = \rho_r + \rho_m + \rho_\Lambda \tag{8.13}$$

(3) The FLE of that uniform scaling can be derived from the time evolution of a **microscopic probe mass** *m as follows:*

(3a) At a density ρ, there is a homogeneous ball of the universe with the same density and generating a field $\vec{G}^*$, and m is at the surface of that ball (Fig. 8.1).

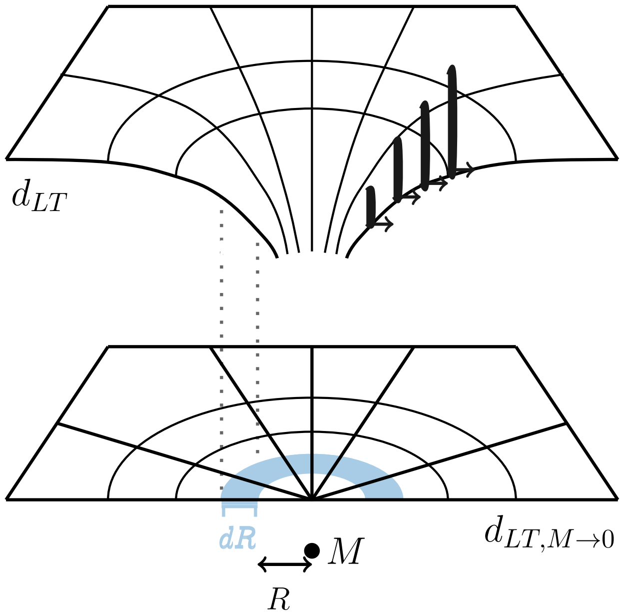

Figure 8.2: A portion of additional volume δV propagates outwards. Thereby, the portion increases. Thus, locally formed volume, LFV, occurs within that portion of additional volume.

(3b) The time evolution of the location of m is derived in the d_{GP} frame and from the position factor, see the DEQ (8.1), and the transformed DEQ (8.7) is derived from the position factor.

(4) Thereby, these above two DEQs use a **structured energy function** *$\bar{E}(R,\dot{R})$ with $\bar{E}(R,\dot{R}) = 0 = k =$ invariant:*

$$\boxed{-k := \frac{2\bar{E}(R,\dot{R})}{m_0 \cdot c^2} \quad \text{with} \quad \bar{E}(R,\dot{R}) = \frac{m_0 \dot{R}^2}{2} - \frac{GMm_0}{R}} \tag{8.14}$$

(5) That **structured energy function** *of m_0 is defined as follows:*

$$\boxed{\frac{E(R,\dot{R})^2 - E_0^2}{2\gamma^2(\dot{R})E_0} =: \bar{E}(R,\dot{R}) = E_0 \cdot \left(\frac{\dot{R}^2}{2c^2} - \frac{G \cdot M}{R \cdot c^2}\right)} \tag{8.15}$$

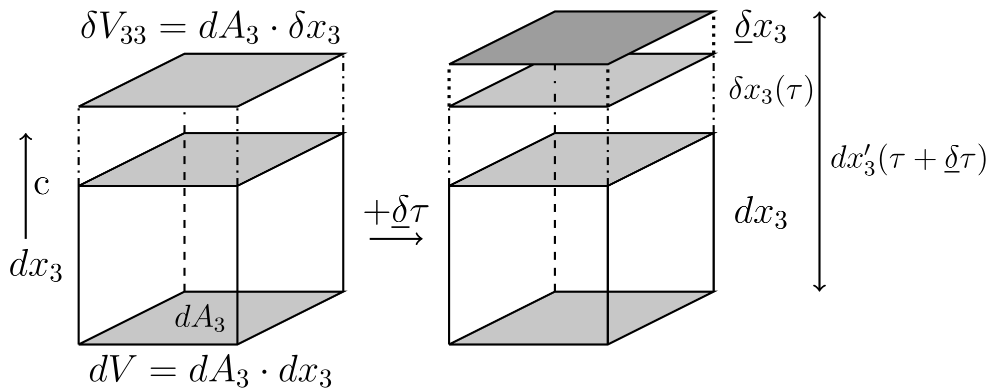

Figure 8.3: Unidirectional formation of volume: Additional volume propagates in x_3-direction at $v = c$. There is additional volume δV_{33}. Thereby, during a time $\underline{\delta}t$, a portion of volume $\underline{\delta}V_{33} = \underline{\delta}x_3 \cdot dA_3$ forms.

8.3 Local formation of volume, LFV

In the vicinity of a mass M, additional volume δV propagates outwards in a radial direction, see Fig. (8.2). Thereby, the amount of additional volume increases. Consequently, there occurs locally formed volume, LFV, $\underline{\delta}V$ or $\underline{\delta}V_{rr}$ in the radial direction r. In principle, that LFV is caused by the mass M.

More precisely, that LFV occurs at each location, see Fig. (8.3). At each location, there is a gradient of the relative additional volume $\varepsilon_{L,jj}$, or of the potential $\Phi_L = -c^2 \cdot \varepsilon_{L,jj}$, or there is a gravitational field $\vec{G}^*$. As a consequence, locally, the LFV is caused by a local gravitational field. In this section, we analyze the relation between the LFV $\underline{\delta}V_{jj}$ and its cause: the field $\vec{G}^*$ or G^*_j.

Definition 6 Locally formed volume, LFV

If additional volume $\underline{\delta}V_{jj}$ forms in a volume dV_L and in a direction $\vec{e}_j$ during a time $\underline{\delta}\tau$, then this process can be described by the following normalized rate of unidirectional LFV, see Figs.

(8.2, 8.3):

$$\dot{\underline{\varepsilon}}_{L,jj} := \frac{\underline{\delta} V_{jj}}{\underline{\delta}\tau \cdot dV_L} \tag{8.16}$$

In the vicinity of a mass M or an effective mass M_{eff}, and at a d_{GP} based distance R from M or M_{eff}, the following holds for the normalized rate:

Theorem 13 Law of locally formed volume, LFV

In the vicinity of a mass M or an effective mass M_{eff}, and at a d_{GP} based distance R from M or M_{eff}, the following holds for the normalized rate, see Fig. (8.3):

(1) In the far distance approximation, FDA, the ratio R_S/R is relatively small compared to 1. Thereby, at first order in that ratio R_S/R, the normalized rate is as follows:

$$\boxed{\dot{\underline{\varepsilon}}_{L,rr}^2 \cdot c^2 = G_{gen,r}^2} \tag{8.17}$$

Hereby, $G_{gen,r}$ is the component parallel to $\vec{e}_r$ of the generalized field. $G_{gen,r}$ is the cause of the rate of LFV $\dot{\underline{\varepsilon}}_{L,rr}$. More generally, each field $\vec{G}_{gen}$ in a direction $\vec{e}_j$ causes a unidirectional normalized rate $\dot{\underline{\varepsilon}}_{L,jj}$ in that direction:

$$\boxed{\dot{\underline{\varepsilon}}_{L,jj}^2 \cdot c^2 = G_{gen,j}^2 \text{ with } \vec{G}_{gen} = -G_{gen,j} \cdot \vec{e}_j} \tag{8.18}$$

(2) As a consequence, the non-diagonal components $\dot{\underline{\varepsilon}}_{L,ij,i\neq j}$ do not provide additional volume.

(3) The exact value of the rate $\dot{\underline{\varepsilon}}_{L,rr}$ is:

$$\dot{\underline{\varepsilon}}_{L,rr} = c \cdot \frac{2}{R} \cdot \varepsilon_E \cdot \left(1 - \varepsilon_E - \frac{1}{4\varepsilon_E^2 \cdot \frac{R}{R_S}}\right) \tag{8.19}$$

Proof: Part (1): At each radial coordinate R, a shell with center at M, and with thickness dL represents the analyzed volume dV_L, see Fig. (8.2). Thereby, the additional volume δV is in a shell with radius R, thickness δR, and volume $\delta V = 4\pi R^2 \delta R$. Similarly, the LFV $\underline{\delta} V$ is in a shell with radius R, thickness $\underline{\delta} R$, and volume $\underline{\delta} V = 4\pi R^2 \underline{\delta} R$.

The LFV is analyzed at leading order in the increment dR. This is exact, as the increment can be made as small as desired. If the additional volume $\delta V(R)$ increases as a function of R, then there occurs additional volume $\underline{\delta} V_{rr}$, whereby the direction r is equal to the radial direction.

We analyze the change of the additional volume:

$$\underline{\delta} V_{rr} := \frac{\partial}{\partial R} \delta V \underline{\delta} R \tag{8.20}$$

Thereby, δV is as follows:

$$\delta V = dV_L - dV_R = 4\pi R^2 \cdot (dL - dR) = 4\pi R^2 dR \cdot (\varepsilon_E^{-1} - 1) \tag{8.21}$$

In the far distance approximation, FDA, in leading order in $\frac{R_S}{R} << 1$, ε_E^{-1} is as follows:

$$\varepsilon_E^{-1} \doteq 1 + \frac{1}{2} \cdot \frac{R_S}{R} \tag{8.22}$$

With it, the derivative in Eq. (8.20) is as follows:

$$\frac{\partial}{\partial R} \delta V = \frac{\partial}{\partial R} \left(4\pi R^2 dR \cdot \frac{1}{2} \cdot \frac{R_S}{R} \right) = 2\pi \cdot dR \cdot R_S \tag{8.23}$$

With it, the change in Eq. (8.20) is as follows:

$$\underline{\delta} V_{rr} := 2\pi \cdot dR \cdot R_S \cdot \underline{\delta} R \tag{8.24}$$

With it, and with $\underline{\delta} \tau = \varepsilon_E \underline{\delta} R / c$, and with $dL = dR / \varepsilon_E$, the normalized rate of unidirectional LFV in Eq. (8.16) is as follows:

$$\underline{\dot{\varepsilon}}_{L,rr} \hat{=} \frac{\underline{\delta} V_{rr}}{\underline{\delta} \tau \cdot dV_L} = \frac{2\pi \cdot dR \cdot R_S \cdot \underline{\delta} R}{\underline{\delta} R / c \cdot 4\pi R^2 dR} = \frac{R_S \cdot c}{2R^2} \tag{8.25}$$

With it, and with $R_S = 2GM/c^2$, the normalized rate of unidirectional LFV in Eq. (8.25) is as follows:

$$c \cdot \dot{\underline{\varepsilon}}_{L,rr} \doteq \frac{G \cdot M}{R^2} \tag{8.26}$$

The above fraction is the absolute value of the field. Thereby, the field is antiparallel to the radial unit vector $\vec{e}_r$:

$$\vec{G}^* = \vec{G}_{gen} = -\frac{G \cdot M}{R^2} \cdot \vec{e}_r = -G_{gen,r} \cdot \vec{e}_r \tag{8.27}$$

Application of the square yields Eq. (8.18):

$$\dot{\underline{\varepsilon}}^2_{L,rr} \cdot c^2 = \vec{G}^2_{gen,r} \tag{8.28}$$

In the case of an effective mass, there is no rotational invariance with respect to M_{eff}, in general. In that case, the proof is performed for each angular direction separately, see for instance (Carmesin, 2023g, chapter 9). Moreover, each field $\vec{G}_{gen}$ parallel to a direction $\vec{e}_j$ causes a unidirectional rate $\dot{\underline{\varepsilon}}_{L,jj}$, according to Eq. (8.18).

Part (3): We analyze the rate in Eq.(8.16):

$$\dot{\underline{\varepsilon}}_{L,rr} = \frac{\underline{\delta} V_{rr}}{\underline{\delta} \tau} \frac{1}{dV_L} \tag{8.29}$$

The change $\underline{\delta} V_{rr}$ is the change of δV_{rr}

$$\dot{\underline{\varepsilon}}_{L,rr} = \frac{1}{dV_L} \frac{\delta V_{rr}(\tau_0 + \underline{\delta}\tau) - \delta V_{rr}(\tau_0)}{\underline{\delta}\tau} \tag{8.30}$$

As the time of propagation $\underline{\delta}\tau$ is incremental, and since a field in the radial direction causes changes in the radial direction only, we can express the difference at linear order in $\underline{\delta}\tau$:

$$\dot{\underline{\varepsilon}}_{L,rr} = \frac{1}{dV_L} \frac{\frac{\partial}{\partial \tau}\delta V_{rr} \cdot \underline{\delta}\tau}{\underline{\delta}\tau} = \frac{1}{dV_L}\frac{\partial}{\partial \tau}\delta V_{rr} = \frac{1}{dV_L} \underbrace{\frac{\partial L}{\partial \tau}}_{c} \frac{\partial}{\partial L}\delta V_{rr} \tag{8.31}$$

Next, L is substituted by R:

$$\dot{\underline{\varepsilon}}_{L,rr} = \frac{c}{dV_L} \underbrace{\frac{\partial R}{\partial L}}_{\varepsilon_E} \frac{\partial}{\partial R} \delta V_{rr} = \frac{\varepsilon_E c}{dV_L} \frac{\partial}{\partial R} \delta V_{rr} \tag{8.32}$$

Next, $\delta V_{rr} = 4\pi R^2 \delta R = 4\pi R^2 (dL - dR) = 4\pi R^2 dR(\varepsilon_E^{-1} - 1)$ is used, as well as $dV_L = 4\pi R^2 dR/\varepsilon_E$:

$$\dot{\underline{\varepsilon}}_{L,rr} = \frac{\varepsilon_E c}{4\pi R^2 dR/\varepsilon_E} 4\pi dR \frac{\partial}{\partial R}[R^2 \cdot (\varepsilon_E^{-1} - 1)] \tag{8.33}$$

The derivative is evaluated:

$$\dot{\underline{\varepsilon}}_{L,rr} = \frac{\varepsilon_E^2 c}{R^2}[2R \cdot (\varepsilon_E^{-1} - 1) - \varepsilon_E^{-3} R_S/2] \text{ or} \tag{8.34}$$

$$\dot{\underline{\varepsilon}}_{L,rr} = \frac{2\varepsilon_E c}{R}\left[1 - \varepsilon_E - \frac{R_S}{4\varepsilon_E^2 R}\right] \text{ q.e.d.} \tag{8.35}$$

8.4 From LFV to global formation of volume

8.4.1 Globally formed volume, GFV

Idea: In GR, the process of the expansion of space is described by a uniform scaling. It is a transformation of space. However, in reality, space is not transformed. Instead, the amount of volume increases. At what rate does the amount of volume increase?

In this section, we derive the corresponding rate, at which volume V has been increasing during the expansion of the universe since the Big Bang. For it, we express the volume V by the scale factor R:

$$V = \frac{4\pi}{3} R^3 \tag{8.36}$$

Using the chain rule, we obtain the derivative:

$$\dot{V} = 3\dot{R}\frac{V}{R} \tag{8.37}$$

So we derive:

$$\frac{\dot{V}}{V} = 3\frac{\dot{R}}{R} = 3 \cdot H \tag{8.38}$$

In order to use the FLE, we apply the square to Eq. (8.38):

$$\left(\frac{\dot{V}}{V}\right)^2 = 9H^2 \tag{8.39}$$

We insert the FLE, Eq. (8.12) with $k = 0$:

$$\left(\frac{\dot{V}}{V}\right)^2 = 24\pi G \cdot \rho \tag{8.40}$$

We summarize our finding:

Theorem 14 Rate of GFV according to the FLE

If the universe expands according to the FLE, Eq. (8.12), and if the curvature parameter is zero, $k = 0$, then the volume increases at the following normalized rate:

$$\boxed{\frac{\underline{\delta} V/\underline{\delta} t}{dV} := \frac{\dot{V}}{V} = \pm\sqrt{24\pi G \cdot \rho}} \tag{8.41}$$

Hereby, the plus-sign corresponds to the case of the expanding universe, whereas the minus sign describes the scenario of a big crunch, Goodstein (1997).

Thereby, the normalized rate $\frac{\underline{\delta} V/\underline{\delta} t}{dV}$ describes the formation of a volume $\underline{\delta} V$ in a volume dV during a time $\underline{\delta} t$.

8.4.2 LFV can cause GFV

Idea: Masses or dynamic masses m_i cause unidirectional formation of LFV in their vicinity. In contrast, isotropic formation of GFV is observed in the expansion of space. In this section, we show how the unidirectional LFV caused by many masses m_i can add up to isotropic GFV.

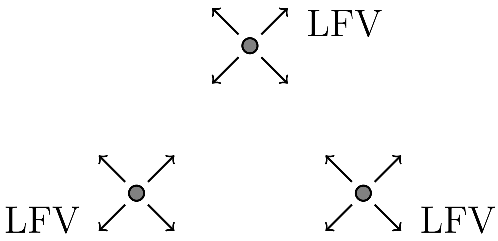

Figure 8.4: Unidirectional LFV at masses or dynamic masses can summarize to isotropic GFV.

Definition 7 Isotropic rate

The isotropic rate $\underline{\dot{\varepsilon}}_{L,iso}(\vec{r})$ is defined as the sum of the Cartesian components of the averaged rates:

$$\boxed{\underline{\dot{\varepsilon}}_{L,iso}(\vec{r}) = \sum_{j}^{3} \left\langle \underline{\dot{\varepsilon}}_{L,i,jj}(\vec{r}) \right\rangle_i = 3 \left\langle \underline{\dot{\varepsilon}}_{L,i,jj}(\vec{r}) \right\rangle_i} \qquad (8.42)$$

Theorem 15 LFV causes GFV

(1) In a natural three-dimensional volume, filled with masses or dynamic masses m_i in a homogeneous and isotropic manner, the m_i cause a gravitational field $\vec{G}_{gen}(\vec{r})$ and a rate of formation of volume at a location $\vec{r}$ as follows, see Fig. (8.4):

(2) In principle, at the location $\vec{r}$, each mass m_i causes a gravitational field $\vec{G}_{gen,i}(\vec{r})$.

(3) An average of quantities q_i with respect to the masses m_i is marked by $\langle q_i \rangle_i$. The average of the field vanishes, according to part (1):

$$\langle \vec{G}_{gen,i}(\vec{r}) \rangle_i = 0 \qquad (8.43)$$

(4) According to THM (11), the average of the squared field causes an energy density $u_{gr.f.}$ and dynamic density $\rho_{gr.f.}$ as follows:

$$\langle \vec{G}^2_{gen,i}(\vec{r}) \rangle_i = 8\pi G \cdot |u_{gr.f.}| = 8\pi G \cdot c^2 \cdot |\rho_{gr.f.}| \qquad (8.44)$$

(5) For each direction $\vec{e}_j$, the following mean field approximation is appropriate:

$$\left\langle \dot{\underline{\varepsilon}}^2_{L,i,jj}(\vec{r}) \right\rangle_i \approx \left\langle \dot{\underline{\varepsilon}}_{L,i,jj}(\vec{r}) \right\rangle_i^2 \tag{8.45}$$

(6) The isotropic rate of formation of volume exhibits the same expansion of space as the rate $\dot{V}/V$ in the FLE in THM (14):

$$\boxed{\dot{\underline{\varepsilon}}_{L,iso}(\vec{r}) = \sqrt{24\pi G \cdot |\rho_{gr.f.}(\vec{r})|} = \frac{\dot{V}}{V}} \tag{8.46}$$

Proof:

Parts (1), (2) and (3) include the derivation.

Part (4): At each location $\vec{r}$, according to THM (11), the average of the squared field has an energy density $u_{gr.f.}(\vec{r})$ and dynamic density $\rho_{gr.f.}(\vec{r})$ as follows:

$$\langle \vec{G}^2_{gen,i}(\vec{r}) \rangle_i = 8\pi G \cdot |u_{gr.f.}(\vec{r})| = 8\pi G \cdot c^2 \cdot |\rho_{gr.f.}(\vec{r})| \tag{8.47}$$

(4.2) According to the law of LFV, that squared field caused by m_i is equal to the unidirectional rate $\dot{\varepsilon}_{L,i,r(i)r(i)}$ multiplied by c^2, whereby the subscript $r(i)$ marks the direction $\vec{e}_{r(i)}$ from the mass m_i to the considered location $\vec{r}$, see Fig. (8.4):

$$\langle \vec{G}^2_{gen,i}(\vec{r}) \rangle_i = c^2 \langle \dot{\underline{\varepsilon}}^2_{L,i,r(i)r(i)}(\vec{r}) \rangle_i = 8\pi G \cdot c^2 \cdot |\rho_{gr.f.}(\vec{r})| \tag{8.48}$$

Note that the directions $\vec{e}_{r(i)}$ ar specific for each mass m_i, at the considered location $\vec{r}$, see Fig. (8.4). In general, these directions $\vec{e}_{r(i)}$ differ from the Cartesian directions $\vec{e}_j$ used by an observer at a location $\vec{r}$.

(4.3) The field $\vec{G}_{gen,i}(\vec{r})$ of a mass m_i is expressed by its Cartesian components $G_{gen,i,j}(\vec{r})$:

$$\left\langle \sum_j^3 G^2_{gen,i,j}(\vec{r}) \right\rangle_i = c^2 \langle \dot{\underline{\varepsilon}}^2_{L,i,r(i)r(i)}(\vec{r}) \rangle_i = 8\pi G \cdot c^2 \cdot |\rho_{gr.f.}(\vec{r})| \tag{8.49}$$

(4.4) According to part (1), the three averaged components are equal:

$$3\left\langle G^2_{gen,i,j}(\vec{r})\right\rangle_i = c^2\langle\underline{\dot{\varepsilon}}^2_{L,i,r(i)r(i)}(\vec{r})\rangle_i = 8\pi G\cdot c^2\cdot|\rho_{gr.f.}(\vec{r})| \quad (8.50)$$

(4.5) The law of LFV is applied to each component $G^2_{gen,i,j}(\vec{r})$. Consequently, $G^2_{gen,i,j}(\vec{r}) = c^2\underline{\dot{\varepsilon}}^2_{L,i,jj}(\vec{r})$. With it, the dynamic density in Eq. (8.50) is as follows:

$$3c^2\left\langle\underline{\dot{\varepsilon}}^2_{L,i,jj}(\vec{r})\right\rangle_i = 8\pi G\cdot c^2\cdot|\rho_{gr.f.}(\vec{r})| \quad (8.51)$$

Part (5): As the standard deviation $\left\langle\underline{\dot{\varepsilon}}^2_{L,i,jj}(\vec{r})\right\rangle_i - \left\langle\underline{\dot{\varepsilon}}_{L,i,jj}(\vec{r})\right\rangle_i^2$ is relatively small, $\left\langle\underline{\dot{\varepsilon}}^2_{L,i,jj}(\vec{r})\right\rangle_i$ is replaced by $\left\langle\underline{\dot{\varepsilon}}_{L,i,jj}(\vec{r})\right\rangle_i^2$. Moreover, the Eq. is divided by c^2:

$$3\left\langle\underline{\dot{\varepsilon}}_{L,i,jj}(\vec{r})\right\rangle_i^2 = 8\pi G\cdot|\rho_{gr.f.}(\vec{r})| \quad (8.52)$$

Part (6): The isotropic rate $\underline{\dot{\varepsilon}}_{L,iso}(\vec{r})$ is the sum of the Cartesian components of the averaged rates, see DEF (7):

$$\underline{\dot{\varepsilon}}_{L,iso}(\vec{r}) = \sum_j^3\left\langle\underline{\dot{\varepsilon}}_{L,i,jj}(\vec{r})\right\rangle_i = 3\left\langle\underline{\dot{\varepsilon}}_{L,i,jj}(\vec{r})\right\rangle_i \quad (8.53)$$

With it, the square of $\underline{\dot{\varepsilon}}_{L,iso}(\vec{r})$ is analyzed:

$$\underline{\dot{\varepsilon}}^2_{L,iso}(\vec{r}) = 3\cdot[3\left\langle\underline{\dot{\varepsilon}}_{L,i,jj}(\vec{r})\right\rangle_i^2] \quad (8.54)$$

Eq. (8.52) is applied to the above rectangular bracket. As a consequence, the isotropic rate is as follows and in accordance with THM (14):

$$\underline{\dot{\varepsilon}}^2_{L,iso}(\vec{r}) = 24\pi G\cdot|\rho_{gr.f.}(\vec{r})| \text{ q.e.d.} \quad (8.55)$$

8.5 Density of kinetic energy

Theorem 16 Density of the generalized kinetic energy

At each location $\vec{r}$, with a density $u_{gr.f.}(\vec{r})$ of gravitational energy, the following holds:

(1) In three-dimensional space, a rate of change of unidirectional relative additional volume $\dot{\varepsilon}_{L,jj}$ has the energy density $u_{gen,kin}$ as follows:

$$u_{gen,kin} := \frac{\dot{\varepsilon}^2_{L,jj} \cdot c^2}{8\pi \cdot G} \tag{8.56}$$

$$u_{gen,kin} = -u_{gr.f.}(\vec{r}) \tag{8.57}$$

(2) Thereby, the term in Eq. (8.56) is identified with a density of kinetic energy for the following reasons:

(2.1) The term in Eq. (8.56) is a density of energy, as it is equal to $-u_{gr.f.}(\vec{r})$.

(2.2) The term in Eq. (8.56) is a density of kinetic energy, as several rates $\dot{\varepsilon}_{L,jj}$ can be averaged to the isotropic rate in THM (15), whereby the isotropic rate in THM (15) corresponds to the macroscopic rate $\dot{V}/V$. Hereby, the macroscopic rate $\dot{V}/V$ corresponds to the Hubble rate $\dot{R}/R$. And $\dot{R}^2$ in the Hubble rate is proportional to a kinetic energy, see section (8.1).

(3) In D - dimensional space, with $D \geq 3$, a rate of change of unidirectional relative additional volume $\dot{\varepsilon}_{L,jj}$ has the energy density $u_{gen,kin}$ as follows:

$$u_{gen,kin} = \frac{\dot{\varepsilon}^2_{L,jj} \cdot c^2}{2A_D \cdot G_D} \tag{8.58}$$

Proof: Part (1): THM (9) with Eqs. (7.8 and 7.11) are multiplied by $\frac{1}{8\pi G}$. Thereby, part (1) is derived.

Part (2): That part includes its derivation.

Part (3): THM (9) with Eqs. (7.8 and 7.11) are multiplied by $\frac{1}{2A_D G_D}$. Thereby, part (3) is derived. q. e. d.

8.6 Rate gravity wave, RGW

Theorem 17 Law of the RGW

(1) A stationary solution of the DEQ of the VD (6.42) describes a localized wave or wavelet of rates and gravity. Accordingly, it is called rate gravity wave, RGW.

(2) The energy density u_{RGW} of a RGW has the following properties:

(2a) The density $u_{gr.f.}$ represents an attractive self - interaction of the RGW, as $u_{gr.f.} \leq 0$.

(2b) The density $u_{gr.f.}$ represents a gravitational field, whereby its average directs to the center of the RGW. This confirms the attractive self - interaction of the RGW.

(2c) The density of the kinetic energy $u_{gen,kin}$ of a RGW is equal to $-u_{gr.f.}$. Thus, $u_{gen,kin} \geq 0$. The sum of both energy densities is zero:

$$u_{RGW} = u_{gen,kin} + u_{gr.f.} = 0 \tag{8.59}$$

(3) Each Gaussian wave packet in THMs (6, 28, 21) represents a RGW. It has the standard deviation $\sqrt{2} \cdot \sigma$, and the absolute square of the wave packet has the standard deviation σ. In the limit σ to infinity, such a Gaussian wave packet becomes a harmonic wave, a harmonic RGW.

(3a) All VPs that are a Fourier sum or a Fourier integral of these harmonic RGWs inherit the energetic structure of the harmonic RGWs.

(3b) All VPs in (3a) are elements of the linear vector space of the solutions of the DEQ of VD. With the usually asymmetric scalar product, see e. g. Ballentine (1998), that space is a Hilbert space $\mathcal{H}$. As these VPs in $\mathcal{H}$ have the same energy, they can be transformed into each other.

Proof: It is included in the theorem.

Chapter 9

VD implies universal quantization

Idea: A light - portion of monochromatic light can be described in two ways:
as a wave with a circular frequency ω
and as a portion E of energy with the momentum $p = E/c$.
Thus, even at the classical level, there is a wave - particle duality, and we derive a universal quantization therefrom.

Process:

We analyze such light - portions that fall towards a mass M. Such light - portions can be represented by Gaussian wave packets in THM (6), for instance, see Fig. (9.1). Each minimal energy light - portion has a (typical) circular frequency ω, a distance R with respect to M and an energy $E_{min}(R, \omega)$. In the limit R to infinity, the quantities are marked by a subscript ∞.

9.1 Quantization of a light - portion

Theorem 18 Law of universal quantization

Light portions with $\omega > 0$ exist already in classical physics, as such light portions have been observed, see e. g. Ye et al. (2015).

A monochromatic light - portion, that is at $R > R_S$, and

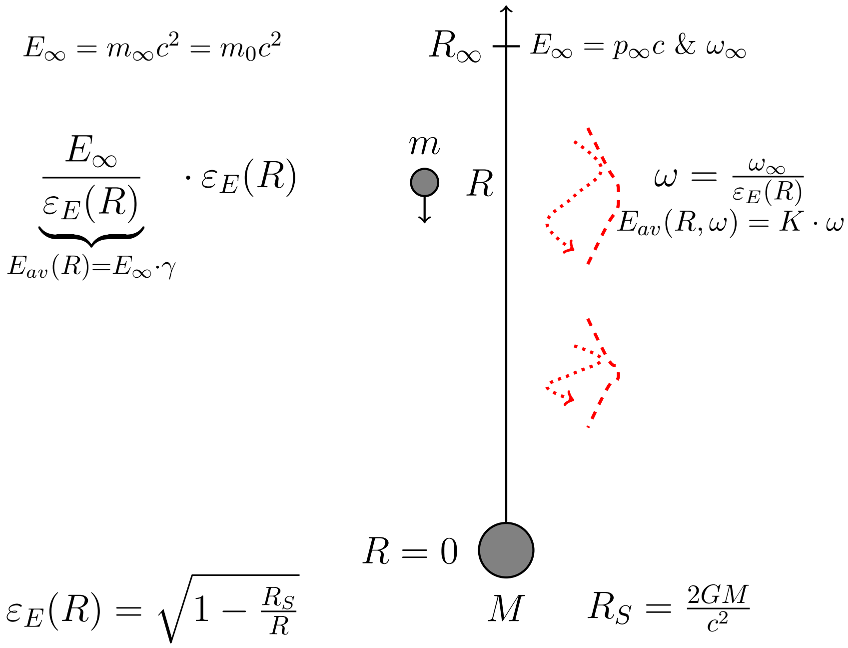

Figure 9.1: Universal quantization: A light - portion (wave packet) falls to a mass M. As a reference, a probe mass (ball) m falls to M.

with a positive circular frequency

$$\omega_\infty > 0 \text{ at } R_\infty, \tag{9.1}$$

and with minimal energy E_{min}, has the following properties:

(1.1) Each light - portion can be described by a change tensor, see THM (7).

(1.2) Moreover, as **special relativity** *is locally applicable at each radial coordinate R, the observable energy is the kinetic energy $E = p \cdot c$.*

(1.3) Furthermore, at each radial coordinate R, an available energy E_{av} is available for a transformation. For instance, the photoelectric effect can transforms the energy of the light portion to a portion of energy of an electron. Correspondingly, the observable energy is $E = p \cdot c$, and it is also equal to the available energy E_{av}, as it is available for a transformation to another form of energy.

(1.4) Altogether, at each radial coordinate R, the minimal observable energy E_{min} of a light - portion is a kinetic energy $E_{kin} = pc$, and it is an available energy E_{av}, and it is an observable energy $E_{observable}$, since it can be observed with help of the transfer of momentum $p = E_{kin}/c$ or with a transfer of energy E_{av}:

$$E_{min} = E_{av} = E_{kin} = p \cdot c = E_{observable} \tag{9.2}$$

(2) As the **dispersion relation** *of a light wave $\omega = k \cdot c$ is applicable, Eq. (9.2) combined with the dispersion relation imply:*

$$\frac{E_{av}}{p} = c = \frac{\omega}{k} \rightarrow \frac{E_{av}}{\omega} = \frac{p}{k} = K(\omega) \tag{9.3}$$

As a consequence, the observable energy of the light - portion is a product of ω and a quantization function $K(\omega)$ of ω.:

$$E_{av}(R, \omega) = \omega \cdot K(\omega) \text{ and} \tag{9.4}$$

$$E_{av}(R_\infty, \omega_\infty) = \omega_\infty \cdot K(\omega_\infty) \tag{9.5}$$

(3) When a light - portion falls, then the **gravitational redshift** *holds, according to THM (10):*

$$\omega(R) = \omega_\infty / \varepsilon_E(R) \tag{9.6}$$

Note that this relation and the position factor in general have also been derived in Landau and Lifschitz (1971) and (Burisch et al., 2022, p. 484 - 489).

(4) The vicinity of M is stationary or invariant with respect to time translation. Thus, the Noether (1918) theorem holds. Hence, **energy is conserved***, see section (4.2.2). This invariance is expressed by expanding with ε_E:*

$$E_{av}(R_\infty, \omega_\infty) = \varepsilon_E(R) \cdot [E_{av}(R_\infty, \omega_\infty)/\varepsilon_E(R)] \tag{9.7}$$

According to the reference mass, the above rectangular bracket is the energy $E_{av}(R,\omega)$ that can be observed at R. The reference mass is applicable to the light - portion, as both can be transformed to each other, for instance by pair formation and annihilation, see e. g. Workman et al. (2022). So the initial energy $E(R_\infty,\omega_\infty)$ is equal to the energy at R. It is the product of ε_E and the observable or available energy $E_{av}(R,\omega)$:

$$E_{av}(R_\infty,\omega_\infty) = \varepsilon_E(R) \cdot E_{av}(R,\omega) \tag{9.8}$$

(5) Application of Eqs. (9.4, 9.5 and 9.6) to Eq. (9.8) implies:

$$K(\omega_\infty) = K(\omega) \tag{9.9}$$

As ω_∞ can be chosen arbitrarily, and since a larger value ω can be chosen freely, the above relation holds for each pair of circular frequencies ω_1 and ω_2:

$$K(\omega_1) = K(\omega_2) \tag{9.10}$$

Thus, we showed that the quantization function does not depend on the circular frequency ω. Thence, the quantization function $K(\omega)$ is a **constant***.*

(6) It is shown that the quantization constant K is nonzero, $K > 0$. For it, we realize $\omega > 0$ (item 1.1), and $p = E_{av}/c > 0$, and we use $K = E_{av}/\omega$ in Eq. (9.4):

(6.1) Eqs. (9.1 and 9.6) imply that the circular frequency ω of a minimal energy light - portion is nonzero.

(6.2) The minimal energy $E_{min} = E_{av}$ of a minimal energy light - portion is nonzero for the following reason:

The light - portion has direction vector $-\vec{e}_3$, as it propagates towards M:

$$\vec{p} = -p_{min} \cdot \vec{e}_3 \tag{9.11}$$

Consequently, the light - portion has nonzero momentum $p_{min} \neq 0$. *Hence, the light - portion has a nonzero available or observable energy:*

$$E_{av} = E_{observable} = E_{min} = p_{min} \cdot c \neq 0 \tag{9.12}$$

(6.3) As a consequence of parts (5), (6.1) as well as (6.2) and Eq. (9.4), the quantization constant K *is positive:*

$$K = \frac{E_{av}}{\omega} > 0 \tag{9.13}$$

As a consequence, the formation of **universal quanta** *has been derived, without any necessity of any process of quantization.*

(6.4) In principle, with a single experiment, the value of $K > 0$ *can be measured. For instance, the experiment can investigate light with a circular frequency* ω *and at a coordinate* R *as follows:*

(6.4.1) In a first part of the experiment, the energy of light E_{av} *is transformed to an energy portion* E_{output}. *Thereby, the minimal achievable value* $E_{output,min}$ *is determined. For instance,* E_{output} *can be the energy of electrons in an illuminated photo diode or in a sufficiently illuminated LED. In the energy transformation, the smallest output* $E_{output,min}$ *is achieved by the smallest input* E_{min}, *namely the minimal energy light - portion. Moreover, in the energy transformation, there can be a loss of energy, so that the used energy* $E_{output,min}$ *is smaller or equal to the input energy* E_{min}:

$$E_{output,min} \leq E_{min} \tag{9.14}$$

(6.4.2) In a second part of the experiment, an energy portion E_{input} *is transformed to the energy of light* E_{av}. *Thereby, the minimal necessary value* $E_{input,min}$ *is determined. For instance,* E_{input} *can be the energy of electrons in a light emitting diode, LED. In the energy transformation, the smallest input* $E_{input,min}$

is necessary for the smallest output E_{min}, namely the minimal energy light - portion. Moreover, in the energy transformation, there can be a loss of energy, so that the input energy $E_{input,min}$ is larger or equal to the output energy E_{min}:

$$E_{min} \leq E_{input,min} \tag{9.15}$$

(6.4.3) The two parts of the experiment are optimized so that the energies $E_{input,min}$ and $E_{output,min}$ are equal. In that case, Eqs. (9.14 and 9.15) and the transitivity of the inequalities imply that E_{min} is equal to $E_{input,min}$:

$$E_{output,min} \leq E_{min} \leq E_{input,min} = E_{output,min}, \tag{9.16}$$

$$\text{consequently,} \;\; E_{input,min} = E_{min} \tag{9.17}$$

As a consequence, the minimal energy of the light portion can be measured in principle. This measurement in principle does not depend on the amount of the error of measurement. Similarly, this measurement in principle does not depend on the present - day availability of the experimental equipment. Consequently, a positive value of K can be measured in principle, see Eq. (9.13):

$$K = \frac{E_{min}}{\omega} > 0 \tag{9.18}$$

This value of K is a constant, as it does not depend on ω, see part (5).

(7) **Generalization:**

(7.1) The quantization constant K holds for each object that propagates at the velocity c and exhibits the gravitational red-shift, as the above analysis applies to each such object. This includes objects propagating at $v = c$ and described by the DEQ of VD.

(7.2) In order to generalize the applicability of the constant K to objects at $v < c$, we need a DEQ describing all objects at $v \leq c$.

We will see in chapter (10), that the DEQ of VD implies a DEQ for objects at $v < c$, whereby the constant of quantization remains the same, see THMs (26 and 27).

As a consequence, the quantization constant K applies to all objects at $v \leq c$, so that K is the **universal constant of quantization**.

(8) The value of K or $K(\omega)$ has been **measured**. $2\pi \cdot K(\omega)$ *takes the following value, the Planck constant:*

$$2\pi \cdot K(\omega) = 6.626\,070\,15 \cdot 10^{-34}\,\mathrm{Js} = h \tag{9.19}$$

The universal constant of quantization $K(\omega)$ is named reduced Planck constant $\hbar$, whereby $2\pi \cdot \hbar$ is named Planck constant h:

$$K(\omega) = \hbar = \frac{h}{2\pi} \tag{9.20}$$

As a convention, the value of the Planck constant is defined to be an exact value, see e. g. Newell et al. (2018) or Workman et al. (2022).

(9) The universal constant $\hbar$ and the light - portion with its wave function of the electric field

$$\vec{E} = \vec{E}_0 \cdot \exp[-i\omega\tau + i\vec{k}\vec{L}] \tag{9.21}$$

provide the momentum $\vec{p}$ in the form of an eigenvalue of an operator $\hat{p}$, called **momentum operator***:*

$$\frac{E}{\omega} = \frac{p}{k} = \hbar \rightarrow p = \hbar k \tag{9.22}$$

Application of $\partial_{\vec{L}}$ to Eq. (9.21) yields:

$$\partial_{\vec{L}}\vec{E} = i\vec{k}\vec{E} \rightarrow -i\hbar\partial_{\vec{L}}\vec{E} = \hbar\vec{k}\vec{E} = \vec{p}\vec{E} \tag{9.23}$$

Thus, the momentum operator $\hat{p}$ has the eigenvalue $\vec{p}$, and it is as follows:

$$\hat{\vec{p}} = -i\hbar\partial_{\vec{L}} \tag{9.24}$$

In flat space, it is as follows:

$$\hat{\vec{p}} = -i\hbar\partial_{\vec{x}} \tag{9.25}$$

(10) Altogether, a minimal energy light - portion at a circular frequency ω has the following observable and kinetic energy:

$$\boxed{E_{min} = E_{kin,min,\omega} = \hbar \cdot \omega} \tag{9.26}$$

Thereby, $\hbar$ is the universal constant of quantization. It is applicable to all localizable and observable objects. Thereby, a localizable object can be represented by one or more wave packets that are solutions of the DEQ of VD or of a DEQ derived therefrom. As the observable minimal energy E_{min} is positive, there is at least one experiment that measures E_{min} in principle for the particular case of that experiment, see part (6). As a consequence, $\hbar > 0$ is derived.

$$\boxed{\hbar = \text{ positive universal quantization constant}} \tag{9.27}$$

Hereby, momentum p and energy E are eigenvalues of the following eigenvalue generating operators p̂ and $\hat{E}$:

$$\boxed{\hat{E} = c \cdot \hat{\vec{p}} \cdot \vec{e}_p = \frac{\hbar c}{i}\partial_{\vec{L}} \cdot \vec{e}_p} \tag{9.28}$$

Hereby, $\vec{e}_p$ is the unit vector in the direction of the momentum.

(11) **Criterion:** *The universal quantization can be applied to objects that fulfill the following criteria:*

(11a) The object is localizable.

(11b) The object has a circular frequency ω and a wave number k with $\omega = c \cdot k$.

(11c) The object has an observable energy E.

(11d) In a free fall to a mass M with $R_S = \frac{2GM}{c^2}$ and $\varepsilon_E(R)$, in the limit R to infinity, the observable energy E has a value E_∞,

and the observable energy E as a function of R is as follows:

$$E(R) = \frac{E_\infty}{\varepsilon_E(R)} \tag{9.29}$$

(11e) The observable energy E can be interpreted as a kinetic energy $E = p \cdot c$.

(11f) Altogether, for each localizable quantum object at $v = c$, this criterion holds.

Proof:

The parts (1) to (11) include their proofs. q. e. d.

Interpretation: Usually, in present - day physics, the constant of quantization is introduced without explanation - as a part of a postulate, for instance, see e. g. Hilbert et al. (1928), Landau and Lifschitz (1965), Ballentine (1998), Scheck (2013). Here, the constant of quantization is explained - this provides a great insight:

Firstly, a light - portion is analyzed: Its observable energy is the kinetic energy $E = pc$. It can be expressed by $E_{av} = K(\omega) \cdot \omega$.

Secondly, during free fall, the kinetic energy is proportional to ω, according to the (derived) gravitational redshift. Thus, $K(\omega)$ is independent of ω, K is a constant for all light - portions - and for all objects propagating at $v = c$.

Thirdly, the constant K is positive, since an observable light - portion with $\omega \neq 0$ has a kinetic energy $pc = E_{av} > 0$.

Fourthly, the same constant K quantizes objects with nonzero rest mass, as their dynamics is derived from the DEQ of VD. We will derive this in chapter (10), see THMs (26 and 27). Thus, K quantizes all objects, with and without nonzero rest mass.

Altogether, the universality of quantization is based on special relativity, on the observability of positive kinetic energy

$E_{av} = pc$ of light - portions, on the gravitational redshift, on energy conservation and on the common dynamics of all objects inherent to the DEQ of VD.

We realize that the full universal quantization emerges: For objects with $v = c$, it emerges from gravity and relativity. This is generalized to objects with $m_0 > 0$ via the common dynamics inherent to the DEQ of VD.

This explanation also shows how the existence of quanta is a natural implication of VD. For instance, the wave property of the electron, see Davisson and Germer (1927) or Fig. (9.2), is now traced back to the DEQ of VD (THM 5), which implies universal quantization (THM 18 as well as the generalized Schrödinger equation (THM 26), which implies the usual Schrödinger equation (THM 27).

Presumably, this is the deep reason for the historic controversies or incompatibilities between relativity and quantum physics, see e. g. Einstein et al. (1935), Einstein (1948), Weinberg (2017). Accordingly, (Feynman, 1965, p. 129) wrote: *'I think I can safely say that nobody understands quantum mechanics.'* Now, you can derive the universal quantization on your own! And in the following of the book, we will see how the universal quantization turns out to be one of the keys to the understanding of advanced theories of quantum physics. Moreover, we show how these theories provide the key to the solution of present - day deep problems of quantum physics - such as the cosmological constant problem. As a consequence, *I think I can safely say that after reading this book, you can explain how the existence of quanta in nature is derived as a property of volume in nature, according to THMs (3, 5, 18, 26, 27).*

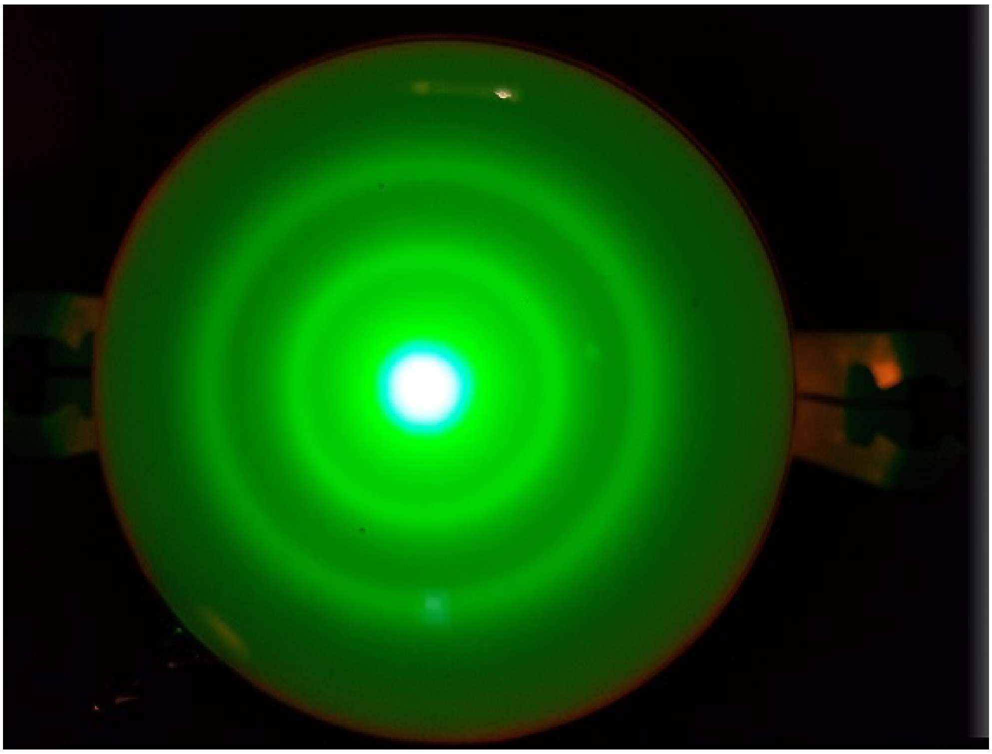

Figure 9.2: The two concentric rings indicate electron waves arriving at a screen: For it, electrons have been accelerated by a voltage of 5000 Volts. Then the electron beam has been diffracted at a slice consisting of many crystals of graphite. Behind that slice, the electrons propagate either at the original direction and form the central light at the screen. Or they propagate at one of two cones around the central beam, whereby these cones cause the two concentric rings when arriving at the screen. This wave property of the electron is now traced back to the dynamics of volume in nature, whereby the existence of quanta is a derived property of volume in nature, see THMs (3, 5, 18, 26, 27).

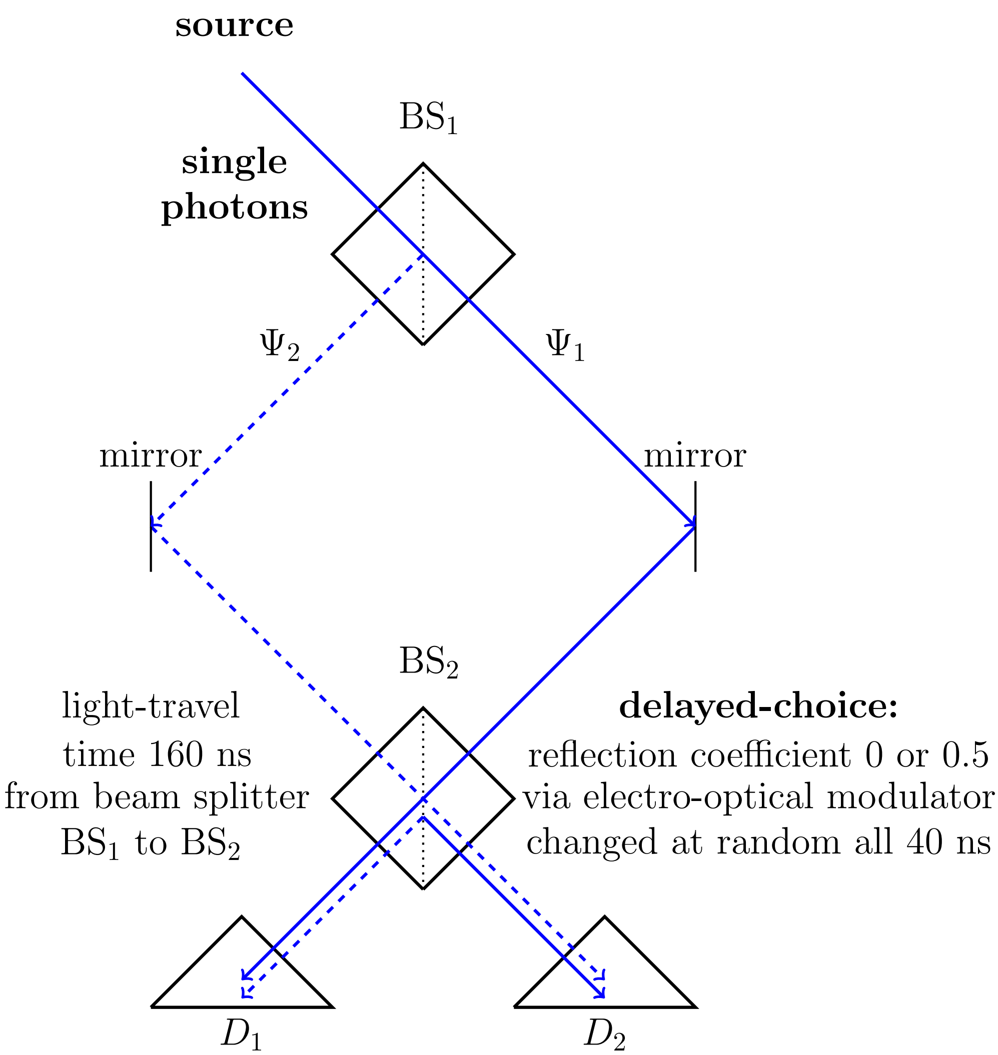

Figure 9.3: Sketch of the delayed-choice experiment: In a Mach-Zehnder interferometer, MZI, the second beam splitter includes an electro-optical modulator that provides a rapid random switching of the reflectivity ρ from $\rho = 0$ to $\rho = 0.5$ and vice versa. The measurements are executed by the detectors D_1 and D_2 Path 1 is marked by Ψ_1. Path 2 is marked by Ψ_2.

9.2 Universality of nonlocality

The first fundamental property of quanta is the fact of minimal energy portions and the universality of the corresponding quantization constant, $E = \hbar\omega$. Both facts are a consequence of the VD, as shown in the previous section (9.1).

The second fundamental property of quanta is the fact the observable nonlocality, see e. g. Jaques et al. (2008), Ye et al. (2010) or Garrisi et al. (2019). In this section, we show that also nonlocaity is a consequence of the VD.

9.2.1 A delayed - choice experiment

Jaques et al. (2008) performed the delayed-choice experiment in Fig. (9.3). It is based on a Mach-Zehnder interferometer, MZI. Single photons at a circular frequency ω enter the MZI. The second beam splitter has one of two reflectivities $\rho = \frac{P_r}{P_0}$, whereby P_0 is the power of all incoming photons, and P_r is the power of the reflected photons. Accordingly, the second beam splitter operates in one of two modes:

(1) If the second beam splitter has the reflectivity $\rho = 0.5$, then the photon exhibits interference. In the experiment, the interference showed a visibility of 94 %.

(2) If the second beam splitter has the reflectivity $\rho = 0$, then the photon is transmitted, and there occurs no interference.

9.2.2 VD implies quantum nonlocality

According to the VD, the following observations are predicted at the detectors and in the two modes:

(1) In the mode with interference, the VD predicts the following:

According to universal quantization in THM (18), the light propagates corresponding to a wave function, it is named Ψ. Hereby, there occur phase differences at mirrors as follows:

In detector D_1, the wave at the solid line in Fig. (9.3) accumulates the phase shift π at the right mirror. Similarly, the wave at the dashed line accumulates the phase shift π at the left mirror plus two phase shifts of $\pi/2$ at each beam splitter. Altogether, the phases of the two paths differ by π. Thus, there occurs destructive interference at D_1.

In detector D_2, the wave at the solid line accumulates the phase shift π at the mirror plus the phase shift $\pi/2$ at the second beam splitter. Moreover, the wave at the dashed line accumulates the phase shift π at the mirror plus the phase shift $\pi/2$ at the first beam splitter. Altogether, the phases of each path is $3\pi/2$. Thus, there occurs constructive interference at D_2.

According to universal quantization in THM (18), the observed minimal energy $E_{min,kin,\omega}$ represents a photon. Photons are measured by the detector with constructive interference. As a consequence of all paths, this is D_2. **Consequently, the photons are measured by D_2 and no photon is measured by D_1:**

$$\text{Interference of single photons is visible.} \tag{9.30}$$

For it, the photon must propagate in both paths in the MZI in Fig. (9.3). Hence, each single photon is not localized to one of the two paths. This is an example for quantum nonlocality.

$$\text{Each single photon shows quantum nonlocality.} \tag{9.31}$$

(2) In the mode, in which the second beam splitter has the reflectivity $\rho = 0$, the VD predicts the following measurements:

According to universal quantization in THM (18), the light propagates according to the wave function Ψ.

At the first beam splitter, Ψ splits into $\Psi_1 = \Psi/\sqrt{2}$ in the solid path and $\Psi_2 = \Psi/\sqrt{2}$ in the dashed path in Fig. (9.3). At the second beam splitter, Ψ_1 and Ψ_2 are transmitted. Thus, there occurs no interference. Hence, there occurs the probability $\Psi_1^2 = \Psi_2^2 = 0.5\Psi^2 = 0.5$ at each detector.

According to universal quantization in THM (18), the observed minimal energy $E_{min,kin,\omega}$ represents a photon.

Though the same part $\Psi_1 = \Psi_2 = \Psi/\sqrt{2}$ arrives at each detector, only one single photon can be detected by both detectors at each time. **Consequently, there is no physical deterministic difference that could determine the detector at which the photon is measured. As a consequence, the photon is measured by one of the detectors in a stochastic manner.**

The same wave function $\frac{\Psi}{\sqrt{2}}$ arrives at each detector. Thus, each detector measures the photon with the same probability 1/2. **Consequently, one of two cases occurs: Either the photon is measured by D_2 and not by D_1. Or the photon is measured by D_1 and not by D_2. Each case occurs with the probability 1/2:**

$$\text{Measured photons are anticorrelated.} \tag{9.32}$$

(Delayed - choice of the mode 1 or 2) In mode 1, the interference pattern verifies that the wave function uses both paths. In mode 2, no interference pattern verifies that the wave function uses both paths.

That verification is achieved by the delayed - choice mode: When the wave passes BS_1, it has not yet been decided whether or not BS_2 will be active. As a consequence, the wave function must use both paths in order to provide the interference pattern in the case in which the BS_2 is activated after the wave passed BS_1.

VD predicts that the wave function uses both paths, so that the interference pattern is provided whenever BS_2 is activated.

9.2.3 Experimental test

Jaques et al. (2008) performed the experiment and confirmed the predictions of the VD within the following errors of measurement:

(1) In the mode with interference, the interference showed a visibility of $v = 94\%$. Thereby, the visibility is defined with help of the intensities I_{max} at a maximum and I_{min} at a minimum of the interference pattern as follows:

$$v = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{9.33}$$

Thus, the interference pattern showed a high contrast.

(2) In the mode without interference ($\rho = 0$), an anticorrelation parameter $\alpha = 0.12$ is observed. Thereby, the anticorrelation parameter α is defined with help of the following quantities:

N_1 is the complete number of detected photons.

N_r is the number of the photons reflected by the first beam splitter. These are detected by D_2.

N_t is the number of the photons transmitted by the first beam splitter. These are detected by D_1.

N_c is the number of measurements with a coincident measurement by D_1 and D_2.

The corresponding probabilities are $p_r = \frac{N_r}{N_1}$, $p_t = \frac{N_t}{N_1}$ and $p_c = \frac{N_c}{N_1}$. The anticorrelation parameter α is as follows:

$$\alpha = \frac{p_c}{p_r \cdot p_t} \tag{9.34}$$

Without error of measurement, there would be no coincident detections, so that $p_c = 0$ and $\alpha = 0$. As the observed value $\alpha = 0.12$ is relatively small, the error of measurement is relatively small, and the predicted anticorrelation in Eq. (9.32) is confirmed in a very clear manner. Moreover, Manning et al. (2015) performed a similar delayed-choice experiment with a single atom.

We summarize our findings:

Theorem 19 Law of universal nonlocality

As a consequence of universal quantization in THM (18), each photon with a circular frequency ω has the following properties:

(1) Each photon propagates at all possible paths according to its wave function. Accordingly, it shows interference.

(2) As a consequence, each photon has the property of quantum nonlocality.

(3) Each photon is measured as a whole photon. Accordingly, detectors at which the photon can be measured show anticorrelated results.

(4) This occurs independently of the value of the universal quantization constant in THM (18), as the measured anticorrelation parameter is independent of the quantization constant. In this sense, these simultaneous properties of quantum nonlocality and wholeness of single photons are universal.

9.3 Universal causality

In the above section (9.2), we derived the nonlocality in quantum physics. In the context, Einstein (1907) proposed a system, in which a negative time dt occurs, and he proposed that this would show a causality violation for velocities larger than c. In this section, we show that such a negative time dt is associated with a negative velocity, in general. As a consequence, such a proposed causality violation does not occur, in general. In fact, in experiments, velocities larger than c do not show causality violation, see Ye et al. (2010), Barbero et al. (2010).

9.3.1 Einstein's proposed system

(Einstein, 1907, p. 381) proposed the system in Fig. (9.4), in order to analyze the possibility of causality violation.

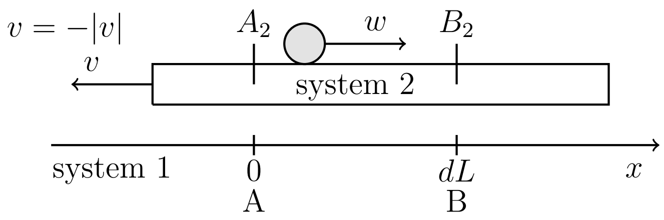

Figure 9.4: Einstein (1907) proposed the following situation: In a system 1, a system 2 moves at a velocity v to the left. In system 2, an object or signal moves at a velocity w to the right. Thus, in system 1, the object moves at a velocity u. Einstein analyzed the time dt, that the object requires for a motion from a point A at $x = 0$ to a point B at $x = dL$. I added points A_2 and B_2 fixed in system 2. At $t = 0$, A and A_2 are at the same location, and B and B_2 are at the same place.

Relative to a first system in Fig. (9.4), there moves a second system with a velocity v to the left. In that system, there moves an object or signal with a velocity w to the right. As a consequence, the object moves with a velocity u relative to the first system. Thereby, u is the following function of v and w, see e. g. (Einstein, 1905b, p. 906) or (Burisch et al., 2022, p. 482):

$$u = \frac{w + v}{1 + \frac{v \cdot w}{c^2}} \tag{9.35}$$

As a consequence, in order to travel a distance dL from a point A to a point B, in (or relative to) the first system, the object requires the following time dt:

$$dt = \frac{dL}{u} \tag{9.36}$$

Application of the velocity in Eq. (9.35) yields the following time:

$$dt = dL \cdot \frac{1 + \frac{v \cdot w}{c^2}}{w + v} \tag{9.37}$$

(Einstein, 1907, p. 381) proposed that the velocity w is positive, and the velocity v is negative, $v = -|v|$. Thus, the required time is as follows:

$$dt = dL \cdot \frac{1 - \frac{|v| \cdot w}{c^2}}{w - |v|} \tag{9.38}$$

9.3.2 Problem: Einstein's causality violation

(Einstein, 1907, p. 381) argued, that the time dt can become negative at appropriate values of the velocity v, and that negative times indicate causality violation.

As an example for it, we use $w = 2c$ and $dL = 3\,\mathrm{m}$ as well as $|v| = 0.8c$, without loss of generality in principle. With it, the time is:

$$dt = 3\,\mathrm{m} \cdot \frac{1 - 1.6}{1.2c} = 3\,\mathrm{m} \cdot \frac{-0.6}{1.2 \cdot 3 \cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}} = -5\,\mathrm{ns} \tag{9.39}$$

(Einstein, 1907, p. 381-382) stated that the negative time dt would imply causality violation: 'Dies Resultat besagt, dass wir einen Übertragungsmechanismus für möglich halten müssten, bei dessen Benutzung die erzielte Wirkung der Ursache vorangeht.' In English: 'This result states, that we must accept a mechanism of transmission, that provides an effect before the cause has taken place.'

Additionally, (Einstein, 1907, p. 381-382) stated the impossibility of $w > c$: ' ..., dass durch dasselbe die Unmöglichkeit der Annahme $w > c$ zu Genüge erwiesen ist.' In English: ' ..., that by this the impossibility of the assumption $w > c$ is sufficiently proven.'

However, such a causality violation would be a severe problem, as effects that spread from A to B faster than the velocity c of light have been shown experimentally, see e. g. Aspect et al. (1982).

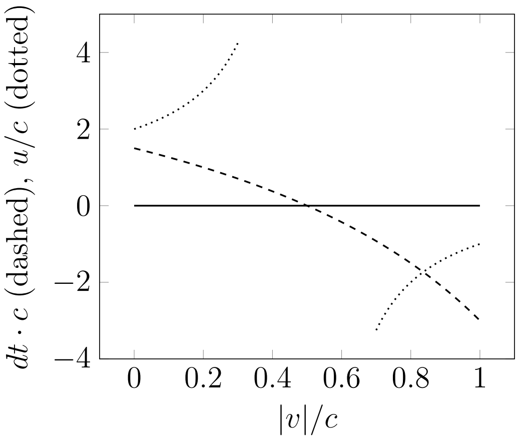

Figure 9.5: Scaled time $dt \cdot c$ (dashed) and velocity u/c (dotted) as a function of $|v|/c$, with $w = 2c$ and $dL = 3$ m. There are three essential cases:
[1] At $|v|/c < 1/2$, the velocity u/c and the time dt are positive.
[2] At $|v|/c = 1/2$, the velocity u/c diverges, and dt is zero.
[3] At $|v|/c > 1/2$, the velocity u/c and dt are negative.

9.3.3 Clarification

We resolve the causality problem (section 9.3.2) and clarify the circumstances: For it, we analyze the example more systematically.

Firstly, we analyze the velocity u and the required time dt as a function of the absolute value of the velocity $|v|$ of system 2. Thereby, we use the above example with $w = 2c$ and $dL = 3$ m, without loss of generality in principle. The result is shown in Fig. (9.5). There are three essential cases:

[1] At $|v|/c < 1/2$, the velocity u/c and the time dt are positive. Thus, no causality violation occurs.

[2] At $|v|/c = 1/2$, the velocity u/c diverges, and dt is zero. Hence, no causality violation occurs.

[3] At $|v|/c > 1/2$, the velocity u/c and dt are negative. As a consequence, the object does not arrive at the point B in Fig. (9.4). Consequently, the event 'arrival of the object at B' does

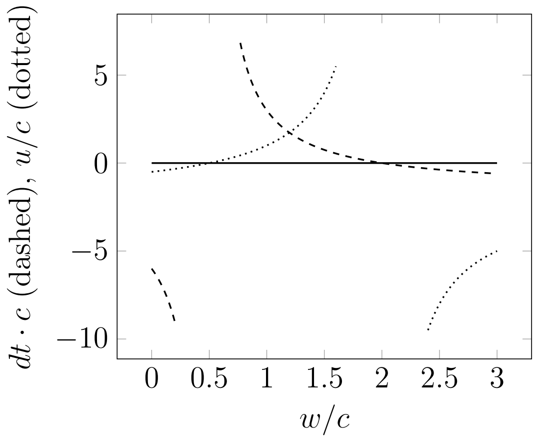

Figure 9.6: Scaled time $dt \cdot c$ (dashed) and velocity u/c (dotted) as a function of $|v|/c$, with $|v|/c = 1/2$ and $dL = 3$ m. There are three essential cases:
[1] If $w/c < 1/2$ or $w/c > 2$, then u/c and dt are negative.
[2] If $w/c = 1/2$ or $w/c = 2$, then u/c diverges, and dt is zero.
[3] At $1/2 < w/c < 2$, the velocity u/c and dt are positive.

not take place. Hence, no causality violation takes place, as dt is the time that the object requires for a motion from A to B.

In general, the event 'arrival of the object at B_2' takes place, but this event is not described by dt.

Altogether, in each of the three cases, no causality violation takes place. (Einstein, 1907, p. 381-382) analyzed the time dt. However, he did not analyze, whether these events that delimit the time dt do take place at all. This analysis clarifies the circumstances of the example proposed by (Einstein, 1907, p. 381-382). Moreover, this analysis resolves the causality violation proposed or stated by (Einstein, 1907, p. 381-382).

In a second clarification, the velocity w is varied, see Fig. (9.6).

Secondly, we analyze the velocity u and the required time dt as a function of the velocity w of the object in system 2. Thereby, we use the above example with $|v|/c = 1/2$ and $dL = 3$ m, without loss of generality in principle. The result is shown in Fig. (9.6). There are three essential cases:

[1] At $w/c < 1/2$ or $w/c > 2$, the velocity u/c and the time dt are negative. As a consequence, the object does not arrive at the point B in Fig. (9.4). Consequently, the event 'arrival of the object at B' does not take place. Hence, no causality violation takes place, as dt is the time that the object requires for a motion from A to B.

In general, the event 'arrival of the object at B_2' takes place, but this event is not described by dt.

[2] At $w/c = 1/2$ or $w/c = 2$, the velocity u/c diverges, and dt is zero. Hence, no causality violation occurs.

[3] At $1/2 < w/c < 2$, the velocity u/c and dt are positive. Hence, no causality violation occurs.

Altogether, in each of the three cases, no causality violation takes place. This analysis clarifies the circumstances of the example proposed by (Einstein, 1907, p. 381-382). Moreover, this analysis resolves the causality violation proposed or stated by (Einstein, 1907, p. 381-382). Next, causality is shown in general.

9.3.4 Universal solution

Theorem 20 Universal causality

(1) Considered system: Relative to a first system in Fig. (9.4), there moves a second system with a velocity v to the left. In that system, there moves an object or signal with a velocity w to the right. As a consequence, the object moves with a velocity u relative to the first system. The time of a motion from a point A to a point B is called dt.

In the system in part (1), the following holds:

(2) A possible causality violation could occur in the case of negative time dt, see e. g. (Einstein, 1907, p. 381-382).

(3) There are two possible cases:

(3.1) Either, the object is analyzed in its own system, system 2. In that case, the motion of the object alone does not imply causality violation.

(3.2) Or the object is analyzed in an external system. System 1 is such a system, without restriction on generality.

There are two cases:

(3.2.1) Either both velocities v and w are positive. Then the time dt is positive, see Eq. (9.37).

(3.2.2) Or the velocities v and w can have negative signs. Then the time dt can become negative, see Eq. (9.37):

In system 1, the velocity u is as follows, see e. g. (Einstein, 1905b, p. 906)) or (Burisch et al., 2022, p. 482):

$$u = \frac{w - |v|}{1 - \frac{|v| \cdot w}{c^2}} \tag{9.40}$$

In system 1, a motion from A to B requires the time $dt = dL/u$:

$$dt = dL \cdot \frac{1}{u} = dL \cdot \frac{1 - \frac{|v| \cdot w}{c^2}}{w - |v|} \tag{9.41}$$

A possible causality violation could occur in the case of negative time dt, see e. g. (Einstein, 1907, p. 381-382). This could occur as the numerator dL as well as the denominator u in Eq. (9.41) could change the sign at a respective singularity. These two cases of sign reversal are shown in Fig. (9.6).

In such a case of negative time dt, the velocity u is also negative, since $dt = dL/u$, *see Eq. (9.41). As a consequence, the object does not arrive at B, as the object moves to the left, but B is at the right, see Fig. (9.4). Consequently, in the analyzed Einstein (1907) example, which is constructed in a general manner, the negative time dt is not a time between two events that take place. Hence,* $dt < 0$ *does not imply any causality violation.*

(4) The obtained results are summarized:

When an object moves from a point A to a point B, then the required time dt is positive or zero. Consequently, each such motion from A to B exhibits causality.

Thereby, there is no restriction with respect to the object or to the systems. As a consequence, the causality is shown in a universal manner.

The case $dt = 0$ is singular, as it corresponds to an infinite velocity u. If $|v| < c$ and $w < c$, then that case is excluded.

Conversely, the case $dt = 0$ can be regarded as nonlocal. Such a nonlocal behavior corresponds to $\frac{|v|}{c} = \frac{1}{w/c}$. In that case, at least one of the velocities is larger or equal c. Such nonlocal and causal cases are conceivable in SR.

Proof: It is included in the proposition. q. e. d.

9.3.5 Three categories of objects in the VD

According to the great importance of the velocity c of light in empty space, (Einstein, 1907, p. 381-382) separated the objects in nature into three categories, see section (2.2.10):

(1) Objects at $v < c$

(2) Objects at $v = c$

(3) Objects at $v > c$

These categories are analyzed with help of the dynamics of volume in nature: In the VD, the objects are described by solutions of the DEQ of VD. In general, such solutions are waves or wave packets, see THM (6). Thereby, the motion of a wave packet is described by a group velocity v_g, and the phase of the wave is described by a phase velocity v_p. As a consequence, in the first two categories, the group velocity v_g is defined and equal to v, whereas the third category summarizes the remaining cases v_g undefined or $v_g > 0$. Consequently, in the VD, the categories are as follows:

(1) Objects at $v_g < c$

(2) Objects at $v_g = c$

(3) Objects with undefined v_g **or** $v_g > c$

The DEQ of VD has harmonic waves as solutions, see THM (6). Thereby, harmonic waves have no localizable local maximum and no group velocity, as a consequence. Consequently, harmonic waves of the VD are an instance of category (3). Thus, these can exhibit effective speeds w_{eff} above c, see DEF (1).

Category (2) and the velocity energy relation imply

$$E^2 - E^2 \cdot \frac{v^2}{c^2} = E_0^2 = m_0^2 c^4 = 0 \text{ or } m_0 = 0 \tag{9.42}$$

Category (1) includes $E > 0$, as otherwise no object would be observable. Category (1), $E > 0$ and the velocity energy relation imply

$$0 < E^2 = \frac{E_0^2}{1 - v^2/c^2}, \text{ consequently, } \frac{E_0}{c^2} = m_0 > 0 \tag{9.43}$$

Altogether, as a consequence of the VD, the three categories are as follows:

(1) Objects at $v_g < c$ **or** $w_{eff} < c$ have nonzero rest mass $m_0 > 0$

(2) Objects at $v_g = c$ **or** $w_{eff} = c$ have zero rest mass $m_0 = 0$ and positive energy $E > 0$. Examples are electromagnetic waves.

Such objects require zero time dt_{own} in their own system in order to travel from A to B, as the time dilation is as follows:

$$dt_{own} = dt \cdot \sqrt{1 - \frac{v^2}{c^2}} \tag{9.44}$$

This own time dt_{own} tends to zero, when v tends to c.

(3) Objects with undefined v_g and $v_p > c$, or $w_{eff} > c$ have an essential representative: A solution of the DEQ of VD in the form of a harmonic wave has an undefined group velocity v_g. Moreover, its phase velocity v_p does not describe the motion of an object. As a consequence, v_p is not restricted by SR. Consequently, v_p can in principle provide nonlocal effects. The corresponding mechanism is elaborated in chapter (13).

Furthermore, such nonlocal effects do not violate Einstein's locality principle in section (2.2.6), as such nonlocal effects are mediated by the volume in nature, but Einstein supposed his locality principle in section (2.2.6) only for the case of unmediated events or 'things'. The mechanism of nonlocality is analyzed in chapter (13).

9.4 Universal minimal fluctuation

In the previous sections, we showed universal quantization (section 9.1), universal nonlocality (9.2) and universal causality (9.3).

In this section, we show how VPs with a center of energy exhibit universal quantization, and how the spatial resolution of volume in nature is limited by the Planck length L_P.

In THM (21), we derive the Heisenberg (1927) uncertainty relation for the case of a mathematical Gaussian wave packet, without the universal constant $\hbar$ of quantization.

In THM (22), we use the fact that VPs provide the triple periodic time, gravity and general relativity. Based on that triple and universal quantization in THM (18), we derive the quantization of VPs as well as the invariance of the energy density of volume, u_{vol}.

In THM (23), we use the mathematical uncertainty relation in THM (21), in order to derive the corresponding mathematical zero-point oscillations, ZPOs in part (1). In part (2), we use the quanta derived in THM (22), in order to derive the ZPOs in general. This result is illuminative, as it shows an essential property of VPs and space, the ZPOs. We will ultimately clarify their energetic structure in the solution of the cosmological constant problem in THM (40).

In THM (24), we use the ZPOs in THM (23) and the above triple, in order to derive the absolute minimum of fluctuations and additionally of spatial uncertainty, the Planck length L_P. This fact is insightful, as it shows an essential property of VPs and space of a minimal measurable length scale: L_P.

Theorem 21 Law of universal minimal fluctuation

(1) For Gaussian wave packets Ψ in THM (6), see Fig. (6.6, with a mean or typical wave number $k_{0,j}$,

$$\varepsilon_{L,\mu,\nu}(\tau, L_j) = t_{n,\mu,\nu} \cdot \hat{\varepsilon}_{L,\mu,\nu} \exp\left(i \cdot k_{0,j} \cdot x_j - \frac{x_j^2}{4\sigma^2} \right), \quad (9.45)$$

$$\text{with} \;\; t_{n,\mu,\nu} = \frac{1}{(2\pi\sigma^2)^{1/4} (\hat{\varepsilon}_{L,\mu,\nu} \cdot \hat{\varepsilon}^*_{L,\mu,\nu})^{1/2}}, \quad (9.46)$$

a center $L_{c,j}$, off-center differences x_j, and coordinates L_j

$$x_j = L_j - L_{c,j}, \quad (9.47)$$

with a center propagating via

$$L_{c,j} = L_{c,j,ini} + c \cdot \tau, \;\; \text{so that} \quad (9.48)$$

$$ik_{0,j}x_j = ik_{0,j}(L_j - L_{c,j,ini} - c\tau) = ik_{0,j}(L_j - L_{c,j,ini}) - i\omega \cdot \tau, \quad (9.49)$$

the following holds:

(2) These wave packets fulfill the following standard deviation relations:

$$\Delta x_j = \sqrt{Var(x_j)} = \sigma > 0 \quad (9.50)$$

$$\Delta k_j = \sqrt{Var(k_j)} = \frac{1}{2\sigma} \quad (9.51)$$

$$\Delta x_j \cdot \Delta k_j = \frac{1}{2} \tag{9.52}$$

$$\Delta|\vec{x}| \cdot \Delta|\vec{k}| = \frac{D}{2}, \quad \text{in } D-\text{dimensional space} \tag{9.53}$$

$$\Delta|\vec{x}| \cdot \Delta|\vec{k}| = \frac{1}{2}, \quad \text{in a one}-\text{dimensional subspace} \tag{9.54}$$

(3) The fluctuation products in Eqs. (9.53, 9.54) are minimal with respect to all functions:

$$\Delta|\vec{x}_{min.\ fluct.\ product}| \cdot \Delta|\vec{k}_{min.\ fluct.\ product}| = \frac{1}{2}, \quad \text{in 1D subspace} \tag{9.55}$$

Wave theory provides the relation $|\vec{k}| = \frac{2\pi}{\lambda}$*, so that the following fluctuation ratio is minimal:*

$$\frac{\Delta|\vec{x}_{min.\ fluct.}|}{\lambda_{min.\ fluct.}} = \frac{1}{4\pi}, \quad \text{in 1D subspace} \tag{9.56}$$

Moreover, the minimal fluctuation products can be explained geometrically, see Fig. (9.7).

Proof:

Part (1) is a condition, it needs no proof.

Part (2): Firstly, the **variance of** x_j is evaluated:

$$Var(x_j) = \int_{-\infty}^{\infty} \varepsilon_{L,\mu,\nu} \cdot \varepsilon^*_{L,\mu,\nu} \cdot x_j^2 dx_j \quad \text{thus,} \tag{9.57}$$

$$Var(x_j) = \int_{-\infty}^{\infty} |\hat{\varepsilon}_{L,\mu,\nu}|^2 t^2_{n,\mu,\nu} \exp\left(-\frac{x_j^2}{2\sigma^2}\right) x_j^2 dx_j = \sigma^2 \tag{9.58}$$

Secondly, the **variance of** k_j is evaluated. The minimal k_j occurs in the own system, since the momentum $p = \hbar k$ tends to zero in the own system. Thus, the typical $k_{0,j}$ is zero. Moreover, the eigenvalue k_j is generated by the operator $\hat{k}_j = -i\partial_{x_j}$, and the absolute square of $\hat{k}_j \varepsilon_{L,\mu,\nu}$ is integrated:

$$Var(k_j) = \int_{-\infty}^{\infty} |\hat{k}_j \varepsilon_{L,\mu,\nu}|^2 dx_j, \quad \text{thus,} \tag{9.59}$$

$$Var(k_j) = t_{n,\mu,\nu}^2 |\hat{\varepsilon}_{L,\mu,\nu}|^2 \int_{-\infty}^{\infty} \left| -i\partial_{x_j} \exp\left(-\frac{x_j^2}{4\sigma^2}\right)\right|^2 dx_j \tag{9.60}$$

$$Var(k_j) = t_{n,\mu,\nu}^2 |\hat{\varepsilon}_{L,\mu,\nu}|^2 \int_{-\infty}^{\infty} \left| i\frac{x_j}{2\sigma^2} \exp\left(-\frac{x_j^2}{4\sigma^2}\right)\right|^2 dx_j \tag{9.61}$$

$$Var(k_j) = t_{n,\mu,\nu}^2 |\hat{\varepsilon}_{L,\mu,\nu}|^2 \int_{-\infty}^{\infty} \frac{x_j^2}{4\sigma^4} \exp\left(-\frac{x_j^2}{2\sigma^2}\right) dx_j \tag{9.62}$$

$$Var(k_j) = \frac{\sigma^2}{4\sigma^4} = \frac{1}{4\sigma^2} \tag{9.63}$$

Thirdly, the **standard deviations** Δx_j of x_j and Δk_j of k_j as well as their product are evaluated.

$$\Delta x_j = \sqrt{Var(x_j)} = \sigma \quad \text{and} \quad \Delta k_j = \sqrt{Var(k_j)} = \frac{1}{2\sigma} \tag{9.64}$$

$$\Delta x_j \cdot \Delta k_j = \frac{1}{2} \tag{9.65}$$

Fourthly, the standard deviations $\Delta|\vec{x}|$ of $\vec{x}$ and $\Delta|\vec{k}|$ of $\vec{k}$ in a D - dimensional space are evaluated. For the case of a vector $\vec{x}$ in a D - dimensional space, the uncertainties of the D coordinates x_j, with $j = 1, 2, ..., D$, accumulate. Thereby, the standard deviation of the absolute value increases by the factor $\sqrt{D}$, see (Olofsson and Andersson, 2012, PROP 2.5.1):

$$\Delta|\vec{x}| = \Delta x_j \cdot \sqrt{D}, \quad \text{analogously,} \tag{9.66}$$

$$\Delta|\vec{k}| = \Delta k_j \cdot \sqrt{D}, \quad \text{thus,} \tag{9.67}$$

$$\Delta|\vec{x}| \cdot \Delta|\vec{k}| = \frac{D}{2}, \quad \text{in } D - \text{dimensional space} \tag{9.68}$$

$$\Delta|\vec{x}| \cdot \Delta|\vec{k}| = \frac{1}{2}, \quad \text{in a one} - \text{dimensional subspace} \qquad (9.69)$$

These Eqs. are called standard deviation relations of a Gaussian wave packet.

Geometrically, the fluctuations of the vector $\vec{x}$ exhibit rotational invariance with respect to the origin. Thereby, the extension of these fluctuations is characterized by the standard deviation $\Delta|\vec{x}|$ of that vector $\vec{x}$. In this sense, the fluctuations of the vector $\vec{x}$ form a ball with the radius:

$$r = \Delta|\vec{x}| \qquad (9.70)$$

Part (3): With respect to all functions, the minimal fluctuation product in Eqs. (9.53, 9.54) is achieved by Gaussian wave packets, as this product has the minimal possible value according to the Heisenberg uncertainty relation, see e. g. Heisenberg (1927), Sakurai and Napolitano (1994). This completes the proof.

Theorem 22 VPs can be quanta in nature

Minimal energy portions of volume in nature can be quanta with the following properties:

(1) Each VP with a center of energy propagates at $v = c$, according to the DEQ of VD. Consequently, the VP has a circular frequency ω and a corresponding periodic time $T = \frac{2\pi}{\omega}$. As a consequence, at a d_{GP} based distance r from a mass or effective mass, each VP exhibits the gravitational redshift:

$$T_{periodic\,time\,of\,VP}(r) = T_{periodic\,time\,of\,VP}(r_\infty) \cdot \varepsilon_E(r) \qquad (9.71)$$

(2) Each VP with a center of energy propagates at $v_g = c$ (see section 2.2.3), and it exhibits gravitational redshift (item 5). This implies, see THM (18 part 7.1): At each ω, the minimal energy volume portion is quantized:

$$dE_{VP,min}(\omega(r)) = \hbar\omega(r) \qquad (9.72)$$

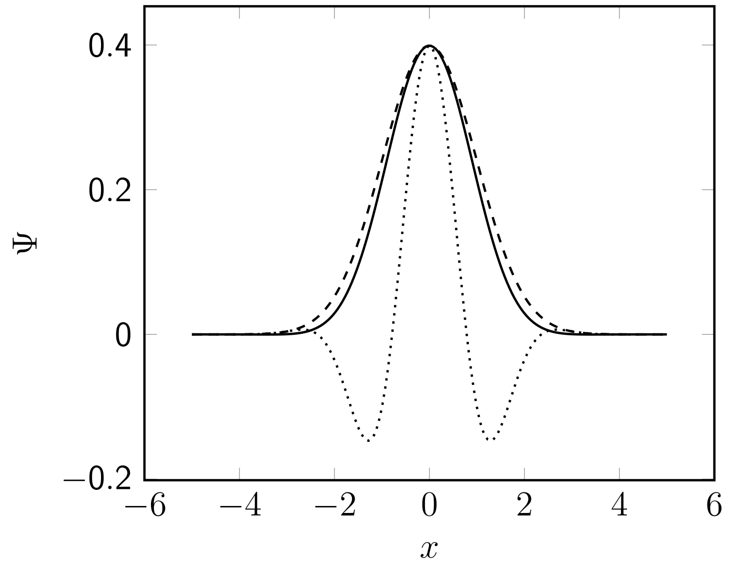

Figure 9.7: Gaussian wave packets $\Psi = \frac{1}{\sqrt{2\pi}\cdot\sigma} \cdot \exp\left[\frac{-x^2}{2\sigma^2}\right] \cdot \cos(k \cdot x)$, see THM (6):
A Gaussian function $\Psi = \frac{1}{\sqrt{2\pi}\cdot\sigma} \cdot \exp\left[\frac{-x^2}{2\sigma^2}\right]$ fluctuates according to $\Delta x = \sigma$. Similarly, a harmonic wave exhibits $k = 2\pi/\lambda$.
A Gaussian wave packet has the minimal fluctuation product at $k = \frac{1}{2\sigma}$ (solid line). In contrast, at a fixed σ, $k < \frac{1}{2\sigma}$ causes a larger λ and a broader maximum, causing a larger fluctuation product (e. g. $k = \frac{1}{4} \cdot \frac{1}{2\sigma}$, dashed). Similarly, $k > \frac{1}{2\sigma}$ causes smaller λ, causing changes of sign of Ψ at relatively large values of Ψ, causing a larger fluctuation product (for instance, $k = 4 \cdot \frac{1}{2\sigma}$, dotted).

(3) In a minimal energy volume portion, the available (see THM 18) energy is proportional to $\frac{1}{\varepsilon_E(r)}$*, see items (5,6):*

$$dE_{VP,min}(\omega(r)) = \hbar\omega(r_\infty) \cdot \frac{1}{\varepsilon_E(r)} \tag{9.73}$$

In a minimal energy volume portion, the volume is proportional to $\frac{1}{\varepsilon_E(r)}$*, see THMs (4, 9, 10):*

$$dV_{L,VP,min}(\omega(r)) = dV_{L,VP,min}(\omega(r_\infty)) \cdot \frac{1}{\varepsilon_E(r)} \tag{9.74}$$

As a consequence, the energy density of a minimal volume portion $u_{vol,E_{min}-VP}$ *does not depend on the gravitational redshift* z_{gr}*. The energy density of each volume portion* u_{vol} *consists of minimal energy volume portions, so that* u_{vol} *does not depend on the gravitational redshift, in general:*

$$u_{vol,E_{min}-VP} = \frac{dE_{VP,min}(\omega(r))}{dV_{L,VP,min}(\omega(r))} = \frac{dE_{VP,min}(\omega(r_\infty))}{dV_{L,VP,min}(\omega(r_\infty))} \tag{9.75}$$

$$u_{vol} \text{ independent of } z_{gr} \tag{9.76}$$

This result has already been derived by different methods in Carmesin (2021d, 2023g).

Proof: Parts (1) - (3) include their derivation.

Theorem 23 Law of the zero - point oscillation

Each VP with a minimal fluctuation product in THM (21) has a radius $r = \Delta x$*, and it has the following momentum and energy:*

(1) Iff a state with minimal fluctuation product, see Eqs. (9.53, 9.54), has minimal spatial and temporal rate of change, then a **minimal fluctuation rate** *is achieved, see Fig. (9.7). Such a state has minimal wave number, minimal circular frequency and maximal wavelength as follows:*

In its own system, the Gaussian wave packet has fluctuations that form a ball with the root mean square (RMS) radius equal to the standard deviation:

$$r = \Delta|\vec{x}| \tag{9.77}$$

In its own system, the Gaussian wave packet has fluctuations that form a <u>ball</u> with the (RMS) <u>radius</u> in Eq. (9.77), formed with the maximal wavelength (as the rate of spatial change is minimal),

$$\lambda_{min\ fluctuation\ rate} = 2\pi \cdot \Delta|\vec{x}| \tag{9.78}$$

with the minimal circular frequency

$$\omega_{min\ fluctuation\ rate} = \frac{c}{\Delta|\vec{x}|} \tag{9.79}$$

and with the minimal wave number:

$$c \cdot k_{min\ fluctuation\ rate,D} = \frac{\omega \cdot D}{2}, \quad \text{in } D - \text{dimensional space} \tag{9.80}$$

$$c \cdot k_{min\ fluctuation\ rate,D=1} = \frac{\omega}{2}, \quad \text{in a one} - \text{dimensional subspace} \tag{9.81}$$

(2) The energy at a minimal fluctuation rate is achieved by multiplication of Eq. (9.81) by the constant of universal quantization, see THM (18):

$$c \cdot \hbar \cdot k_{min\ fluctuation\ rate,D=1} = \frac{\hbar \cdot \omega}{2} = c \cdot p_{min\ fluctuation\ rate,D=1} \tag{9.82}$$

We apply $E = p \cdot c$:

$$\frac{\hbar \cdot \omega}{2} = E_{min\ fluctuation\ rate,D=1} \tag{9.83}$$

This energy is identified with a kinetic energy, as the states in universal quantization represent a kinetic energy. This energy is identified with a zero-point energy, ZPE, as it is at the minimal fluctuation rate at a single mode.

Proof:

Part (1): Modes with the **minimal wave number** $k_{min\ fluctuation\ rate}$ or **minimal circular frequency** $\omega_{min\ fluctuation\ rate} = k_{min\ fluctuation\ rate} c$ provide the minimal rate of spatial and temporal change:

Fluctuations can be characterized by waves with wavelengths λ and circular frequencies ω:

$$\omega = \frac{2\pi c}{\lambda} \tag{9.84}$$

At a ball with a radius r, the wave mode with the lowest k is the mode with the longest wavelength λ, see e. g. Jackson (1975). It corresponds to the spherical harmonic function $Y_1^0(\vartheta, \varphi) = \sqrt{\frac{3}{4\pi}} \cos \vartheta$. It is the mode with $\lambda = 2\pi r$. Thus, the fluctuation mode with the longest wavelength is the fluctuation mode with λ equal to the circumference $2\pi r$ of the ball in Eq. (9.70):

$$\lambda_{min\ fluctuation\ rate} = 2\pi \cdot \Delta|\vec{x}| \tag{9.85}$$

The average - fluctuation with this wavelength has the minimal circular frequency $\omega_{min\ fluctuation\ rate}$ and the minimal $k_{min\ fluctuation\ rate}$:

$$\omega_{min\ fluctuation\ rate} = \frac{c}{\Delta|\vec{x}|} \tag{9.86}$$

$$c \cdot k_{min\ fluctuation\ rate,D} = c \cdot k = c \cdot \Delta|\vec{k}| \tag{9.87}$$

The standard deviation relation in Eq. (9.68) yields:

$$c \cdot k_{min\ fluctuation\ rate,D} = c \cdot \frac{D}{2\Delta|\vec{x}|} \tag{9.88}$$

The minimal circular frequency in Eq. (9.86) implies:

$$c \cdot k_{min\ fluctuation\ rate,D} = \frac{\omega_{min\ fluctuation\ rate} \cdot D}{2} \tag{9.89}$$

The fluctuations in orthogonal one - dimensional subsystems are statistically independent. Thus, their minimal standard deviations $k_{min\ fluctuation\ rate,D=1}$ add up. Thus, the minimal k in such a one - dimensional subsystem is $k_{min\ fluctuation\ rate,D}$ in Eq. (9.86) divided by D:

$$c \cdot k_{min\ fluctuation\ rate,D=1} = \frac{\omega_{min}}{2} \tag{9.90}$$

Part (2): This part includes its derivation. This completes the proof of the theorem.

Theorem 24 Law of the absolute minimum of fluctuation

The VPs in THM (23) with the minimal fluctuation products $\Delta x_{j,min}(\omega)$ as a function of ω in THM (21) can be minimized further by variation of ω. The resulting absolute minimum of the standard deviation is named $\Delta x_{j,min,abs}$. It can be derived with help of the universal quantization of light - portions in THM (18) combined with gravity in THMs (9, 10), and it is the Planck length L_P:

$$\Delta x_{j,min,abs} = L_P = \sqrt{\frac{\hbar G}{c^3}} \tag{9.91}$$

With it, the quantization ratio $K(\omega)$ of VPs can be derived. It is the same as the universal constant of quantization:

$$K(\omega) = \hbar \tag{9.92}$$

Proof:

We analyze a Gaussian wave packet in a one-dimensional system with direction $\vec{e}_j$.

Part (1): Analysis of Δp_j:

For each VP with an energy E, momentum p, circular frequency ω and wave vector k holds $E = p_j c$ and $\omega = k_j c$, so that

$$\frac{E}{p_j} = c = \frac{\omega}{k_j} \tag{9.93}$$

This relation implies:

$$\frac{E}{\omega} = c = \frac{p_j}{k_j} \tag{9.94}$$

The above ratio $\frac{E}{\omega}$ is identified with the ratio $K(\omega)$ in THM (18), and the above Eq. is solved for p:

$$p_j = K(\omega) \cdot k_j \tag{9.95}$$

The standard deviation is applied:

$$\Delta p_j = K(\omega) \cdot \Delta k_j \tag{9.96}$$

The standard deviation in part (1) is used:

$$\Delta p_j = K(\omega) \cdot \frac{1}{2 \cdot \Delta x_j} \tag{9.97}$$

Part (2): The smallest possible standard deviation Δx_j is analyzed:

For it, the energy is analyzed. For it, Eq. (9.96) is multiplied by c:

$$\Delta E = K(\omega) \cdot \frac{c}{2 \cdot \Delta x_j} \tag{9.98}$$

When Δx_j is decreased, the ΔE is increased. This process comes to an end, when the energy of a black hole is reached. Thereby, the Schwarzschild radius marks the x_j coordinate in the considered direction:

$$R_S = \frac{2GE}{c^4} = x_j \tag{9.99}$$

The above Eq. is solved for E, and the standard deviation is applied:

$$\Delta E = \Delta x_j \cdot \frac{c^4}{2G} \tag{9.100}$$

As a consequence, at the absolute minimum $\Delta x_{j,min,abs}$ is described by Eqs. (9.98) and (9.100), so that these terms are equal:

$$K(\omega) \cdot \frac{c}{2 \cdot \Delta x_{j,min,abs}} = \Delta x_{j,min,abs} \cdot \frac{c^4}{2G} \tag{9.101}$$

The above Eq. is solved for $K(\omega)$:

$$K(\omega) = \Delta x_{j,min,abs}^2 \cdot \frac{c^3}{G} \tag{9.102}$$

Based on universal quantization of light - portions in THM (18), and on gravity in THMs (9, 10), the Planck length has been derived, see e. g. Carmesin (2019b, 2021d,a):

$$\Delta x_{j,min,abs} = L_P = \sqrt{\frac{\hbar G}{c^3}} \tag{9.103}$$

With it, the ratio $K(\omega)$ in Eq. (9.102) is derived:

$$K(\omega) = \frac{\hbar G}{c^3} \cdot \frac{c^3}{G} = \hbar \tag{9.104}$$

As a consequence, the VPs have the absolute minimum of the standard deviation in Eq. (9.103), and the ratio of quantization in VPs is the same as the universal constant of quantization. This completes the proof of the theorem.

9.5 On the self - interaction of a quantum

In this section, we analyze the self - interaction of a quantum. Moreover, we compare its energy density u_{field} with its kinetic energy density u_{kin}.

Observable energy:

In universal quantization in THM (18), the observable energy is the available energy $E_{av} = p \cdot c$.

For the purpose of a clear analysis, we analyze a quantum at a far distance from any field generating mass or charge. This analysis is without loss of generality, as the effect of a field generating mass or charge can be investigated later.

In such a quantum without external field, the observable energy is the kinetic energy, see THM (18). As a consequence, each quantum has a kinetic energy density $u_{kin} = \frac{c^2 \cdot \dot{\varepsilon}_L^2(\vec{L})}{4\pi G} > 0$, see THM (16).

Attractive self - interaction:

Each quantum has a gravitational self - interaction with a field $\vec{G}^*(\vec{L})$. As a consequence, each quantum has an energy density $u_{field} = \frac{(\vec{G}^*)^2}{4\pi G}(\vec{L}) < 0$, see THM (11). Since the self - interaction is attractive, it contributes to a stabilization of each quantum.

Balanced self - interaction:

In each quantum and at each location $\vec{L}$, the kinetic energy density exactly compensates the field energy density $u_{kin}(\vec{L}) = -u_{field}(\vec{L})$, see THMs (11, 16). As a consequence, the complete energy of each quantum is zero at each location $\vec{L}$:

$$u_{complete}(\vec{L}) := u_{kin}(\vec{L}) + u_{field}(\vec{L}) \tag{9.105}$$

Balanced self - interaction enables nonlocality: If the absolute value of the field energy would exceed the kinetic energy, then the quantum would form a bound state. As a consequence, the wave function Ψ of the quantum could not split in a beam splitter. Consequently, the universal nonlocality would not be possible. As a further consequence, the quantum could not exhibit its full properties of nonlocality. As consequence of the above, for each quantum, the field energy does not exceed the kinetic energy:

$$u_{kin}(\vec{L}) \geq |u_{field}(\vec{L})| \tag{9.106}$$

Balanced self - interaction enables stability: Quanta are very stable. For instance, the quanta of light emitted by the star Vega at a distance of 26 light years or 26 years ago, arrive at Earth with their precise spectral properties, (Karttunen et al., 2007, p. 210, p. 456), and with their abilities of interference and of energy quantization, see e. g. Carmesin et al. (2020).

If the absolute value of the kinetic energy would exceed the field energy, then parts of the quantum would propagate apart. This would be similar to a nonperiodic comet that leaves the solar system, see e. g. comet 2I/Borisov, (Carmesin et al., 2021, p. 110) - note that the self - interaction of quanta is very different from the external field of the sun that enables the binding of a comet, such as Halley's comet, see Karttunen et al. (2007).

However, according to universal quantization (THM 18), a quantum at a circular frequency ω has the invariant energy $E = \hbar\omega$, so that a quantum does not loose any energy. Consequently, the binding by the self - interaction must be sufficiently large.

As a consequence, for each quantum, the energy densities must fulfill the following relation:

$$u_{kin}(\vec{L}) \leq |u_{field}(\vec{L})| \tag{9.107}$$

Necessary balanced self - interaction: Altogether, the stability and nonlocality of quanta imply Eqs. (9.106 and 9.107). These imply a fully balanced self interaction:

$$u_{kin}(\vec{L}) = |u_{field}(\vec{L})| \tag{9.108}$$

More generally, such self - interaction of quanta includes generalized potentials, fields and charges, see section (7.4). This balanced self - interaction is provided by the VD. We summarize our results as follows:

Theorem 25 Law of self - interaction of quanta

Each quantum has self - interaction with the following properties:

(1) At each location $\vec{L}$, the self - interaction provides a binding energy with an energy density $u_{field}(\vec{L}) < 0$, and the field energy is compensated by the kinetic energy:

$$u_{kin}(\vec{L}) = |u_{field}(\vec{L})| \tag{9.109}$$

This binding energy enables the simultaneous stability and nonlocality of each quantum.

(2) The observable energy of a quantum is its kinetic energy, see THM (18).

(3) A quantum can be transformed to another quantum, or absorbed by another quantum. For instance, in the photo - effect, a photon can be absorbed by an electron. In such a transformation, the available energy E_{av} is observed. The energy density u_{field} of the self - interaction is not observed, since u_{field} is inherent to the transformed or absorbing quantum.

(4) We will show that quanta can also be described with ladder operators and number operators, see THMs (34, 35). As a consequence, quanta can form and vanish, so that a many quantum system occurs in general, including quantum fluctuations such as derived in THMs (21, 22, 23, 24). In such many quantum systems, the compensation in Eq. (9.109) remains fundamental, whereby observable kinetic energies can be observed in appropriate measurement devices, see e. g. Casimir (1948), Klimchitskaya et al. (2009), or in appropriate systems, see e. g. Lamb and Retherford (1947), Dyck et al. (1987).

Chapter 10

VD implies time evolution of quanta

10.1 Generalized Schrödinger Eq., GSEQ

For each localizable quantum object at $v = c$, the criterion in THM (18) holds. Additionally, the DEQ of VD in THM (5) holds. It is informative and utile, to combine these two key results as follows:

Theorem 26 Law of the generalized Schrödinger Eq.

For each localizable quantum object at $v = c$, criterion (11) in THM (18) is fulfilled, so that the following holds:

(1) The object exhibits universal quantization in THM (18).

(2) The object fulfills the DEQ of VD in THM (5).

(3) The following generalized Schrödinger equation, holds:

$$i\hbar\frac{\partial}{\partial\tau}\Psi = c\cdot\hat{\vec{p}}\cdot\vec{e}_v\Psi = \hat{H}\Psi \quad \text{or} \tag{10.1}$$

$$i\hbar\frac{\partial}{\partial\tau}\Psi = \hat{E}\Psi \tag{10.2}$$

Hereby, the energy operator $c\cdot\hat{\vec{p}}\cdot\vec{e}_v = \hat{E}$ is also called Hamilton operator $\hat{H}$.

(4) The traditional wave function is provided by the unidirectional rate in a direction r or j as follows:

$$\Psi = t_n \cdot \dot{\varepsilon}_{L,rr} \tag{10.3}$$

In Eqs. (10.3 and 10.4), t_n is a normalization factor.

(5) **Generalization:** *In general, each polarization p in the DEQ of VD in THM (5) provides a corresponding wave function as follows:*

$$\Psi_p = t_n \cdot \dot{\varepsilon}_{L,p} \tag{10.4}$$

Proof:

Parts (1) and (2) include their derivations.

Parts (3) and (4): The DEQ of VD is used for the polarization $p = rr$. Thus, the direction vector $\vec{e}_v$ points to direction of r and of L. Moreover, $v = c$ is inserted, and the time derivative is applied:

$$\frac{\partial}{\partial \tau}\dot{\varepsilon}_{L,rr} = -c \cdot \vec{e}_L \cdot \frac{\partial}{\partial \vec{L}}\dot{\varepsilon}_{L,rr} \tag{10.5}$$

In an equivalence transformation of the above Eq., the factor $i\hbar$ is multiplied. This is also physically founded, as the universal quantization applies, see part (1):

$$i\hbar \cdot \frac{\partial}{\partial \tau}\dot{\varepsilon}_{L,rr} = c \cdot \vec{e}_L \cdot \left[-i\hbar \cdot \frac{\partial}{\partial \vec{L}}\right]\dot{\varepsilon}_{L,rr} \tag{10.6}$$

The rectangular bracket is identified with the momentum operator, see THM (18):

$$i\hbar \cdot \frac{\partial}{\partial \tau}\dot{\varepsilon}_{L,rr} = \vec{e}_L \cdot c \cdot \hat{\vec{p}} \cdot \dot{\varepsilon}_{L,rr} \tag{10.7}$$

The rate is normalized by a factor t_n, and the normalized rate is called wave function Ψ:

$$i\hbar \cdot \frac{\partial}{\partial \tau}\Psi = c \cdot \vec{e}_L \cdot \hat{\vec{p}} \cdot \Psi \tag{10.8}$$

The product $\vec{e}_L \cdot \hat{\vec{p}}$ is the operator of the absolute value of the momentum $\hat{p}$:

$$i\hbar \cdot \frac{\partial}{\partial \tau}\Psi = c \cdot \hat{p} \cdot \Psi \tag{10.9}$$

The product of c and the momentum operator is the energy operator, see THM (18):

$$i\hbar \cdot \frac{\partial}{\partial \tau}\Psi = \hat{E} \cdot \Psi \tag{10.10}$$

Part (5): Each polarization p describes a change tensor. For instance, a change tensor of rank two is written as follows $\varepsilon_{L,i,j}$. For each such change tensor, the parts (1) to (4) can be shown in the same manner as above. q. e. d.

Interpretation: As the appropriate wave function Ψ is proportional to the derivative $\dot{\varepsilon}_{L,p}$ of the relative additional volume, $\varepsilon_{L,p}$, the volume and its change tensors represent the underlying structure of the wave function. Conversely, the wave function represents the rate $\dot{\varepsilon}_{L,p}$, which contains less information than relative additional volume, $\varepsilon_{L,p}$.

10.2 Original Schrödinger Equation, SEQ

The DEQ of VD in THM (5) implies the generalized Schrödinger Eq., GSEQ in THM (26). It holds for objects that propagate at $v = c$. The particular case of objects with $v < c$ is derived from the GSEQ with help of a non-relativistic power series with respect to v/c. The result is useful, as many essential objects have nonzero rest mass, corresponding to $v < c$. The result is instructive, as it shows how objects at $v < c$ are dynamically related to objects at $v = c$.

Theorem 27 Law of the derived original Schrödinger equation

For each localizable quantum object at $v \leq c$*, THM (26) implies the following:*

(1) For the case of **ultrafast objects***, with* $p^2c^2 >> m_0^2c^4$*, see e. g. (Workman et al., 2022, Eq. 14.38), an energy eigenvalue is obtained by the following linear approximation of the energy momentum relation* $E = \sqrt{p^2c^2 + m_0^2c^4}$ *with respect to the small ratio* $\frac{m_0^2c^4}{p^2 \cdot c^2}$*:*

$$E \doteq p \cdot c + \frac{m_0^2c^4}{2 \cdot p \cdot c} \tag{10.11}$$

The corresponding linear approximation of the GSEQ in Eq. (10.2) is obtained by replacing the eigenvalue p *by its operator* $\hat{p}$*:*

$$i\hbar\frac{\partial}{\partial\tau}\Psi \doteq \left(\hat{p} \cdot c + \frac{m_0^2c^4}{2 \cdot \hat{p} \cdot c}\right) \cdot \Psi \tag{10.12}$$

(2) For the case of **slow objects***, with* $p^2c^2 << m_0^2c^4 =: E_0^2$*, the following holds:*

(2a) An energy eigenvalue of the energy is obtained by the following linear approximation of the energy momentum relation $E = \sqrt{p^2c^2 + m_0^2c^4}$ *with respect to the small ratio* $\frac{p^2 \cdot c^2}{E_0^2}$*:*

$$E \doteq E_0 + \frac{p^2}{2 \cdot m_0} \tag{10.13}$$

(2b) The corresponding linear approximation of the GSEQ in Eq. (10.2) is obtained by replacing the eigenvalue p *by its operator* $\hat{p}$*.*

$$i\hbar\frac{\partial}{\partial\tau}\Psi_{E_0} \doteq \left(E_0 + \frac{\hat{p}^2}{2 \cdot m_0}\right) \cdot \Psi_{E_0} \tag{10.14}$$

Hereby, the wave function includes the rest energy E_0*. Accordingly, the wave function is named* Ψ_{E_0}*. In general, the following* *squares are equal:*

$$\hat{p}^2 = \hat{\vec{p}}^{\,2} \tag{10.15}$$

(2c) The wave function is factorized:

$$\Psi_{E_0} = \Psi \cdot \exp\left(\frac{E_0 \tau}{i\hbar}\right) \tag{10.16}$$

The left hand side of Eq. (10.14) is evaluated with the product rule:

$$E_0 \Psi_{E_0} + \exp\left(\frac{E_0 \tau}{i\hbar}\right) i\hbar \frac{\partial \Psi}{\partial \tau} \doteq \left(E_0 + \frac{\hat{p}^2}{2m_0}\right) \cdot \Psi_{E_0} \tag{10.17}$$

(2d) In the above DEQ, $E_0 \Psi_{E_0}$ is subtracted. The resulting DEQ is divided by $\exp\left(\frac{E_0 \tau}{i\hbar}\right)$. As a consequence, the SEQ proposed or postulated by Schrödinger is derived:

$$i\hbar \frac{\partial}{\partial \tau} \Psi \doteq \frac{\hat{p}^2}{2 \cdot m_0} \cdot \Psi = \hat{H} \Psi \tag{10.18}$$

Additionally, $\hat{H}$ can include a potential energy:

$$i\hbar \frac{\partial}{\partial \tau} \Psi \doteq \frac{\hat{p}^2}{2 \cdot m_0} \Psi + E_{pot} \Psi = \hat{H} \Psi \tag{10.19}$$

Proof: Each part includes its proof. q. e. d.

10.3 Stationary solution of the GSEQ

We derived stationary solutions of the DEQ of VD in THMs (21, 22, 23, 24). In this section, we derive similar stationary solutions for the GSEQ.

Theorem 28 Law of a stationary solution of the GSEQ

(1) A Gaussian wave packet $\Psi(\tau, x_j)$ that extends in a one - dimensional subspace with a coordinate x_j, with a characteristic

wave vector $k_{0,j}$, with the term

$$\Psi(\tau, x_j) = t_n \cdot \exp\left(i \cdot k_{0,j} \cdot x_j - \frac{x_j^2}{4\sigma^2}\right), \tag{10.20}$$

with the normalization factor, corresponding to the normalization $1 = \int_{-\infty}^{-\infty} \Psi \cdot \Psi^* dx_j$,

$$t_n = \frac{1}{(2\pi\sigma^2)^{1/4}}, \tag{10.21}$$

with a center $L_{c,j}$, with an off-center difference x_j

$$x_j = L_j - L_{c,j} \tag{10.22}$$

and with a center propagation

$$L_{c,j} = L_{c,j,ini} + c \cdot \tau, \tag{10.23}$$

has the following properties:

(1) The wave packet solves the GSEQ (10.1).

(2) In particular, the Gaussian wave function $\Psi(\tau, L)$ does not broaden as a function of time. As a consequence, the Gaussian wave packet is a stationary solution of the GSEQ. In contrast, in the original SEQ, a Gaussian wave packet broadens as a function of time, see e. g. Scheck (2013), Greiner (1979). The reason is that the dispersion relation is $\omega = k \cdot c$ in the case of the GSEQ, but this is not the case in the original SEQ.

(3) **Generalization:** *More generally, for each polarization p, there are stationary solutions $\dot{\varepsilon}_{L,p}$ of the GSEQ in THM (26) in the form of such Gaussian wave packets or of Gaussian wave packets in a D-dimensional space $\dot{\varepsilon}_{L,p}(\tau, \vec{x})$.*

Proof: Part (1): The solution is confirmed by inserting into the GSEQ. This is done analogously as in THM (6). The normalization is confirmed by integrating the absolute square $|\Psi|^2$. This is done analogously as in THM (6). q. e. d.

Parts (2) and (3): These parts are shown analogously as in THM (6). q. e. d.

10.4 Universal minimal energy solution

Theorem 29 Law of universal minimal energy solutions
(1) The Gaussian wave packets Ψ in THM (28) fulfill the uncertainty relations of a Gaussian wave packet:

$$\Delta|\vec{x}| \cdot \Delta|\vec{p}| = \frac{\hbar \cdot D}{2}, \quad \text{in } D - \text{dimensional space} \tag{10.24}$$

$$\Delta|\vec{x}| \cdot \Delta|\vec{p}| = \frac{\hbar}{2}, \quad \text{in a one} - \text{dimensional subspace} \tag{10.25}$$

(2) In its own system, the Gaussian wave packet has fluctuations that form a ball with the root mean square (RMS) radius equal to the standard deviation or uncertainty:

$$r = \Delta|\vec{x}| \tag{10.26}$$

(3) In its own system, the Gaussian wave packet has fluctuations that form a ball with the (RMS) radius in part (2), formed with the maximal wavelength,

$$\lambda_{max} = 2\pi \cdot \Delta|\vec{x}| \tag{10.27}$$

with the minimal circular frequency

$$\omega_{min} = \frac{c}{\Delta|\vec{x}|} \tag{10.28}$$

and with the corresponding observable and kinetic and minimal energy of universal quantization, see THM (18):

$$E_{kin,min,D} = \frac{\hbar \cdot \omega \cdot D}{2}, \quad \text{in } D - \text{dimensional space} \tag{10.29}$$

$$E_{kin,min,D=1} = \frac{\hbar\omega}{2}, \quad \text{in a one} - \text{dimensional subspace} \tag{10.30}$$

(4) **Generalization:** *More generally, for each polarization p, there are stationary minimal energy solutions $\dot{\varepsilon}_{L,p}$ of the GSEQ in THM (26) in the form of such Gaussian wave packets or of Gaussian wave packets in a D-dimensional space $\dot{\varepsilon}_{L,p}(\tau, \vec{x})$.*

Proof: The proof is implied by THMs (21, 22, 23, 24).

10.5 Derived principle of least action

Action: A physical system or object is described by the SEQ:

$$i\hbar\frac{\partial}{\partial\tau}\Psi = \hat{H}\Psi \tag{10.31}$$

Each solution Ψ of the SEQ has the following time evolution:

$$\Psi(\tau) = \exp\left(\frac{i}{\hbar}\cdot(-)\hat{H}\cdot\tau\right)\Psi_0 \tag{10.32}$$

Hereby, Ψ_0 is the initial value of the solution:

$$\Psi(\tau_0) = \Psi_0 \tag{10.33}$$

Thereby, the term $(-)\hat{H}\tau$ is an action:

$$\hat{S} = \cdot(-)\hat{H}\cdot\tau \tag{10.34}$$

With it, the solution in Eq. (10.32) takes the following form:

$$\Psi(\tau) = \exp\left(\frac{i}{\hbar}\hat{S}\right)\Psi_0 \tag{10.35}$$

Paths from A to B: It is convenient to analyze a process in space (or phase space), in which the system starts at a point A and propagates to a point B. In quantum physics, QP, the system is not restricted to one of these paths. Consequently, the following consideration makes sense in a semiclassical limit only, see e. g. (Landau and Lifschitz, 1971, paragraph 6).

Paths in a semiclassical limit: A semiclassical limit can be described as a limit $\hbar$ to zero. Thereby, the action in Eq. (10.35) divided by $\hbar$ describes a real phase φ in the exponential, as the Hamilton operator has real eigenvalues only:

$$\varphi = \frac{S}{\hbar} \tag{10.36}$$

Thereby, in a semiclassical limit, a wave packet (see Carmesin (2023g)) is considered, and the operator $\hat{S}$ is replaced by the corresponding eigenvalues or values, see e. g. (Landau and Lifschitz, 1971, paragraph 6). In principle, the paths corresponding to the same wave packet correspond to amplitudes $\exp\left(\frac{i}{\hbar}S\right)$. These can cancel out. In the limit $\hbar$ to zero, this cancellation can be minimized, if the phase in Eq. (10.36) is minimized. Correspondingly, the action is minimized. Accordingly, such a semiclassical limit provides the principle of least action, PLA or stationary action, see e. g. (Landau and Lifschitz, 1971, paragraph 6).

10.6 A path to the Einstein field equation

Einstein (1915) proposed the DEQ of GR, the Einstein field equation, EFE.

Great success of the EFE: On one hand, the EFE describes essential features of spacetime, for instance curvature of spacetime, gravitational waves and a global expansion of space.

High generality of the VD: On the other hand, the EFE does not describe quanta. However, the VD describes quanta, gravity and curvature of spacetime. Accordingly, we will derive a limiting case of the VD that provides the EFE. This limiting case is achieved by the following steps:

(1) Averaged volume portions describe the metric tensor: In the VD, the states are described by propagating volume portions, see THM (3). In GR, the states can be described by metric tensors. Other tensors, such as the curvature tensor, can be calculated from the metric tensor, see e. g. Hobson et al. (2006), Landau and Lifschitz (1971).

Volume portions include the metric tensors as an average. For instance, in the vicinity of a mass M, the relative additional volume $\varepsilon_{L,jj}$ can be regarded as an average of volume portions. Moreover, that average $\varepsilon_{L,jj}$ provides the position factor $\varepsilon_E = 1 - \varepsilon_{L,jj}$. Furthermore, the position factor provides the metric tensor, see THM (10). Accordingly, in a first limitation, only an average of the volume portions is used, and it provides the metric tensor.

(2) Symmetric change tensors describe the metric tensors: In VD, symmetric & antisymmetric tensors are included. In contrast, in GR, states are described by the symmetric metric tensors. As a consequence, in a second limitation, only symmetric change tensors are used in order to describe GR.

(3) The VD includes nonlocal physics. In order to provide the EFE, a restriction to local physics is applied to the VD: The VD includes quanta. As a consequence, the VD includes nonlocal physics. In a third limitation, the VD is restricted to local physics, in order to provide the EFE. With that restriction, the methods of the local QFT can be applied. These methods include the use of an action S.

(4) Only the metric tensor and its first and second derivative are used in the action: The EFE includes an additional restriction: The EFE can be derived from an action S. In a fourth limitation, the most simple action is used that includes the metric tensor g_{ij} and first and second derivatives of g_{ij}. That action is the Einstein-Hilbert action S_{EH}, see (Hobson et al., 2006, section 19.8). Accordingly, this action is used.

(5) The semiclassical limit provides the EFE: In a fifth limitation, the semiclassical limit, the universal quantization constant $\hbar$ tends to zero, see section (10.5). As a consequence, the principle of least action, PLA, holds. The EFE can be derived by application of the PLA to the S_{EH}, see (Hobson et al., 2006, section 19.8).

Altogether, the limiting case of the VD, that is described by the above steps (1) to (5), provides the EFE. We summarize:

Theorem 30 The VD implies EFE as a limiting case:

(1) The VD implies the Einstein (1915) field equation, EFE, as a limiting case consisting of the following five limitations described above:

(1.1) Averaged volume portions describe the metric tensor. In order to provide the EFE, a restriction to these average values is applied to the VD.

(1.2) Symmetric change tensors describe the metric tensors. In order to provide the EFE, a restriction to symmetric tensors is applied to the VD.

(1.3) The VD includes nonlocal physics. In order to provide the EFE, a restriction to local physics is applied to the VD.

(1.4) Only the metric tensor and its first and second derivative are used in the action, in order to derive the EFE.

(1.5) The semiclassical limit is applied, in order to derive the EFE.

(2) The above five limitations or restrictions (1.1) to (1.5) are sufficient, in order to derive the EFE from the VD.

(3) The above five limitations or restrictions (1.1) to (1.5) are necessary, in order to derive the EFE from the VD. The reason is that the lack of one of these limitations or restrictions would provide a more general description of nature than that provided by the EFE.

(4) Altogether, the above five limitations or restrictions (1.1) to (1.5) are **necessary and sufficient**, *in order to derive the EFE from the VD.*

10.7 Periodicity φ_{per} of a spin 2 wave function

Proposition 4 Periodicity of a spin 2 wave function

(1) If a wave function $\Psi(\tau)$ of an object with Spin S, and with an observed component S_z, is rotated by an angle $\varphi = \omega \cdot \tau$, then $\Psi(\tau)$ is the following function of φ:

$$\Psi(\tau) = \exp\left(-i \cdot \frac{S_z}{\hbar} \cdot \varphi\right) \cdot \Psi_0 \;\text{ with }\; \Psi_0 = \Psi(\tau_0) \qquad (10.37)$$

(2) The periodicity is $\varphi_{per} = \frac{2\pi\hbar}{S_z}$. In particular, $\varphi_{per}(S_z = 2\hbar) = \pi$, or $\varphi_{per} = \pi$ for spin 2:

$$\Psi(\tau \cdot \omega = \varphi_{per}) = \Psi_0, \text{ with } \varphi_{per} = \frac{2\pi\hbar}{S_z}, \qquad (10.38)$$

Proof: Part (1): For the prototypical case of an electron, the Hamiltonian or energy is, see (Sakurai and Napolitano, 1994, section 3.2):

$$H = -\frac{e}{m_e}\vec{B} \cdot \vec{S} = \omega \cdot S_z \;\text{ with }\; \omega = \frac{|e| \cdot B}{m_e} \qquad (10.39)$$

In the SEQ or GSEQ, the following time evolution of the wave function is confirmed by insertion into these DEQs:

$$\Psi(\tau) = \exp\left(\frac{-i}{\hbar}\hat{H} \cdot \tau\right) \cdot \Psi_0 \qquad (10.40)$$

Insertion of H in Eq. (10.39) into the abve Eq. implies Eq. (10.37) shows part (1).

Part (2): When the exponent $-i \cdot \frac{S_z}{\hbar} \cdot \varphi$ in Eq. (10.37) is equal to $-2\pi \cdot i$, then the periodicity φ_{per} is achieved. These two Eqs. are set equal and solved for the angle. As a consequence, the periodicity $\varphi_{per} = 2\pi \cdot \frac{h}{S_z}$ in Eq. (10.38) is derived. q. e. d.

Chapter 11

Derivation of quantum postulates

In this chapter, we derive the postulates of quantum physics[1]. For it, we use the evident properties of volume in nature, see chapter (4), and we apply consequences of these evident properties, see Fig. (1.4).

This derivation is informative helpful, as the postulates become founded and explained rules. Moreover, this is very valuable, as fundamental problems are solved with help of the more general VD - and the solutions are related to QP. Furthermore, this is very useful, as the derivation provides generalizations of QP, additionally.

The derived postulates[2] include the five postulates presented by Kumar (2018) as well as the postulate about mixed states in (Ballentine, 1998, p. 46). We begin with the postulate about the time evolution.

11.1 Time evolution

In this section, we derive the postulate about the time evolution, (Kumar, 2018, p. 170):

[1]Earlier derivations, see Carmesin (2022d), Carmesin (2022a), Carmesin (2022f), include the FDA at some points. In this book, we provide an exact derivation.

[2]Hilbert et al. (1928) proposed an early system of postulates, for instance.

Postulate 1 Time evolution

'The time evolution of the state vector is governed by the time-dependent Schrödinger equation, SEQ, see (Schrödinger, 1926d, Eq. (4")):

$$i\hbar\partial_t|\psi\rangle = \hat{H}|\psi\rangle, \tag{11.1}$$

where $\hat{H}$ is the Hamilton operator corresponding to the total energy of the system.'

Hereby, the wave function $|\psi\rangle$ represents a complex valued function. Thus, the complex conjugate function fulfills the DEQ with an opposite sign, see (Schrödinger, 1926d, p. 112):

$$-i\hbar\partial_t|\psi\rangle^* = \hat{H}|\psi\rangle^*, \tag{11.2}$$

Thus, in quantum physics, the sign in the SEQ is a convention. Thence, more adequately, the square of the SEQ holds, this corresponds to the Lorentz invariant form of the DEQ of VD in THM (5).

Derivation: This postulate has been derived from the SEQ in THM (27), whereby the SEQ has been direved from the DEQ of VD in THM (5).

11.2 Hilbert space

In this section, we derive the following postulate about Hilbert space (Kumar, 2018, p. 168):

Postulate 2 Hilbert space

'The state of a quantum mechanical system, at a given instant of time, is described by a vector $|\Psi(t)\rangle$, in the abstract Hilbert space $\mathcal{H}$ of the system.'

Derivation: In general, volume and matter propagate according to the GSEQ. In the case of relatively small momentum,

$\frac{p^2}{m_0^2c^2} << 1$, in leading order in $\frac{p^2}{m_0^2c^2}1$, the GSEQ becomes the SEQ. In that case, objects propagate according to the SEQ, see THMs (5, 26, 27). So, volume and matter propagate according to a linear DEQ. The time derivative of a solution of that DEQ, $\dot{\varepsilon}(\tau, \vec{L})$, is used as a wave function Ψ, see section (11.1). In the Dirac notation, a wave function Ψ is named a state $|\Psi\rangle$.

Thereby, two wave functions, $\Psi_1(\tau, \vec{L})$ and $\Psi_2(\tau, \vec{L})$, form a scalar product as follows:

$$\langle \Psi_1 | \Psi_2 \rangle = \int d^3 L \Psi_1^*(\tau, \vec{L}) \cdot \Psi_2(\tau, \vec{L}) \tag{11.3}$$

Hereby, the superscript $*$ marks the complex conjugate value, this is nowadays usual in quantum physics, see e. g. Griffiths (1994), Ballentine (1998), Scheck (2013), Kumar (2018).

Mathematically, in principle, many scalar products are available, see Lay et al. (2016). In general, wave functions can have complex values. Accordingly, the scalar product in Eq. (11.3) is adequate.

Based on that scalar product, a state $|\Psi\rangle$ is multiplied by a normalization factor t_n so that the following scalar product is equal to one:

$$\langle \Psi \cdot t_n | \Psi \cdot t_n \rangle = \int d^3 r \Psi^*(\vec{r}, t) \cdot \Psi(\vec{r}, t) \cdot |t_n|^2 = 1 \tag{11.4}$$

Next, we show that these states form a Hilbert space $\mathcal{H}$:

The states $\Psi(\vec{r}, t)$ form a **complete vector space**, as they are solutions of the (linear DEQ) SEQ. As a consequence, they form a linear vector space, whereby they include all linear combinations of states $\Psi(\vec{r}, t)$, including Fourier integrals, for instance. These form a complete Hilbert space $\mathcal{H}$, see e. g. (Teschl, 2014, p. 47) or (Sakurai and Napolitano, 1994, p. 57).

Generalization: The above derivation holds as well for the GSEQ, including rates of change tensors $\dot{\varepsilon}_{L,p} \cdot t_n$. Consequently, such wave functions form a Hilbert space as well.

Altogether, the solutions of the DEQ of volume, radiation or matter form a Hilbert space $\mathcal{H}$.

Interpretation: It is insightful to compare this traditional postulate with the VD: The postulate restricts the set of possible results of quantum physics, as the information about the volume and its changes $\varepsilon_{L,p}$ and its inherent self - interaction is only partially included in QP. In particular, as the self - interaction is not included, the corresponding energy density is not included, so that QP generates the cosmological constant problem, for instance, see e. g. Zeldovich (1968), Cugnon (2012), Nobbenius (2006). The VD solves that problem and clarifies the role of the zero - point oscillations, see THM (40).

Moreover, the understanding of the universal quantization and of the marginal stabilization are not included, see chapter (9).

11.3 Observables and operators

In this section, we derive the following postulate about the relation between observables and operators (Kumar, 2018, p. 169):

Postulate 3 Observables correspond to operators

'A measurable physical quantity A (called an observable or dynamic physical quantity), is represented by a linear and hermitian operator $\hat{A}$ acting in the Hilbert space of state vectors.'

Correspondence principle: We remind the correspondence of operators and physical quantities, see THM (18): **If the application of an operator to the wave function provides a physical quantity, then that operator can be used to represent that physical quantity.** The postulate extends the above correspondence by an additional requirement: To each measurable physical quantity A, there corresponds a hermitian operator $\hat{A}$ acting in the Hilbert space.

Derivation:

(1) The VD provides linear DEQs (THMs 5, 26, 27) and corresponding solutions in respective Hilbert spaces. These DEQs and solutions describe objects. These objects include the known species in the universe that are characterized by the FLE, see Eqs. (8.12 and 8.13) or Hobson et al. (2006), Workman et al. (2022), Carmesin (2019b, 2021a):

relativistic species, see (Workman et al., 2022, section 25.2.3), including radiation or electromagnetic waves,

non-relativistic species, including matter,

species of the density ρ_Λ, such as portions of volume in nature with the energy density u_{vol}, see C. (14).

(2) In these DEQs, universal quantization (THM 18) provides the universal constant of quantization.

(3) As a consequence, each physical state is a state vector $|state\rangle$ in the Hilbert space $\mathcal{H}$ of these DEQs.

(4a) Consequently, each **physical function** A is a function $A(|state\rangle)$ of at least one state vector $|state\rangle$.

(4b) At first order, a physical function A is a linear function or operator $\hat{A}|state\rangle$ of at least one state vector $|state\rangle$. Accordingly, these linear functions play the role of eigenvalue generating operators, whereby the eigenvalues are possible outcomes of measurements. That case is covered by the postulate (3). Thus, the present postulate includes the convention that a physical function of first order is regarded as a **measurable physical quantity or observable**.

(4bα) Hereby, the linear function is represented by a linear operator $\hat{A}$ acting in the Hilbert space of state vectors.

(4bβ) As each outcome of a measurement is real and member of a discrete or continuous set, the linear operator is a hermitian (or self-adjoint) operator, as such operators provide real eigenvalues that are members of a discrete or continuous set of eigenvalues (Teschl, 2014, THM 3.6 or spectral theorem):

$$\hat{A} = \hat{A}^\dagger \tag{11.5}$$

(4c) At second order, a physical function A is a quadratic function $A(|state\rangle^2)$ of at least one state vector $|state\rangle$. That case is covered by the postulate (11.5) below. It will turn out that the second order gives rise to a variety of probability densities ρ, including mixed states in postulate (6) and entanglement in section (11.6) below.

(4d) At order $n \geq 3$, a physical function A is an $n-th$ power $A(|state\rangle^n)$ of at least one state vector $|state\rangle$. That case is essential in dimensional phase transitions in Carmesin (2023g, 2021d, 2019b).

Generalization: The above derivation holds as well for the GSEQ, including rates of change tensors $\dot{\varepsilon}_{L,p} \cdot t_n$. Consequently, in such wave functions, measurable quantities can be described with help of operators as well.

Interpretation: In everyday life and in many experimental settings, the experimental determination of a probability would be probably also be regarded as a measurement, see for instance the measurements of probabilities in the Rutherford experiment, see e. g. (Tipler and Llewellyn, 2008, p. 153), or see the probability of a molecule in a velocity distribution in statistical physics, see (Karttunen et al., 2007, p. 146) or the probability of a collision of an asteroid with Earth, see (Karttunen et al., 2007, p. 189).

Moreover, in everyday life and science, increments of time dt can be measured. Furthermore, since space is globally flat, see Carmesin (2023h,g), Planck-Collaboration (2020), increments dt of time can be combined to a global time t ranging from the

Big Bang to the present - day time t_0, and beyond $t \in [0, t_0[$. However, QP has no time operator.

As a consequence, this postulate introduces a particular and restricted meaning of the words 'measurement' and 'measurable physical quantity'.

More general experimental determinations including probabilities are treated in later postulates. Moreover, the VD includes incremental time dt, relativistic time $d\tau$, as well as cosmological time $t \in [0, t_0[$, including the dynamics of local and global formation of volume since the Big Bang. In general, that cosmological dynamics includes dimensional phase transitions of volume or space, see Carmesin (2017c, 2019b, 2021d,a,b, 2023g).

11.4 Outcomes of measurements

In this section, we derive the following postulate about the relation between the possible outcomes of a measurement and the eigenvalues (Kumar, 2018, p. 169):

Postulate 4 Outcomes of measurements are eigenvalues

'The measurement of an observable A in a given state may be represented formally by the action of an operator $\hat{A}$ on the state vector $|\Psi(t)\rangle$. The only possible outcome of such a measurement is one of the eigenvalues, $\{a_j\}, j = 1, 2, 3, \ldots,$ of $\hat{A}$.'

Derivation: That postulate has already been derived in section (11.3).

Generalization: The above derivation holds as well for the GSEQ, including rates of change tensors $\dot{\varepsilon}_{L,p} \cdot t_n$. Consequently, in such wave functions, outcomes of measurements can be described with help of eigenvalues of operators as well.

11.5 Probabilistic outcomes

In this section, we derive the following postulate about probabilistic outcomes of measurements (Kumar, 2018, p. 169, 170). Hereby, the used eigenfunctions form an orthonormal basis, (Kumar, 2018, p. 169):

Postulate 5 Probabilistic outcomes of measurements

If a measurement of an observable A is made in a state $|\Psi(t)\rangle$ of the quantum mechanical system, then the following holds:

(1) The probability of obtaining one of the non-degenerate discrete eigenvalues a_j of the corresponding operator $\hat{A}$ is given by

$$P(a_j) = \frac{|\langle\phi_j|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle}, \tag{11.6}$$

where $|\phi_j\rangle$ is the eigenfunction of $\hat{A}$ with the eigenvalue a_j. If the state vector is normalized to unity, $P(a_j) = |\langle\phi_j|\Psi\rangle|^2$.

(2) If the eigenvalue a_j is m_j-fold degenerate, this probability is given by

$$P(a_j) = \frac{\Sigma_{i=1}^{m_j}|\langle\phi_{j,i}|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle}, \tag{11.7}$$

(3) If the operator $\hat{A}$ possesses a continuous eigenspectrum $\{a\}$, the probability that the result of a measurement will yield a value between a and $a + da$ is given by

$$P(a) = \frac{|\langle\phi(a)|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle} \cdot da = \frac{|\langle\phi(a)|\Psi\rangle|^2}{\int_{-\infty}^{\infty}|\Psi(a')|^2 da'} \cdot da \tag{11.8}$$

Derivation: In the following, we derive the probabilistic outcomes of measurements, and we derive the probabilities.

11.5.1 Necessity of probabilistic outcomes

In volume in nature and in the VD, randomness is a necessary consequence for the following reason:

The VD implies universal quantization, see THM (18). As a consequence, an emitted minimal portion of light E_{min} with circular frequency ω has the observable energy $E_{min} = E_{av} = E = \hbar\omega$. Thereby, ω is invariant in flat space. However, the wave function can spread in space: For instance, a quantum object emitted at a time t_0 in all directions typically has a wave function Ψ with a maximum $\Psi(R_{max}(t))$ at a sphere with a radius $R_{max}(t)$. Thereby, $\Psi(R_{max}(t))$ decreases as a function of time t and tends to zero for t tending to infinity. Thus, $\Psi(R_{max}(t))$ can reach arbitrarily small values. So the question arises, how can an arbitrarily small $\Psi(R_{max}(t))$ provide the constant minimal observable energy E_{min} at a location? Correspondingly, $\Psi(R_{max}(t))$ has the same small value at many locations. These locations have equal properties, as a consequence of translation invariance and isotropy in section (4.2.2). Consequently, the object with its energy E_{min} can be observed at many locations with the same probability. But E_{min} is observed at one location only, as there is only one emitted quantum object.

As a consequence, in general, the location at which the quantum object is measured is determined in a stochastic or probabilistic manner.

11.5.2 Probability of observation of an object

The probability $p(\vec{r}, dV)$ to observe an object described by a wave packet within a volume dV and at a location $\vec{r}$ is determined as shown below. Hereby, the volume dV can be used with its volumetric property, without analyzing the additional properties that each portion dV of volume in nature has.

The observable energy is the kinetic energy, see THM (18).

Each wave packet has a positive density $u_{kin}(\vec{r})$ of kinetic energy, see THM (16).

Thus, the probability $p(\vec{r}, dV)$ of measuring the object at $\vec{r}$ is proportional to the energy density $u_{kin}(\vec{r})$ of the wave packet at that volume:

$$\boxed{p(\vec{r}, dV) \propto u_{kin}(\vec{r})} \tag{11.9}$$

This relation is founded additionally as follows:

(1) If N objects are emitted at $\vec{r} = \vec{0}$, and if N_{obs} objects are detected at the location $\vec{r}$ and within a volume dV, the **relative frequency** $\bar{N}_{obs}$ in statistics is as follows, see (Olofsson and Andersson, 2012, p. 2):

$$\bar{N}_{obs} = \frac{N_{obs}}{N} \tag{11.10}$$

(2) According to the (empirical) law of large numbers, see e. g. Kolmogorov (1956), in the limit N to infinity, the expectation value and the probability describe experiments in a precise manner. Correspondingly, the probability $p(\vec{r}, dV)$ is the limit N to infinity of the relative frequency $\bar{N}_{obs}$:

$$\lim_{N \to \infty} \bar{N}_{obs} = p(\vec{r}, dV) \tag{11.11}$$

(3) The expectation value of the energy at the location $\vec{r}$ and within a volume dV is the probability $p(\vec{r}, dV)$ to measure an object multiplied with its energy E_{packet}:

$$\langle dE(\vec{r}, dV) \rangle = p(\vec{r}, dV) \cdot E_{packet} \tag{11.12}$$

(4) The energy density $u_{kin}(\vec{r})$ of the wave packet is the ratio of the expectation value $\langle dE(\vec{r}, dV) \rangle$ and the volume dV:

$$u_{kin}(\vec{r}) = \frac{\langle dE(\vec{r}, dV) \rangle}{dV} = \frac{p(\vec{r}, dV) \cdot E_{packet}}{dV} \propto p(\vec{r}, dV) \tag{11.13}$$

This shows the relation in Eq. (11.9).

11.5.3 Probability $p(t,\vec{r})$ proportional to $|\Psi(t,\vec{r})|^2$

Question: How is the energy density $u_{kin}(\vec{r})$ related to the wave function?

(1) The energy density $u_{kin}(\vec{r})$ of the kinetic energy is proportional to the square of the rates, see THM (16):

$$u_{kin}(\vec{r}) \propto \dot{\varepsilon}^2_{L,p}(\vec{r}) \tag{11.14}$$

As the rate is proportional to the wave function Ψ, the energy density is proportional to the absolute square of Ψ:

$$u_{kin}(\vec{r}) \propto \langle\Psi|\Psi\rangle(\vec{r}) \tag{11.15}$$

(2) The probability $p(t,\vec{r},dV)$ to measure an object described by a wave packet within a volume dV at a location $\vec{r}$ and at a time t is equal to a probability density function $f_R(t,\vec{r})$ at $\vec{r}$ and at t multiplied by dV, Olofsson and Andersson (2012), Jahnke et al. (2005), Müller (1975), Ash (2008):

$$p(t,\vec{r},dV) = f_R(t,\vec{r}) \cdot dV \tag{11.16}$$

Thereby, the probability density function is proportional to the energy density u_{kin} (Eq. 11.13), which is proportional to the absolute square of Ψ (Eq. 11.15):

$$f_R(\vec{r}) \propto |\Psi(\vec{r})|^2 \tag{11.17}$$

Next, we normalize the above probability density function:

$$f_R(\vec{r}) = \frac{|\Psi(\vec{r})|^2}{\langle\Psi|\Psi\rangle} \text{ with } \langle\Psi|\Psi\rangle = \int d\vec{r}\,'\Psi^*(\vec{r}\,')\Psi(\vec{r}\,')$$
$$\Psi^* = \text{ complex conjugated of } \Psi \tag{11.18}$$

11.5.4 Probability $P(a_j)$ in item (1) in postulate (5)

In this section, we derive item (1) of the postulate (5).

Derivation of the probability: For it, we use the probability density function in Eq. (11.18), and we apply the Dirac notation (Kumar, 2018, section 4.2):

$$f_R(\vec{r}) = \frac{|\Psi(\vec{r})|^2}{\langle\Psi|\Psi\rangle} \quad \text{with} \quad |\Psi(\vec{r})|^2 = \Psi^*(\vec{r}) \cdot \Psi(\vec{r})$$
$$\text{and} \quad \Psi(\vec{r}) = |\Psi(\vec{r})\rangle \quad \text{and} \quad \Psi^*(\vec{r}) = \langle\Psi(\vec{r})|$$
$$\text{and} \quad \int d\vec{r}|\Psi(\vec{r})|^2 = \langle\Psi|\Psi\rangle \qquad (11.19)$$

In order to obtain the traditional form of the postulate, we transform the probability density function to the probability. For it, we multiply by dV:

$$p(\vec{r}, dV) = f_R(\vec{r}) \cdot dV = \frac{|\Psi(\vec{r})|^2}{\langle\Psi|\Psi\rangle} \cdot dV \qquad (11.20)$$

Next, we apply the integral $\int d\vec{r}$ to Eq. (11.20), and we divide by dV:

$$\int d\vec{r} p(\vec{r}, dV)/dV = \int d\vec{r} f_R(\vec{r}) = \frac{\int d\vec{r}|\Psi(\vec{r})|^2}{\langle\Psi|\Psi\rangle} = 1 \qquad (11.21)$$

Transformation of the numerator: The numerator in the above Eq. (11.21) is expressed as a scalar product:

$$\int d\vec{r}|\Psi(\vec{r})|^2 = \langle\Psi|\Psi\rangle \qquad (11.22)$$

Next, we transform the state $|\Psi\rangle$ with the orthonormal[3] eigenvectors $|\Phi_j\rangle$ in item (1). So, we expand the ket $|\Psi\rangle$ as follows (Kumar, 2018, Eq. 4.7.1):

$$|\Psi\rangle = \sum_j |\Phi_j\rangle\langle\Phi_j|\Psi\rangle \qquad (11.23)$$

[3]The eigenvectors of a hermitian operator are orthogonal (Kumar, 2018, THM 4.6.2), and we normalize them.

Similarly, we transform the bra $\langle\Psi|$:

$$\langle\Psi| = \sum_k \langle\Psi|\Phi_k\rangle\langle\Phi_k|, \quad \text{thus,} \tag{11.24}$$

$$\int d\vec{r}|\Psi(\vec{r})|^2 = \langle\Psi|\Psi\rangle = \sum_k \sum_j \langle\Psi|\Phi_k\rangle\langle\Phi_k|\Phi_j\rangle\langle\Phi_j|\Psi\rangle \tag{11.25}$$

As the eigenfunctions are orthonormal, the above scalar product $\langle\Phi_k|\Phi_j\rangle$ is equal to the Kronecker delta δ_{kj} (Kumar, 2018, example 4.11.1):

$$\int d\vec{r}|\Psi(\vec{r})|^2 = \sum_k \sum_j \langle\Psi|\Phi_k\rangle\delta_{kj}\langle\Phi_j|\Psi\rangle \tag{11.26}$$

Using the Kronecker delta δ_{kj}, we evaluate one of the sums:

$$\int d\vec{r}|\Psi(\vec{r})|^2 = \sum_j \langle\Psi|\Phi_j\rangle \cdot \langle\Phi_j|\Psi\rangle \tag{11.27}$$

The product of the two scalar products in the above Eq. is equal to the absolute square of one scalar product:

$$\int d\vec{r}|\Psi(\vec{r})|^2 = \sum_j |\langle\Phi_j|\Psi\rangle|^2 \tag{11.28}$$

Transformed probability: Next, we apply the above integral to the integral of the probability in Eq. (11.21):

$$\int d\vec{r}p(\vec{r}, dV)/dV = \int d\vec{r}f_R(\vec{r}) = \sum_j \frac{|\langle\Phi_j|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle} \tag{11.29}$$

The integral of the probability density function provides the complete probability one, $\int d\vec{r}f_R(\vec{r}) = 1$. Similarly, the integral $\int d\vec{r}p(\vec{r}, dV)/dV = 1$ provides the whole probability. Correspondingly, the sum $\sum_j$ is a sum of probabilities P_j as follows:

$$1 = \sum_j P_j \ \text{ with } \ P_j = \frac{|\langle\Phi_j|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle} \tag{11.30}$$

Hereby, the scalar product $\langle\Phi_j|\Psi\rangle$ describes the projection of a wave function $|\Psi(t,\vec{r})\rangle$ to the new basis vector $|\Phi_j\rangle$ in Eq. (11.23). In the following, we do not mark the time, according to the postulate (5). Correspondingly, P_j is the probability to find the eigenvalue a_j of the new basis vector $|\Phi_j\rangle$ in the projection of $|\Psi(\vec{r})\rangle$. So, we mark that probability by $P_{|\Psi(\vec{r})\rangle}(a_j)$:

$$P_j = P_{|\Psi(\vec{r})\rangle}(a_j) = \frac{|\langle\Phi_j|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle} \tag{11.31}$$

For comparison, the probability $P(a_j)$ in item (1) of the postulate describes the probability to find the eigenvalue a_j of the new basis vector $|\Phi_j\rangle$ in the projection of $|\Psi\rangle$. We mark the respective state, and we remind the term in that item (1):

$$P_{|\Psi\rangle}(a_j) = \frac{|\langle\Phi_j|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle} \tag{11.32}$$

Thus, we derived the probability in item (1).

The items (2) and (3) are derived in (Carmesin, 2023g, chapter 14). q. e. d.

Generalization: The above derivation holds as well for the GSEQ, including rates of change tensors $\dot{\varepsilon}_{L,p} \cdot t_n$, see e. g. THM (18) for the case of electromagnetic radiation. Consequently, in such wave functions, the above probabilistic properties occur as well.

11.6 Mixed states

In this section, we derive the following postulate about the expectation value of an observable in the case of a mixed state.

What is a mixed state?

Firstly, a pure state is a vector in Hilbert space.

However, if we have insufficient knowledge about the objects of a system, then we can describe the system in the framework of statistical physics, see e. g. Landau and Lifschitz (1980). In the case of a quantum system, such a system is in a mixed state.

A mixed state can be characterized as follows, see e. g. (Ballentine, 1998, p. 46, 50):

Definition 8 Mixed state

(1a) In a mixed state, the orthonormal eigenfunctions ϕ_j of an operator $\hat{A}$ of an observable A and of a non-degenerate or degenerate eigenvalue a_j occur at corresponding probabilities p_j.

(1b) The probabilities are non-negative and their sum is one:

$$p_j \geq 0 \text{ and } \sum_{j=1}^{N_A} p_j = 1 \tag{11.33}$$

Hereby, N_A denotes the number of eigenvectors of $\hat{A}$.

(2) A mixed state can be described by a density operator as follows (Sakurai and Napolitano, 1994, p. 180, 181):

$$\hat{\rho} = \sum_{j=1}^{N_A} |\phi_j\rangle p_j \langle\phi_j| \tag{11.34}$$

The density operator is also called state operator or statistical operator, and the above form of that operator $\hat{\rho}$ is called spectral representation (Ballentine, 1998, p. 46 and section 2.3).

Postulate 6 Expectation value of a mixed state

To each state there corresponds a unique state operator. The expectation (or average) value of a dynamic variable (or observable) A, represented by the operator $\hat{A}$, in the virtual ensemble of events that may result from a preparation procedure for the state, represented by an operator $\hat{\rho}$ is:

$$\langle A\rangle = \frac{Tr(\hat{\rho}\cdot\hat{A})}{Tr(\hat{\rho})} \text{ Hereby, Tr marks the trace.} \tag{11.35}$$

Derivation: It is presented in (Carmesin, 2023g, section 15.7).

Theorem 31 The VD implies the quantum postulates:

(1) The VD implies the usual quantum postulates (1, 2, 3, 4, 5, 6).

(2) Additionally, the VD provides a deeper understanding of quantum physics. For instance, the VD explains the wave function in terms of the normalized rate $t_n \cdot \dot{\varepsilon}_{L,p}$ of change of volume or, more generally, of change tensors of volume portions.

(3) Hereby, the rates $\dot{\varepsilon}_{L,jj}$ are confirmed by the derivation of the energy density of volume u_{vol} and by the time evolution of the Hubble constant $H_0(z_{em})$ or $H_0(t_{em})$. Thereby, u_{vol} as well as $H_0(z_{em})$ are confirmed by observation, see THMs (38, 43) and Fig. (16.4).

(4) Moreover, the VD provides generalizations of these quantum postulates. For instance, the VD provides the generalized Schrödinger equation, GSEQ, and change tensors $t_n \cdot \dot{\varepsilon}_{L,p}$.

Chapter 12

VD implies spectra and correlations

Present-day quantum field theory, QFT, is very successful in deriving spectra and correlations with help of algebra and ladder operators, see e. g. Ballentine (1998), Peskin and Schroeder (1995).

However, present-day QFT, is a 'set of ideas and tools', see (Peskin and Schroeder, 1995, p. xi). Similarly, such relativistic quantum physics has been regarded as a calculation tool rather than a theory, see (Thirring, 1978, p. V, line 17-21). Correspondingly, there is no generally accepted system of postulates that would define present - day QFT or one of its main users, elementary particle physics, see e. g. Feynman (1985), Peskin and Schroeder (1995), Schwartz (2014), Workman et al. (2022). Furthermore, one of the 'essential features of present-day QFT is locality', see (Peskin and Schroeder, 1995, p. 393), but the Einstein's principle of locality is hardly founded and limits the explanatory power with respect to the experimentally founded nonlocality in nature, see chapters (2 and 9). Moreover, present-day QFT exhibits problems, such as the cosmological constant problem, see e. g. Nobbenius (2006) and chapters (9 and 13).

In this book, we show that the VD implies the algebra of ladder operators, which provides spectra and correlations, similarly as in the present-day QFT. Moreover, we show that the

VD implies essential results beyond the present-day QFT, so that the cosmological constant problem is solved, see chapter (13).

This is revealing, as the derivation clarifies the source of results, so that these become traceable. This is very valuable, as the spectra and correlations are provided.

12.1 Transformed wave packet

A general wave packet is transformed via a Fourier transform. This is physically founded, as harmonic waves are solutions of the VD. This is very useful, as the used waves can be analyzed in a systematic and transparent manner.

Theorem 32 Transformation of a wave packet

A wave packet of relative additional volume or change $\varepsilon_{L,p}(\tau, \vec{L})$, whereby p describes the polarization or the change tensor, that is transformed according to part (1), is represented by part (2):

(1) The following **orthonormal harmonic functions** *are utilized:*

$$b_\mu := \nu_\mu \cdot \exp[-i\omega_\mu \tau] \tag{12.1}$$

Thereby, ν_μ is a normalization factor. Hereby, the following scalar product is applied:

$$\langle b_\mu | b_{\mu'} \rangle_t := \int_{-\infty}^{\infty} b_\mu(\tau) \cdot b^*_{\mu'}(\tau) d\tau = \delta(\mu - \mu') \tag{12.2}$$

*Hereby, * marks the conjugate complex.*

In general, the index μ describes the mode: The mode describes the circular frequency ω_μ, and possible polarization directions, and the subscripts of change tensors. Thereby, possible polarization directions, and the subscripts of change tensors, are represented as parts of the amplitudes $\hat{\varepsilon}_\mu$ and $\hat{\Phi}_{gen,\mu}$ in part (2).

Moreover, $\langle b_\mu | b_{\mu'} \rangle$ is a scalar product in the Hilbert space of the solutions of the DEQ of VD, and $\delta(\mu - \mu')$ is the Dirac delta *function (or delta distribution), see e. g. Kumar (2018). The spatial Fourier components are as follows:*

$$f_\mu := \nu_{\mu,f} \cdot \exp[i\vec{k}_\mu \vec{L}] \tag{12.3}$$

Hereby, the scalar product is as follows, and it is proportional to the delta function:

$$\langle f_\mu | f_{\mu'} \rangle := \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_\mu \cdot f_{\mu'}^* d^3L \propto \delta(\vec{k}_\mu - \vec{k}_{\mu'}) \tag{12.4}$$

This Eq. is transformed to polar coordinates:

$$\langle f_\mu | f_{\mu'} \rangle = 4\pi \int_{-\infty}^{\infty} f_\mu \cdot f_{\mu'}^* L^2 dL \tag{12.5}$$

According to Eq. (12.4), the result is proportional to the delta function. Corresponding to Eq. (12.5), we normalize so that the integral in Eq. (12.4) represents the following orthonormality of the integral in Eq. (12.5):

$$\langle f_\mu | f_{\mu'} \rangle = 4\pi \delta(\vec{k}_\mu - \vec{k}_{\mu'}) \tag{12.6}$$

(2) The transformed wave packet has the following relative additional volume,

$$\varepsilon_{L,p}(\tau, \vec{L}) = \int d\mu \hat{\varepsilon}_\mu \cdot b_\mu(\tau) \cdot f_\mu(\vec{L}) \tag{12.7}$$

the following potential with its amplitude $\hat{\Phi}_{gen,\mu}$,

$$\Phi_{gen}(\tau, \vec{L}) = \int d\mu \hat{\Phi}_{gen,\mu} \cdot b_\mu(\tau) \cdot f_\mu(\vec{L}) \tag{12.8}$$

the following energy E

$$E = \int d\mu E_\mu \tag{12.9}$$

and the following energy E_μ of a mode μ:

$$E_\mu = \frac{c^2}{2G} \cdot \hat{\varepsilon}_\mu \hat{\varepsilon}_\mu^* \cdot b_\mu b_\mu^* \cdot (\omega_\mu^2 - c^2 k_\mu^2) \tag{12.10}$$

Hereby, $\hat{\varepsilon}_\mu$ is the amplitude or Fourier transformed function of $\varepsilon_{L,p}$, and $\int d\mu$ summarizes the integrals with respect to ω_μ and $\vec{k}_\mu$.

Proof:

Part (2a): According to the theory of Fourier transformations, see e. g. Schiff (1991), the functions $\varepsilon_{L,p}(\tau, \vec{L})$ and $\Phi_{gen}(\tau, \vec{L})$ are expressed with help of the transformed functions $\hat{\varepsilon}_\mu$ and $\hat{\Phi}_{gen,\mu}$ in terms of the integrals in Eqs. (12.7 and 12.8).

Part (2b): In order to derive the energy in Eqs. (12.9 and 12.10), the terms in the energy density in THM (17) are transformed:

$$u = \frac{c^2}{8\pi G}|\dot{\varepsilon}^2_{L,p}| - \frac{1}{8\pi G}|\partial_{\vec{L}}\Phi_{gen}|^2 \tag{12.11}$$

Next, the absolute squares in the above Eq. are evaluated as the products of the term with its conjugate complex:

$$|\dot{\varepsilon}^2_{L,p}| = \int d\mu \int d\mu' \omega_\mu \omega_{\mu'} \cdot \hat{\varepsilon}_\mu \hat{\varepsilon}^*_{\mu'} \cdot b_\mu b^*_{\mu'} \cdot f_\mu f^*_{\mu'} \tag{12.12}$$

Hereby, the factor $\omega_\mu \omega_{\mu'}$ is provided by the time derivatives.

$$|\partial_{\vec{L}}\Phi_{gen}|^2 = \int d\mu \int d\mu' \vec{k}_\mu \vec{k}_{\mu'} \cdot \hat{\Phi}_{gen,\mu} \hat{\Phi}^*_{gen,\mu'} \cdot b_\mu b^*_{\mu'} \cdot f_\mu f^*_{\mu'} \tag{12.13}$$

Hereby, the factor $\vec{k}_\mu \vec{k}_{\mu'}$ is provided by the spatial derivatives. The potential is expressed with help of the relative additional volume $\Phi_{gen} = \varepsilon_{L,p} c^2$:

$$|\partial_{\vec{L}}\Phi_{gen}|^2 = c^4 \int d\mu \int d\mu' \vec{k}_\mu \vec{k}_{\mu'} \cdot \hat{\varepsilon}_\mu \hat{\varepsilon}^*_{\mu'} \cdot b_\mu b^*_{\mu'} \cdot f_\mu f^*_{\mu'} \tag{12.14}$$

The transformed terms in Eqs. (12.12 and 12.14) are inserted in the energy density of the wave packet in Eq. (12.11):

$$u = \int d\mu \int d\mu' \hat{\varepsilon}_\mu \hat{\varepsilon}^*_{\mu'} \cdot b_\mu b^*_{\mu'} \cdot f_\mu f^*_{\mu'} \cdot \left(\frac{\omega_\mu \omega_{\mu'} c^2}{8\pi G} - \frac{c^4 \vec{k}_\mu \vec{k}_{\mu'}}{8\pi G} \right) \tag{12.15}$$

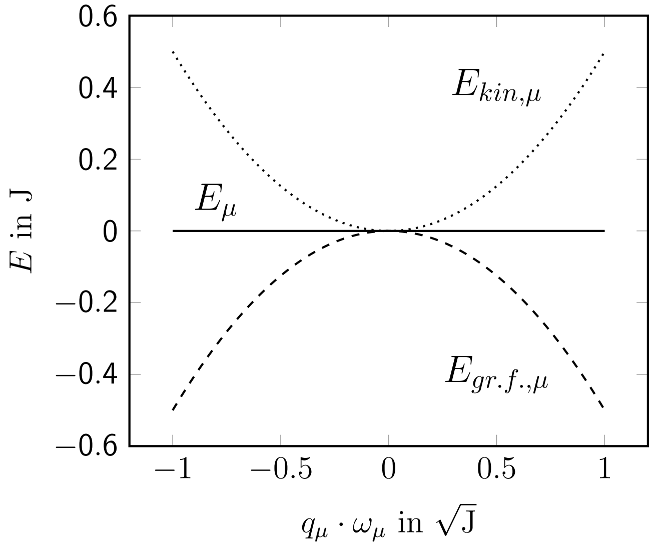

Figure 12.1: Sketch of the energy E_μ of a mode of the wave packet, with its parts, kinetic energy $E_{kin,\mu}$ and energy of the gravitational field $E_{gr.f.,\mu}$, as a function of the product $q_\mu \cdot \omega_\mu$.

We derive the energy with help of the spatial integral and the average or scalar product with respect to time:

$$E = \left\langle \int u d^3L \right\rangle_t \tag{12.16}$$

Thus, we derive:

$$E = \int d\mu \int d\mu' \hat{\varepsilon}_\mu \hat{\varepsilon}^*_{\mu'} \cdot b_\mu b^*_{\mu'} \cdot \int f_\mu f^*_{\mu'} d^3L \cdot \left(\frac{\omega_\mu \omega_{\mu'} c^2}{8\pi G} - \frac{c^4 \vec{k}_\mu \vec{k}_{\mu'}}{8\pi G} \right) \tag{12.17}$$

We evaluate the spatial integral with help of Eqs. (12.4 to 12.6), $\int f_\mu f^*_{\mu'} d^3L = \langle f_\mu | f_{\mu'} \rangle = 4\pi\delta(\vec{k}_\mu - \vec{k}_{\mu'})$. With that delta function, we evaluate the integral with respect to μ':

$$E = \int d\mu E_\mu = \frac{c^2}{2G} \int d\mu \hat{\varepsilon}_\mu \hat{\varepsilon}^*_\mu \cdot b_\mu b^*_\mu \cdot \left(\omega^2_\mu - k^2_\mu c^2 \right) \tag{12.18}$$

This completes the proof of part (2).

12.2 Eigenvalue generating operator

In quantum physics, the possible outcome of a measurement of an observable quantity is an eigenvalue of the operator corresponding to the observable quantity, see chapter (11). Accordingly, it is useful to analyze eigenvalue generating operators and their eigenvalues. This is elaborated in this section.

Theorem 33 Law of eigenvalue generating operator

(1.1) The energy in Eq. (12.10),

$$E_\mu = \frac{1}{2} \cdot \frac{c^2}{G} \cdot \hat{\varepsilon}_\mu \hat{\varepsilon}^*_\mu \cdot b_\mu b^*_\mu \cdot (\omega^2_\mu - c^2 k^2_\mu) \tag{12.19}$$

is expressed with transformed variables, a formal coordinate q_μ *and momentum* p_μ*,*

$$q_\mu = \frac{c}{\sqrt{G}} \cdot \hat{\varepsilon}_\mu \cdot b_\mu \text{ with} \tag{12.20}$$

$$p_\mu = c \cdot k_\mu \cdot q_\mu, \tag{12.21}$$

so that the energy is expressed by as follows:

$$E_\mu = \frac{1}{2} \cdot \left(q_\mu q^*_\mu \omega^2_\mu - p_\mu p^*_\mu \right) \tag{12.22}$$

Hereby, the complex parts are inherent to q_μ*, whereas the wave vector* $\vec{k}_\mu$ *is real.*

(1.2) Eigenvalue generating operators $\hat{p}_\mu$ *and* $\hat{k}_\mu$ *are used. For it, the operator is applied to Eq. (12.21),*

$$\hat{p}_\mu = \hat{k}_\mu \cdot q_\mu \cdot c, \tag{12.23}$$

$\hat{k}_\mu$ *is in accordance with the momentum operator in THM (18),*

$$\hat{k}_\mu = -i \cdot \vec{e}_v \cdot \frac{\partial}{\partial \vec{L}}, \text{ so that : } \hat{k}_\mu q_\mu = \vec{k}_\mu \cdot \vec{e}_v \cdot q_\mu, \tag{12.24}$$

$$\text{and so that : } \hat{p}_\mu = -i \cdot \vec{e}_v \cdot \frac{\partial}{\partial \vec{L}} q_\mu \cdot c = \vec{k}_\mu \cdot \vec{e}_v \cdot q_\mu \cdot c \tag{12.25}$$

In the analyzed volume in nature, the DEQ of VD in THM (5 part 2) is applicable. With it, the derivative $\frac{\partial}{\partial \vec{L}}$ in Eqs. (12.24 and 12.25) is substituted:

$$\hat{k}_\mu = \frac{i}{c} \cdot \frac{\partial}{\partial \tau} \text{ or} \tag{12.26}$$

$$\hat{p}_\mu = i \cdot \frac{\partial}{\partial \tau} q_\mu \tag{12.27}$$

With the operator $\hat{p}_\mu$ in Eq. (12.27), the energy operator takes the following form:

$$\hat{E}_\mu = \frac{1}{2} \cdot \left(q_\mu q_\mu^* \omega_\mu^2 - \hat{p}_\mu \hat{p}_\mu^* \right) \tag{12.28}$$

(1.3) The use of the operator includes a derivative. Its inverse operation, integration or solution of a DEQ, includes boundary values. In this sense, the operator is under-determined. And the introduction of the operator is not fully reversible.

(2) **Commutator and complementarity:**

The coordinate q_μ and momentum p_μ have a nonzero commutator:

$$\langle [q_\mu, \hat{p}_{\mu'}^*] \rangle_t = i\omega_\mu q_\mu q_\mu^* \cdot \delta(\mu' - \mu) \tag{12.29}$$

As a consequence, they cannot be measured exactly in a simultaneous manner. In this sense, they are complementary.

(3) **Explanation.**

In general, an eigenvalue generating operator of a physical quantity A that contains a derivative $\frac{\partial}{\partial A}$ can provide differences ΔA of the quantity A.

(3.1) However, a derivative can not provide the zero-point of A, as the zero-point would require the integration constant or boundary value.

(3.2) Many operators in present-day quantum physics are eigenvalue generating operators. These eigenvalues of the wave function are correct. However, the wave function is proportional to

the rate $\dot{\varepsilon}_{L,ij}$ *of relative additional volume. But the amount of relative additional volume is essential for gravity, for the expansion of space and for the energy density* u_{vol} *of volume. Thus, also the eigenvalues do not provide the full information.*

(3.3) The VD provides the above results and more:

The VD provides the results based on the derivative in part (3.1) and the results based on the eigenvalue generating operators in part (3.2).

Moreover, the VD provides the results based on the relative additional volume, such as gravity, curvature and the expansion of space.

Furthermore, the VD provides the derivation and meaning of the wave functions, of the postulates of quantum physics, and of the essential operators, see THMs (32, 34, 35).

Additionally, the VD provides all change tensors and the corresponding generalizations of the SEQ.

Proof:

Parts (1.1), (1.2) and (1.3) include their derivations.

Part (1.4): **Test of the operator $\hat{p}_\mu$:**

In order to obtain the eigenvalue p_μ, the derivative in the operator $\hat{p}_\mu$ in Eq. (12.27) is evaluated. For it, the time derivative $\partial_\tau b_\mu = -i\omega_\mu b_\mu$ or $\partial_\tau q_\mu = -i\omega_\mu q_\mu$ is used:

$$p_\mu = i\partial_\tau \cdot q_\mu = \omega_\mu \cdot q_\mu \tag{12.30}$$

The dispersion relation $\omega_\mu = c \cdot k_\mu$ is used:

$$p_\mu = c \cdot k_\mu \cdot q_\mu \tag{12.31}$$

Thus, the relation between p_μ and q_μ in Eq. (12.21) is reproduced and confirms the derivation of the operator $\hat{p}_\mu$.

Part (2): The scalar product or time derivative of a commutator is analyzed:

$$\langle [q_\mu, \hat{p}^*_{\mu'}] \rangle_t = \langle q_\mu \hat{p}^*_{\mu'} - \hat{p}^*_{\mu'} q_\mu \rangle_t = i \langle q_\mu \partial_\tau q^*_{\mu'} - \partial_\tau q^*_{\mu'} q_\mu \rangle_t \text{ so,} \quad (12.32)$$

$$\langle [q_\mu, \hat{p}^*_{\mu'}] \rangle_t = i \langle q_\mu \partial_\tau q^*_{\mu'} - q^*_{\mu'} \partial_\tau q_\mu - q_\mu \partial_\tau q^*_{\mu'} \rangle_t \text{ thence,} \quad (12.33)$$

$$\langle [q_\mu, \hat{p}^*_{\mu'}] \rangle_t = i \langle - q^*_{\mu'} \partial_\tau q_\mu \rangle_t = -\omega_\mu \langle q^*_{\mu'} q_\mu \rangle_t \quad (12.34)$$

The scalar product $\langle q_{\mu'} q^*_\mu \rangle_t$ is equal to the term $q_\mu q^*_\mu \delta(\mu' - \mu)$. Thus, we derive the commutator:

$$\langle [q_\mu, \hat{p}^*_{\mu'}] \rangle_t = -\omega_\mu q_\mu q^*_\mu \cdot \delta(\mu' - \mu) \quad (12.35)$$

In particular, for the case of equal modes $\mu = \mu'$, the commutator is as follows:

$$\langle [q_\mu, \hat{p}^*_\mu] \rangle_t = -\omega_\mu q_\mu q^*_\mu \quad (12.36)$$

Part (3) includes its derivation. This completes the proof.

12.3 Excitation generating operators

In order to analyze spectra, it is very useful to investigate excitation generating operators or ladder operators. Hereby, it is informative to consider a mode μ of a one-dimensional subsystem.

Theorem 34 Law of excitation generating operator

The operators q_μ and $\hat{p}_\mu$ transform to excitation generating operators or ladder operators $\hat{a}^+_\mu$ and $\hat{a}_\mu$ as follows:

$$q_\mu = \frac{\alpha}{\omega_\mu} \left(\hat{a}^+_\mu + \hat{a}_\mu \right) \quad (12.37)$$

$$\hat{p}_\mu = \frac{\alpha}{\omega_\mu} \left(\hat{a}^+_\mu - \hat{a}_\mu \right) \quad (12.38)$$

Hereby, α is determined so that the commutator of the ladder operators is as follows:

$$\left[\hat{a}_\mu, \hat{a}^+_{\mu'}\right] = \delta(\mu' - \mu) \tag{12.39}$$

In the case of a one - dimensional subsystem, these operators have the following properties:

(1) A mode μ has the following first summand of the energy in Eq. (12.22):

$$\hat{E}_{\mu,1D,first} = \frac{1}{2} \cdot q_\mu q^*_\mu \omega^2_\mu, \text{ with } \alpha^2 = \frac{1}{2}\omega^2_\mu q_\mu q^*_\mu \tag{12.40}$$

(2) The ladder operators act as follows:

$$\boxed{\langle n'_\mu | \hat{a}^+_\mu | n_\mu \rangle = \sqrt{n_\mu + 1} \cdot \delta_{n'_\mu, n_\mu + 1}} \tag{12.41}$$

Accordingly, $\hat{a}^+_\mu$ is called **raising operator**.

$$\boxed{\langle n'_\mu | \hat{a}_\mu | n_\mu \rangle = \sqrt{n_\mu} \cdot \delta_{n'_\mu, n_\mu - 1} \;\; for \;\; n_\mu > 0} \tag{12.42}$$

Correspondingly, $\hat{a}_\mu$ is called **lowering operator**.

(3) Number operators are introduced with normalized eigenstates $|n_\mu\rangle$ and eigenvalues n_μ:

$$\hat{N}_\mu := \hat{a}^+_\mu \hat{a}_\mu \tag{12.43}$$

$$\hat{N}_\mu \cdot |n_\mu\rangle = n_\mu \cdot |n_\mu\rangle \tag{12.44}$$

Proof:

Ad (1.1): With the linear transformation in Eqs. (12.37, 12.38, 12.39), we evaluate a commutator:

$$\langle [q_\mu, \hat{p}^*_{\mu'}] \rangle_t = -\frac{\alpha^2}{\omega_\mu} \left[\left(\hat{a}^+_\mu + \hat{a}_\mu \right), \left(\hat{a}^+_{\mu'} - \hat{a}_{\mu'} \right) \right] \tag{12.45}$$

For the case of equal modes $\mu = \mu'$ we evaluate the commutator. Moreover, we apply the commutator in Eq. (12.39):

$$\langle [q_\mu, \hat{p}^*_\mu] \rangle_t = -\frac{2\alpha^2}{\omega_\mu} \left[\hat{a}_\mu, \hat{a}^+_\mu\right] = -\frac{2\alpha^2}{\omega_\mu} \tag{12.46}$$

Application of the commutator in Eq. (12.36) yields:

$$-\frac{2\alpha^2}{\omega_\mu} = -\omega_\mu q_\mu q^*_\mu, \quad \text{hence,} \quad \alpha^2 = \frac{1}{2}\omega^2_\mu q_\mu q^*_\mu \tag{12.47}$$

Ad (1.2): As a mathematical method, the energy in Eq. (12.22) can be analyzed for the case that two Cartesian components are zero, $\hat{p}_{\mu,2} = 0 = \hat{p}_{\mu,3}$:

$$\hat{E}_{\mu,1D} = \frac{1}{2} \cdot \left(q_\mu q^*_\mu \omega^2_\mu - \hat{p}_\mu \hat{p}^*_\mu\right) \tag{12.48}$$

The ladder operators in Eqs. (12.37 and 12.38) are inserted in the above energy:

$$\hat{E}_{\mu,1D} = \frac{\alpha^2}{2} \cdot \left(\left(\hat{a}^+_\mu + \hat{a}_\mu\right)^2 - \left(\hat{a}^+_\mu - \hat{a}_\mu\right)^2\right) \tag{12.49}$$

Evaluation of the squares yields:

$$\hat{E}_{\mu,1D} = \alpha^2 \cdot \left(\hat{a}^+_\mu \cdot \hat{a}_\mu + \hat{a}_\mu \cdot \hat{a}^+_\mu\right) \tag{12.50}$$

The commutator Eq. (12.39) yields the energy:

$$\hat{E}_{\mu,1D} = 2\alpha^2 \cdot \left(\hat{a}^+_\mu \cdot \hat{a}_\mu + \frac{1}{2}\right) \tag{12.51}$$

Ad (1.3): The lowest energy $\hat{E}_{\mu,1D,min}$ does not depend on the ladder operators. Hence, it is the second summand in the above Eq. (12.51):

$$\hat{E}_{\mu,1D,min} = 2\alpha^2 \cdot \frac{1}{2} = \alpha^2 \tag{12.52}$$

Universal quantization in THM (18) implies that the observable energy of the wave packet is its kinetic energy. It is equal to the minimal energy of the positive term in Eq. (12.48):

$$\hat{E}_{\mu,1D,min,kin} = \frac{1}{2} \cdot q_\mu q_\mu^* \omega_\mu^2 = \alpha^2 \tag{12.53}$$

Ad (2.1): **Essential operator in $\hat{E}_\mu$ or $\hat{H}_\mu$**: We analyze the operator $\hat{a}_\mu^+ \hat{a}_\mu$ in $\hat{H}_\mu$, usually called **number operator**:

$$\hat{N}_\mu = \hat{a}_\mu^+ \hat{a}_\mu \tag{12.54}$$

Moreover we call its normalized eigenstates $|n_\mu\rangle$ and the eigenvalues n_μ:

$$\hat{N}_\mu \cdot |n_\mu\rangle = n_\mu \cdot |n_\mu\rangle \tag{12.55}$$

Ad (2.2): **Eigenstates of the number operator:** We apply $\hat{N}_\mu$ to $\hat{a}_\mu \cdot |n_\mu\rangle$, and we use the commutator. So we get:

$$\hat{N}_\mu \cdot \hat{a}_\mu \cdot |n_\mu\rangle = \hat{a}_\mu \cdot (n_\mu - 1) \cdot |n_\mu\rangle = (n_\mu - 1) \cdot \hat{a}_\mu \cdot |n_\mu\rangle \tag{12.56}$$

This Eq. shows that $\hat{a}_\mu \cdot |n_\mu\rangle$ is an eigenstate with the eigenvalue $n_\mu - 1$.

Similarly we apply $\hat{N}_\mu$ to $\hat{a}_\mu^+ \cdot |n_\mu\rangle$, and we utilize the commutator. So we obtain:

$$\hat{N}_\mu \cdot \hat{a}_\mu^+ \cdot |n_\mu\rangle = \hat{a}_\mu^+ \cdot (n_\mu + 1) \cdot |n_\mu\rangle = (n_\mu + 1) \cdot \hat{a}_\mu^+ \cdot |n_\mu\rangle \tag{12.57}$$

This Eq. shows that $\hat{a}_\mu^+ \cdot |n_\mu\rangle$ is an eigenstate to the eigenvalue $n_\mu + 1$.

Ad (2.3): **Effect of ladder operators:** Firstly, we analyze the matrix element $\langle n_\mu|\hat{a}_\mu \hat{a}_\mu^+|n_\mu\rangle$. Here we identify the number operator:

$$\langle n_\mu|\hat{a}_\mu \hat{a}_\mu^+|n_\mu\rangle = \langle n_\mu|\hat{N}_\mu + 1|n_\mu\rangle = (n_\mu + 1)\langle n_\mu|n_\mu\rangle = n_\mu + 1 \tag{12.58}$$

Secondly, we analyze the matrix element $\langle n_\mu|\hat{a}^+_\mu \hat{a}_\mu|n_\mu\rangle$. As above, we identify the number operator:

$$\langle n_\mu|\hat{a}^+_\mu \hat{a}_\mu|n_\mu\rangle = \langle n_\mu|\hat{N}_\mu|n_\mu\rangle = (n_\mu)\langle n_\mu|n_\mu\rangle = n_\mu \qquad (12.59)$$

Both Eqs. (12.58, 12.59) are fulfilled by the following relations:

$$\hat{a}^+_\mu|n_\mu\rangle = \sqrt{n_\mu + 1}|n_\mu + 1\rangle \qquad (12.60)$$

$$\hat{a}_\mu|n_\mu\rangle = \sqrt{n_\mu}|n_\mu - 1\rangle \qquad (12.61)$$

We summarize the matrix elements of the ladder operator $\hat{a}^+_\mu$ as follows:

$$\boxed{\langle n'_\mu|\hat{a}^+_\mu|n_\mu\rangle = \sqrt{n_\mu + 1} \cdot \delta_{n'_\mu, n_\mu+1}} \qquad (12.62)$$

Accordingly, the ladder operator $\hat{a}^+_\mu$ is called **raising operator**. Similarly, the matrix elements of $\hat{a}_\mu$ are represented as follows:

$$\boxed{\langle n'_\mu|\hat{a}_\mu|n_\mu\rangle = \sqrt{n_\mu} \cdot \delta_{n'_\mu, n_\mu-1} \ \text{ for } \ n_\mu > 0} \qquad (12.63)$$

Correspondingly, the ladder operator $\hat{a}_\mu$ is called **lowering operator**.

Ad (3): This part represents a definition, so that it needs no proof. This completes the proof.

12.4 Number - and energy spectrum

Theorem 35 Law of derived number - and energy spectrum

(1.1) The lowest number state is named $|n_\mu\rangle = |0\rangle$*, with* $\hat{N}_\mu|n_\mu\rangle = 0 \cdot |0\rangle$.

(1.2) The number spectrum of eigenvalues of the number operator is as follows:

$$\boxed{n_\mu \in \{0, 1, 2, 3, ...\}} \qquad (12.64)$$

(2.1) The energy spectrum is as follows, see Eq. (12.51):

$$\langle n_\mu|\hat{E}_{\mu,1D}|n_\mu\rangle = 2\alpha^2 \cdot \left(\langle n_\mu|\hat{N}_\mu|n_\mu\rangle + \frac{1}{2}\right) = 2\alpha^2 \cdot \left(n_\mu + \frac{1}{2}\right) \tag{12.65}$$

Consequently, the number states provide an energy spectrum with the zero - point energy, ZPE_μ*:*

$$ZPE_\mu := \langle 0|\hat{E}_{\mu,1D}|0\rangle = 2\alpha^2 \cdot \left(0 + \frac{1}{2}\right) = \alpha^2 \tag{12.66}$$

(2.2) The observable energy is the available energy, see THM (18). Thus, the lowest observable energy is as follows:

$$ZPE_{\mu,av} = \langle 0|\hat{E}_{\mu,1D,av}|0\rangle = \alpha^2 \tag{12.67}$$

(2.3) It is revealing to realize the **source of quantization***: The operators* q_μ*,* $\hat{p}_\mu$*,* $\hat{a}_\mu$*,* $\hat{a}_\mu^+$ *provide the number spectrum and the energy spectrum without the key factor* $ZPE_{\mu,av}$*. That factor represents the source of quantization. It is provided by universal quantization in THMs (18, 21, 22, 23, 24).*

(2.4) The observable zero-point energy of a mode μ *is determined by universal quantization in THMs (18, 21, 22, 23, 24) as follows:*

$$ZPE_{\mu,av} = \frac{\hbar\omega_\mu}{2} \tag{12.68}$$

(2.5) For each mode, the classical energy in Eq. (12.10) is zero, as the following difference is zero:

$$\omega_\mu^2 - c^2 k_\mu^2 = 0 \tag{12.69}$$

Accordingly, the observable zero-point energy of a mode μ *is a kinetic energy* $p \cdot c$*, see THM (18), and it is compensated by the corresponding energy density of the field, see THMs (11, 16, 21, 22, 23, 24).*

(3) The energy spectrum of a mode μ is as follows:

$$\hat{E}_{\mu,1D} = \hbar\omega_\mu \cdot \left(\hat{N}_\mu + \frac{1}{2}\right) = 2 \cdot ZPE_{\mu,av} \cdot \left(\hat{N}_\mu + \frac{1}{2}\right) \quad (12.70)$$

$$E_{\mu,1D} = \hbar\omega_\mu \cdot \left(n_\mu + \frac{1}{2}\right) = 2 \cdot ZPE_{\mu,av} \cdot \left(n_\mu + \frac{1}{2}\right) \quad (12.71)$$

(4) Altogether, the **source of quantization** *is traced back to universal quantization, which is traced back to gravity and curvature (THMs 9, 10) and a Gaussian or harmonic solution (THMs 6, 28, 17), these are traced back to the DEQ of VD (THM 5), see Fig. (1.4).*

Proof:

Ad (1): **Number spectrum:**

In order to derive the full spectrum of the number operator, we show that the lowering of states ends at the state $|n_\mu\rangle = 0$:

$$\hat{a}_\mu|1\rangle = \sqrt{1}|0\rangle \text{ and } \hat{a}_\mu|0\rangle = \sqrt{0}|-1\rangle = 0 \quad (12.72)$$

Starting at this state, the raising operator can successively create the states with all positive natural numbers:

$$n_\mu \in \{0, 1, 2, 3, ...\} \quad (12.73)$$

Ad (2.1), (2.2), (2.3) and (2.4): **Energy spectrum:** These parts include their derivations.

Ad (3): **Energy spectrum of a mode μ:**

As a consequence, the energy operator of a mode μ of a one - dimensional subsystem is as follows:

$$\hat{E}_{\mu,1D} = \hbar\omega_\mu \cdot \left(\hat{N}_\mu + \frac{1}{2}\right) = 2E_{\mu,kin,min,D1} \cdot \left(\hat{N}_\mu + \frac{1}{2}\right) \quad (12.74)$$

Hence, the spectrum is:

$$E_{\mu,1D} = \hbar\omega_\mu \cdot \left(n_\mu + \frac{1}{2}\right) = 2E_{\mu,kin,min,D1} \cdot \left(n_\mu + \frac{1}{2}\right) \quad (12.75)$$

Thereby, the minimum value $E_{\mu,kin,min,D1}$ is founded by the universal quantization and by the minimal energy stationary state of the DEQ of VD and of the GSEQ.

Using the ladder operators, number operators and their eigenvalues, expectation values of rates $\dot{\varepsilon}_{L,p}$, potentials Φ_{gen} and of fields $\vec{G}_{gen}$ can be derived. For the case of electromagnetism, similar derivations can be found in Ballentine (1998), for instance. Accordingly, the present theory provides and generalizes results of a quantum field theory. It has been applied in order to solve fundamental problems, such as the Hubble tension, see chapter (16) or Carmesin (2021d,a, 2023d,g, 2024c).

Ad (4): This part includes its derivation. q. e. d.

Chapter 13

VD implies mechanism of nonlocality

So far, locality (THMs 5, 7), the existence of quanta (THM 18), the existence of nonlocality (THM 19), and causality (THM 20) are derived as universal properties of the volume dynamics. In this chapter, we derive the mechanism that provides nonlocal phenomena.

13.1 Propagation of harmonic solutions

In this section, we analyze the propagation of portions of volume in nature on the basis of the DEQ of VD in THM (5). In particular, we analyze how that DEQ provides solutions that are capable of nonlocal phenomena:

(1) **The DEQ:**

A volume portion propagates according to the DEQ of VD as follows, see THM (5) and Fig. (6.4):

$$\frac{\partial}{\partial \tau}\varepsilon_{L,p} = -v \cdot \vec{e}_v \cdot \frac{\partial}{\partial \vec{L}}\varepsilon_{L,p} \text{ with } v = c \tag{13.1}$$

These VPs are also essential for objects that move at $v < c$, and that are described by the SEQ in THM (27), as the SEQ is derived from the above DEQ of VD as a special case for masses $m_0 > 0$.

(2) **The solutions:**

Solutions of the DEQ of VD with a center of energy are Gaussian wave packets in THM (6), for instance. More generally, also VPs without a center of energy are included. For instance, such VPs are described by harmonic solutions of that DEQ of VD, see THM (17). Consequently, such solutions have no wave group and no group velocity v_g. Instead, such solutions have a phase velocity v_p. Hence, in full generality, phase velocity v_p and group velocity v_g must be distinguished, similarly as in the analysis of causality, see PROP (20). In general, that distinction of v_g and v_p is essential for many questions in quantum physics, see THMs (6, 28, 21, 22, 23, 24) or (Scheck, 2013, section 1.3).

As a consequence, that distinction of v_g and v_p must be applied to the DEQ (13.1) of VD:

(3) **Distinction of v_g and v_p in the DEQ of VD:**

So far, we derived the DEQ of VD for the case of VPs with a center of energy and without rest mass m_0, see Fig. (6.4) and section (6.2). For that case, the velocity is equal to the velocity of light, $v = c$, see THM (3). Thereby, this holds for the phase velocity and for the group velocity, $v_p = v_g = c$.

(4) **Evident properties provide generalization:**

More generally, only a propagating group or center of energy provides an unequivocal and unique observation of the propagating object. Accordingly, a group velocity $v_g \leq c$ is required only for such a group. In contrast, in general, a phase φ cannot be distinguished from a phase $\varphi + 2\pi$. Correspondingly, in general, a phase velocity is not restricted by c.

As a matter of fact, the derivation of the DEQ of VD does not require any restriction of the velocity, see the proof of THM (5). As a consequence, the evident properties in section (4.2.2) are a basis for a generalization of the DEQ of VD to the same

DEQ with solutions with phase velocity $v_p \in]0, \infty[$.

(5) **Empirical evidence:**

Phase velocities larger than c have been measured. For instance, Ye et al. (2010) investigated microwaves at a frequency of 9.447 GHz. These waves propagated in a metamaterial from a microwave emitter to a microwave receiver at a distance of $d = 0{,}012\,\mathrm{m}$. Thereby, a phase velocity of $v_p = 5814 \cdot c$ has been measured.

In this experiment, the absolute phase can be controlled as follows: The antennas A_{em} of the emitter and A_{rec} of the receiver are placed at a table at variable distance d, and the signal of both antennas are displayed at a two channel oscilloscope.

$d = 0\,\mathrm{m}$: In a first step, the distance between A_{em} and A_{rec} is zero. Thus, the two channels show the same signal at the oscilloscope. In this state, a counter Z is set to zero.

In a second step, the distance d between A_{em} and A_{rec} is increased gradually, and at each time the phase difference at the oscilloscope reaches $\Delta\varphi = 2\pi$, the counter Z is increased by one. Thereby, d is increased, until the final value $d = 0.012\,\mathrm{m}$ is reached.

In a third step, the phase difference $\Delta\varphi(d)$ at the oscilloscope is read out. Consequently, the true phase difference is as follows: $\Delta\varphi = \Delta\varphi(d) + Z \cdot 2\pi$.

(6) **Summary:**

Altogether, only the group velocity is restricted by c. As a consequence, in the DEQ of VD, the phase - and group velocities are distinguished as follows:

Theorem 36 Law of propagation of relative additional volume or change

THM (5) is generalized: VPs or changes propagate as follows.

(1) If the relative additional volume or change, described by a change tensor $\varepsilon_{L,p}$ of rank two or larger, and described by a phase velocity v_p, whereby a possible center of energy is described by a group velocity v_g, is analyzed as a function of time τ and location $\vec{L}$, see Fig. (6.4), then it fulfills the following differential equation, DEQ:

$$\boxed{\frac{\partial \varepsilon_{L,p}}{\partial \tau} = -v\vec{e}_v \frac{\partial \varepsilon_{L,p}}{\partial \vec{L}};\ v = \begin{cases} v_g = c, & \text{for center of energy} \\ v_p \geq c, & \text{without center of energy} \end{cases}} \tag{13.2}$$

This DEQ is the **DEQ of VD for v_g and v_p**. *It holds at each location in empty space, according to the principle of translation invariance.*

More generally, objects at $v_g < c$ have nonzero rest mass $m_0 > 0$, and they are described by the SEQ, which is implied by the DEQ of VD:

$$\boxed{\text{DEQ of VD} \rightarrow SEQ, \text{describing}\, m_0 > 0\, \&\, v_g < c} \tag{13.3}$$

(2) Each VP or change propagates according to the following Lorentz invariant DEQ:

$$\dot{\varepsilon}^2_{L,p} - v^2 \left(\frac{\partial \varepsilon_{L,p}}{\partial \vec{L}} \right)^2 = 0;\ v = \begin{cases} v_g = c, & \text{for center of energy} \\ v_p \geq c, & \text{without center of energy} \end{cases} \tag{13.4}$$

(3) Objects moving at $v < c$ according to the SEQ are included, as the SEQ is derived from the above DEQ of VD for v_g and v_p, for the special case of objects with nonzero mass $m_0 > 0$, see THM (27).

13.2 Solutions in a subspace of Hilbert space

In general, a physical object can be described by a mixture of solutions of the DEQ (13.2) of VD $\varepsilon_{L,p}$. In particular, the

DEQ of VD implies the corresponding GSEQ, and it implies the respective SEQ, for the case of objects at $v < c$, so that these DEQs can be appropriate in a particular case.

In each case, a solution spans a subspace $\mathcal{H}_1$ of Hilbert space $\mathcal{H}$. In the following, we analyze the case of a pure state, not a mixed state, as a pure state is sufficient for the investigation of nonlocal phenomena, such as the delayed-choice experiment in Fig. (9.3).

For instance, in the delayed-choice experiment in Fig. (9.3), a photon entering the experiment is described by a wave function Ψ that spans a subspace $\mathcal{H}_1$.

At the first beam splitter BS_1, the wave splits into two parts:

Part 1: $\Psi_1 = \Psi/\sqrt{2}$ takes the right path (solid line in Fig. 9.3).

Part 2: $\Psi_2 = \Psi/\sqrt{2}$ takes the left path (dashed in Fig. 9.3).

13.3 Subspace change

The subspace $\mathcal{H}_1$ of $\mathcal{H}$ can be changed to a subspace $\mathcal{H}_2$ by a preparation of the experiment or by a measurement or by a similar preparation of the physical situation.

For instance, the subspace $\mathcal{H}_1$ of the mode (1) with the reflectivity $\rho = 1/2$ of the second beam splitter BS_2 can be changed to the subspace $\mathcal{H}_2$ of mode (2) with the reflectivity $\rho = 0$ of BS_2.

Another example occurs in mode (2): At each detector D_j, there arrives the part $\Psi_j = \Psi/\sqrt{2}$. When the detector D_1 detects the photon, then the part Ψ_2 becomes zero, so that no photon can be detected at D_2.

This is described with the wave function Ψ_{D_1} as follows:

Part 1 has the full amplitude of the original wave packet Ψ, whereas part 2 has the amplitude zero:

$$\Psi_{D_1} = \sqrt{2} \cdot \Psi_1 + 0 \cdot \Psi_2 \tag{13.5}$$

That wave function spans another subspace $\mathcal{H}_{D_1}$ of $\mathcal{H}$. Such a change of the subspace is sometimes called 'collapse of the wave function', see e. g. Garrisi et al. (2019).

In principle, such a change of the subspace must take place in an arbitrarily short time, so that the observed anticorrelation in mode (2) is provided. In fact, Garrisi et al. (2019) performed an experiment that exhibits a 'speed' of such a change of the subspace (collapse of the wave function) of $speed = w_{eff} = 1550 \cdot c$. As a consequence, in general, such a change of the subspace can take place at a speed of $w_{eff} \geq 1550 \cdot c$.

Similarly, Manning et al. (2015) performed a delayed - choice experiment for a single atom.

In the following, we show how the VD can provide such a rapid change of a subspace of Hilbert space $\mathcal{H}$.

13.4 Rapid subspace change

13.4.1 Omnipresent transient phenomena

In general, a change of a subspace of a solution of a linear DEQ occurs by a transient phenomenon, see Schiff (1991). Essential examples are as follows:

(1) When a resonant circuit is switched on, then a nearly harmonic oscillation gradually forms in a transient phenomenon, and a nearly harmonic oscillation occurs after the transient phenomenon.

(2) When a swing is initially at rest, and when the swing is then excited by a periodic motion of the user, then a nearly harmonic oscillation gradually forms in a transient phenomenon, and a nearly harmonic oscillation occurs after the transient phenomenon.

(3) When a swing is initially at rest, and when the swing at a point A is then excited from a distant point B with a periodic

motion at B, and via a soft elastic rope, then a nearly harmonic oscillation gradually forms in a transient phenomenon, and a nearly harmonic oscillation occurs after the transient phenomenon, whereby the duration of the transient phenomenon depends on the softness or stiffness of the elastic rope.

(4) In the examples (3.1), (3.2) and (3.3), an oscillator can be described by a linear differential equation, and the set of solutions can be described by a Hilbert space $\mathcal{H}$. Thereby, an excitation causes a change of the state in $\mathcal{H}$. Hereby, the change of the state in $\mathcal{H}$ is a transient phenomenon. Thereby, the excitation is the cause, and the harmonic oscillation of the oscillator is the effect.

13.4.2 Harmonic waves transmit transient phenomena

In general, such a transient phenomenon can be provided in terms of a linear combination of harmonic wave solutions in the form of a Laplace transform, see Schiff (1991). The respective detailed calculations are provided in (Carmesin, 2023g, chapter 16). As a consequence, the velocity, at which such a transient phenomenon takes place is determined by the velocity of propagation of the harmonic wave solutions. This velocity is not restricted by c, and it can be arbitrarily large, see section (13.1), THM (36).

As a consequence, the change of the subspace of $\mathcal{H}$ in a measurement or preparation can take place at an arbitrarily short time, and this is explained by the following mechanism of the VD:

Theorem 37 Explanation of nonlocality

(1) The harmonic wave solutions of the DEQ (13.2) in THM (36) of VD provide rapid transient phenomena. These can change the subspace of $\mathcal{H}$, that can be caused by a measurement or preparation, in an arbitrarily rapid manner. (2) Details are

derived in (Carmesin, 2023g, chapter 16).

(2) Mechanisms in a transient phenomenon:

In general, a change of the state in the Hilbert space in a transient phenomenon (see section 13.4.2) is described by the theory of Laplace transforms, see Schiff (1991), Carmesin (2023g). Such a transient phenomenon is constituted by the following elements:

(2.1) The transient phenomenon is transmitted by a linear combination (Fourier integral) of harmonic solutions of the respective DEQ (see section 13.4.2).

(2.2) Consequently, the time dt required by a transient phenomenon from one state in $\mathcal{H}$ compatible with the considered physical system to another state in $\mathcal{H}$ compatible with the considered physical system depends on the distance dL of cause and effect, and it depends on the phase velocity v_p of the harmonic solutions.

(2.3) As a consequence, the duration dt of the transient phenomenon is characterized by the ratio $\frac{dL}{v_p}$.

(2.4) In particular, in the VD, the phase velocity $v_p \in]0; \infty[$ is not restricted. As a consequence, the duration dt of the transient phenomenon tends to zero.

Proof: The proof is included in the theorem, and mathematical details are presented in (Carmesin, 2023g, chapter 16).

Chapter 14

VD implies energy density of volume

14.1 A great question

The energy density u_Λ corresponds to the cosmological constant Λ, see Einstein (1917). It is the energy density that is the same for each cosmological redshift. As a consequence, u_Λ can be regarded as a property of empty space, see e. g. Friedmann (1922), Lemaître (1927), Hobson et al. (2006). Thereby, empty space is space without any additional objects such as radiation or matter.

In outer space, Perlmutter et al. (1998) and Riess et al. (2000) discovered the accelerated rate of expansion of the universe. Based on the FLE, this implies a nonzero energy density of empty space, see e. g. Friedmann (1922), Lemaître (1927), Hobson et al. (2006), Carmesin (2019b). An actual value is $u_{\Lambda,obs} = 5.133 \cdot 10^{-10}\,\frac{\mathrm{J}}{\mathrm{m}^3}$, see Planck-Collaboration (2020).

The nature of an energy density of empty space is an important question and mystery: Zeldovich (1968) proposed that such an energy density would be based on the zero-point energy, ZPE, of the electromagnetic field, and in that context, Huterer and Turner (1999) suggested the name 'dark energy'. However, that proposed ZPE is by far too large, and that discrepancy is called cosmological constant problem. A solution to that prob-

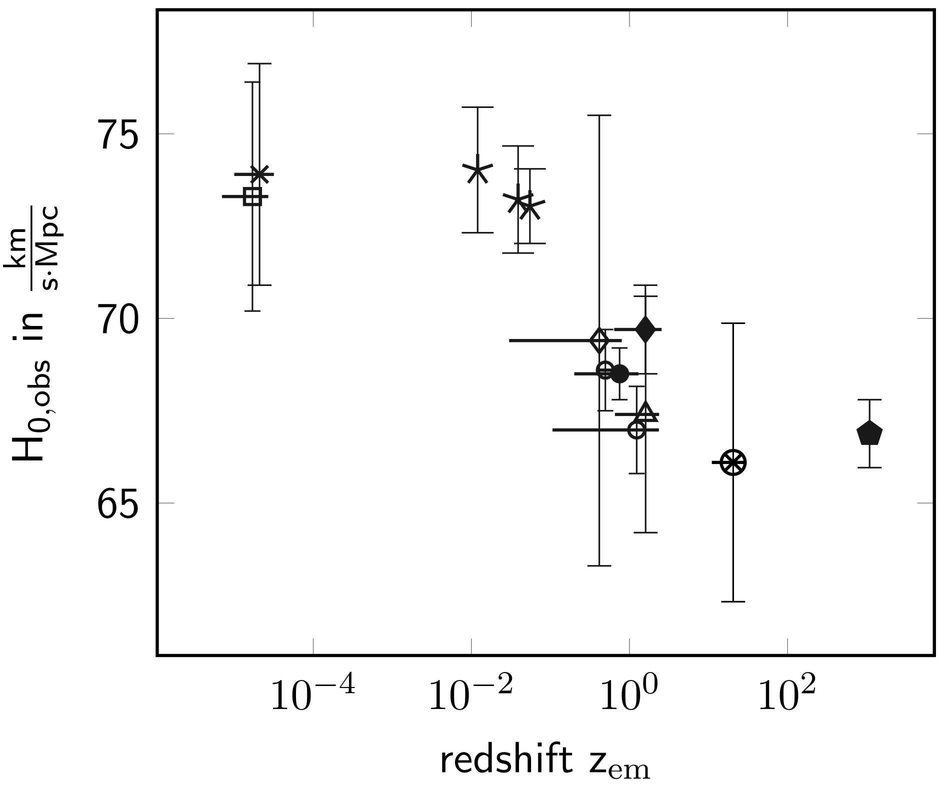

Figure 14.1: Observed values $H_{0,obs}$ of H_0 as a function of z_{em}. **Probes:** $\times$, megamaser, Pesce et al. (2020). $\star$, distance ladder with SN type Ia, Riess et al. (2022), Galbany et al. (2023), (Uddin et al., 2024, most precise evaluation). full $\diamond$, starburst galaxies, Cao et al. (2021). o, baryonic acoustic oscillations, BAO, Philcox et al. (2020), Addison et al. (2018)). $\bullet$, weak gravitational lensing and galaxy clustering, Abbott et al. (2020)). Δ, strong gravitational lensing, Birrer et al. (2020). $\diamond$, gravitational wave, Escamilla-Rivera and Najera (2022). $\otimes$, old galaxies or stars, Cimatti and Moresco (2023), (Tab. 1). Square, surface brightness, Blakeslee et al. (2021). Pentagon, CMB, Planck-Collaboration (2020).

lem is presented in chapter (15).

Also in cosmology, the energy density of empty space remains a mystery: Riess et al. (2022) used local galaxies as a probe, in order to measure the rate of expansion of the universe. Thus, the local galaxies emitted the observed radiation in the late universe. On the basis of that probe, a local value $H_{0,local}$ or late value $H_{0,late}$ of the Hubble constant has been discovered. Thereby, the late value $H_{0,late}$, with probes emitted at an averaged redshift $\langle z_{em} \rangle = 0.055$, differs significantly from the value measured in the early universe $H_{0,early}$, with probes emitted at the redshift $z_{CMB} = 1090.3$, see Fig. (14.1). Thus, H_0 apparently is a heterogeneous quantity.

However, in the usual ΛCDM model of cosmology, see e. g. (Workman et al., 2022, chapters 21, 22, 25-29), Hobson et al. (2006), H_0 should be a constant and homogeneous quantity. Such a discrepancy is named Hubble tension or H_0 tension, see Fig. (14.1) or (Planck-Collaboration, 2020, p. 46), Riess et al. (2022); Galbany et al. (2023). This problem is solved in chapter (16).

In order to analyze the above question of the heterogeneity of H_0, and of space more generally, we derived the homogeneous energy density u_{vol} of volume in nature in THM (8).

In this chapter, we derive the energy density of volume in nature u_{vol}, from the VD. Thereby, we show that u_{vol} is in precise accordance with the observed value $u_{\Lambda,obs}$.

14.2 Process of formation of volume

In order to derive the energy density u_{vol} of volume in nature, we analyze the process of formation of volume in nature in three - dimensional space.

For it, we remind that a good approximation of volume in nature is outer space, as there are only few molecules that could disturb that volume in nature.

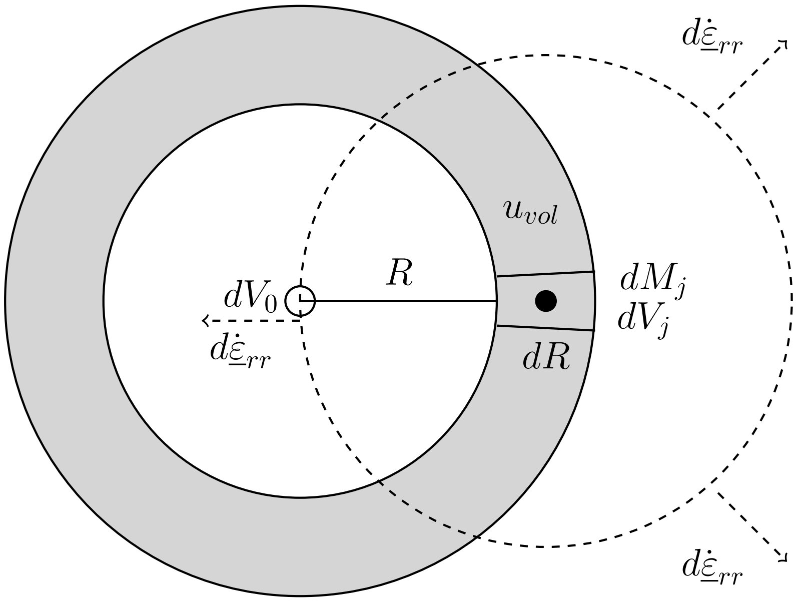

Figure 14.2: The energy density u_{vol} in a shell at a distance R from dV_0 has a $j-th$ dynamic mass dM_j. It generates rates $d\dot{\varepsilon}_{rr}$ propagating towards all directions in the same manner.

(1) **Formation of volume in nature in empty space:**

In order to derive the energy density u_{vol} of volume in nature, we use outer space as an approximation, and we improve that approximation by removing all content in outer space. As a consequence, we analyze the formation of volume in nature in an empty space.

(2) **Formation of volume in nature since the Big Bang:**

The present-day volume in nature has formed since the Big Bang. The duration of that process is essentially described by the Hubble time t_{H_0}, the inverse of the Hubble constant H_0:

$$t_{H_0} = \frac{1}{H_0} \tag{14.1}$$

More precise details occur in a universe with content, see e. g. Carmesin (2019b) or chapter (16).

(3) **Global flatness:**

Carmesin (2023h,g) showed the global flatness of space. As a consequence, the dynamical density of the universe has the critical value, see e. g. Hobson et al. (2006), Carmesin (2019b), or see section (16.1):

$$\rho_{cr} = \frac{3 \cdot H_0^2}{8\pi G} \quad \text{or} \quad u_{cr} = \frac{3 \cdot H_0^2 c^2}{8\pi G} \tag{14.2}$$

As a usual convention, the density parameter of u_{vol} is as follows, see section (16.1):

$$\Omega_{vol} := \frac{u_{vol}}{u_{cr}} \tag{14.3}$$

As a further consequence, space and time can be analyzed separately, and time increases at a constant rate, at a global level.

(4) **Probe volume dV_0:**

As a consequence of translation invariance in section (4.2.2), the formation of volume in nature can be analyzed with help of the formation of a probe volume dV_0. Thereby, dV_0 describes a fixed amount of size that fills with volume in nature during the Hubble time:

$$dV_0 \ \text{ fills with volume in nature during } t_{H_0} \tag{14.4}$$

(5) **Local formation of volume in dV_0:**

The formation of volume in nature in dV_0 is described by locally formed volume in dV_0, since the propagation of volume through dV_0 does not describe the formation of that LFV. That LFV has a normalized rate $\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}$ of unidirectional LFV at dV_0, see THM (13):

$$\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0} = \frac{\underline{\delta} V_{rr}}{\underline{\delta} t \cdot dV_0} \tag{14.5}$$

(6) **LFV in dV_0 is caused at increments dV_j:**

As the considered universe is empty, the only source for the LFV in dV_0 is the volume in nature in increments dV_j, see Fig. (14.2).

(7) $\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}(t_0)$ **is identified with** H_0**:**

The present-day normalized rate $\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}(t_0)$ of LFV at dV_0 is equal to one third of the normalized isotropic rate of formation of volume, see DEF (7):

$$\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0} = \frac{1}{3} \cdot \dot{\underline{\varepsilon}}_{L,iso,\ at\ dV_0} \tag{14.6}$$

The present-day normalized rate H_0 is equal to one third of the normalized isotropic rate of formation of volume, see Eq. (8.38):

$$H_0 = H(t_0) = \frac{1}{3} \cdot \frac{\dot{V}}{V} \tag{14.7}$$

As the rates $\frac{\dot{V}}{V}$ and $\dot{\underline{\varepsilon}}_{L,iso,\ at\ dV_0}$ describe the same formation of volume in nature at dV_0, these rates are equal, see Eqs. (8.46):

$$H_0 = \frac{1}{3} \cdot \frac{\dot{V}}{V} = \frac{1}{3} \cdot \dot{\underline{\varepsilon}}_{L,iso,\ at\ dV_0} = \dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0} \tag{14.8}$$

(8) **The emitted RGWs do not cancel each other:**

The RGWs emitted by quanta of volume in nature at increments dV_j do not cancel each other on their way to dV_0. This is formally derived in chapter (16). Similarly, the light - portions emitted by the Sun do not cancel each other on their way to Earth. As another analogy, in a hollow ball with light emitting walls, the light portions do not cancel each other.

14.3 Derived rate of formation

In this section, we derive the present-day normalized rate of formation $\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}(t_0)$ of volume in nature in Eq. (14.6).

14.3.1 Formation of volume by a volume portion

An incremental volume dV_j has the dynamic mass dM_j:

$$dM_j = dV_j \cdot \rho_{vol,h}. \tag{14.9}$$

According to exact gravity in THMs (9, 10), each dynamic mass dM_j causes the following exact field in its near vicinity:

$$|d\vec{G}_j^*|(R) = \frac{G \cdot dM_j}{R^2} \tag{14.10}$$

As a part of a RGW, this field can propagate. Thereby, the RGWs of different dynamic masses dM_j do not cancel each other, see part (8) in section (14.2), see Fig. (14.2).

According to the law of **unidirectional** formation of volume, see THM (13), at the probe volume dV_0, each dynamic mass dM_j at a distance R causes the following normalized rate of unidirectional formation of volume:

$$d\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}(R, dM_j) = \frac{|d\vec{G}_j^*|(R)}{c} = \frac{G \cdot dM_j}{R^2 \cdot c} \tag{14.11}$$

14.3.2 Formation of volume by one shell

The volume that is caused by the dM_j in the shell in Fig. (14.2) is derived next:

Shell around dV_0: The dynamic masses dM_j at a distance R from dV_0 constitute a shell with center dV_0, and with a distance R from dV_0, and with a thickness dR, so that the dynamic mass of the shell is the sum of the dynamic masses dM_j in the shell, see Fig. (14.2):

$$dM(R, dR) = \sum_{j,\ R_j \in \text{shell}} dM_j \tag{14.12}$$

Rate caused by shell around dV_0: The rates in Eq. (14.11), that are caused in the shell in Eq. (14.12), are added:

$$\sum_{j,\ R_j \in \text{shell}} d\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}(R, dM_j) = \frac{G \cdot \sum_{j,\ R_j \in \text{shell}} dM_j}{R^2 \cdot c} \tag{14.13}$$

In the above Eq., the sum of the masses is identified with $dM(R, dR)$ in Eq. (14.12). Consequently, the rate is as follows:

$$d\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}(R, dR) = \frac{G \cdot dM(R, dR)}{R^2 \cdot c} \tag{14.14}$$

Application of the dynamic density: According to the global flatness, the dynamic mass $dM(R, dR)$ is equal to the product of the dynamic density ρ_{vol} and the volume $dV = 4\pi \cdot R^2 \cdot dR$ of the shell, see Fig. (14.2):

$$dM(R, dR) = \rho_{vol} \cdot 4\pi \cdot R^2 \cdot dR \tag{14.15}$$

As a consequence, the rate in Eq. (14.14) is as follows:

$$d\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}(R, dR) = \frac{4\pi \cdot G \cdot \rho_{vol} \cdot dR \cdot R^2}{R^2 \cdot c} \tag{14.16}$$

It is instructive to realize that the rate caused by a shell is not a function of the radius of the shell, as the terms R^2 cancel out. This cancellation does also take place, if R is replaced by the comoving distance, see (Condon and Mathews, 2018, section 4.3). Consequently, the rate is as follows:

$$d\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}(dR) = \frac{4\pi \cdot G \cdot \rho_{vol} \cdot dR}{c} \tag{14.17}$$

14.3.3 Light - travel time of one shell

In this section, the shell in Fig. (14.2) is described with help of light - travel times t_{LT}:

The shell in Fig. (14.2) is described with help of a smallest radius R, and by a largest radius $R + dR$.

These radii are described by the light - travel time of a light - portion that travels from these radii to dV_0:

$$t_{LT,R} = \frac{R}{c} \tag{14.18}$$

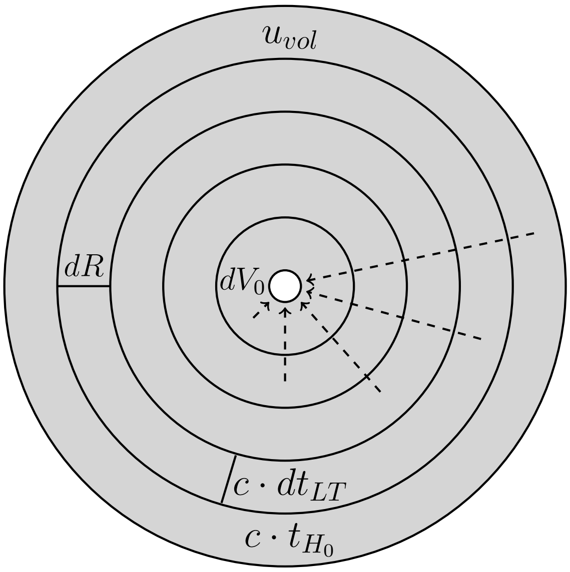

Figure 14.3: Ball with center dV_0 and Hubble radius $c \cdot t_{H_0}$: That ball is partitioned into shells with the same center dV_0 and thickness $c \cdot dt_{LT}$. Each shell causes the same rate (dashed), $\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}(c \cdot t_{LT,R}, c \cdot dt_{LT,dR})$, arriving at dV_0.

$$t_{LT,R+dR} = \frac{R+dR}{c} \quad \text{or} \quad dt_{LT} = \frac{dR}{c} = t_{LT,R+dR} - t_{LT,R} \tag{14.19}$$

The RGWs, that transfer the gravity in Fig. (14.2), propagate at $v = c$. As a consequence, the largest light - travel time of these RGWs, is the Hubble time t_{H_0}:

$$t_{LT,max} = t_{H_0} \tag{14.20}$$

Altogether, the sources dM_j that contribute to the LFV at dV_0, can be partitioned into shells, whereby each shell has light - travel times in an interval $[t_{LT,R}; t_{LT,R} + dt_{LT}]$, see Fig. (14.3).

As a consequence, the rate in Eq. (14.17) is expressed in terms of the light travel time as follows:

$$d\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}(dt_{LT}) = 4\pi \cdot G \cdot \rho_{vol} \cdot dt_{LT} \tag{14.21}$$

14.3.4 Integration of the shells

In this section, the shells in Figs. (14.2, 14.3) are integrated: For each shell, the time increments are integrated from zero to the maximum t_{H_0}, and the rate increments are integrated from zero to the resulting integrated rate $\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}$. These integrals are applied to the rate in Eq. (14.21):

$$\int_0^{\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0}} d\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0} = 4\pi \cdot G \cdot \rho_{vol} \cdot \int_0^{t_{H_0}} dt_{LT} \qquad (14.22)$$

The above integral is evaluated:

$$\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0} = 4\pi \cdot G \cdot \rho_{vol} \cdot t_{H_0} \qquad (14.23)$$

The above rate is identified with H_0, see Eq. (14.8):

$$H_0 = 4\pi \cdot G \cdot \rho_{vol} \cdot t_{H_0} \qquad (14.24)$$

The above Eq. is divided by t_{H_0}:

$$\frac{H_0}{t_{H_0}} = \rho_{vol} \cdot 4\pi \cdot G \qquad (14.25)$$

As the right hand side of the above Eq. is a constant (see THM 8), the fraction is the same for each value t_{H_1} of the Hubble time, and for the corresponding value of the Hubble constant $H_1 = 1/t_{H_1}$:

$$\frac{H_0}{t_{H_0}} = \rho_{vol} \cdot 4\pi \cdot G = \frac{H_1}{t_{H_1}} \qquad (14.26)$$

The above Eq. is solved for the dynamic density $\rho_{vol} = u_{vol}/c^2$. Consequently, the dynamic density of volume is as follows:

$$\rho_{vol} = \frac{H_0}{t_{H_0}} \cdot \frac{1}{4\pi \cdot G} = \frac{H_1}{t_{H_1}} \cdot \frac{1}{4\pi \cdot G} \qquad (14.27)$$

Next, the Hubble constant is identified with the inverse Hubble time in Eqs. (14.25, 14.26, 14.27). Moreover, we use the rate

$\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0} = H_0$ in Eq. (14.8),

$$\boxed{\rho_{vol} = \frac{H_0^2}{4\pi \cdot G} \quad \text{or} \quad u_{vol} = \frac{H_0^2 c^2}{4\pi \cdot G} = \frac{H_1^2 c^2}{4\pi \cdot G} = \frac{\dot{\underline{\varepsilon}}^2_{L,rr,\ at\ dV_0} c^2}{4\pi \cdot G}} \tag{14.28}$$

14.3.5 Amount of formed volume

Next, we analyze the amount $\underline{\delta}V$ of formed volume. For it, we solve the rate in Eq. (14.5) for the volume:

$$\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0} = \frac{\underline{\delta}V_{rr}}{\underline{\delta}t \cdot dV_0} \tag{14.29}$$

$$\text{thus,}\ \ \underline{\delta}V_{rr} = \dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0} \cdot \underline{\delta}t \cdot dV_0 \tag{14.30}$$

We use the rate $\dot{\underline{\varepsilon}}_{L,rr,\ at\ dV_0} = H_0$ in Eq. (14.8), and we analyze the full duration of the process of formation $\underline{\delta}t = t_{H_0}$:

$$\underline{\delta}V_{rr} = H_0 \cdot t_{H_0} \cdot dV_0 \tag{14.31}$$

We apply the identity $H_0 = 1/t_{H_0}$:

$$\underline{\delta}V_{rr} = dV_0 \tag{14.32}$$

Consequently, during the Hubble time t_{H_0}, the full amount of the probe volume dV_0 forms in terms of volume in nature.

We summarize the derived results:

Theorem 38 Law of the derived energy density of volume in an empty universe

(1) In a universe consisting of volume only, the process of GFV from LFV causes the following energy density of volume:

$$u_{vol} = \frac{c^2 H_0^2}{4\pi G} \tag{14.33}$$

The energy density of volume has the following amount:

$$u_{vol} = \frac{c^2 H_0^2}{4\pi G} = 5.040\,(\pm 0.1395) \cdot 10^{-10}\,\frac{\text{J}}{\text{m}^3} \tag{14.34}$$

Hereby, the Hubble constant H_0 is taken from observation, see Planck-Collaboration (2020), as H_0 represents a calendar date, the age of the universe $t_0 \approx 1/H_0$. A precise relation is provided in chapter (16) or Carmesin (2019b). The observed value $u_{\Lambda,obs}$ is as follows, see Planck-Collaboration (2020):

$$u_{\Lambda,obs} = 5.133\,(\pm 0.2432) \cdot 10^{-10}\,\frac{\mathrm{J}}{\mathrm{m}^3} \qquad (14.35)$$

The derived result u_{vol} is in precise accordance with the observed value $u_{\Lambda,obs}$.

For the first time, a fundamental and very general theory provides the functional relation for u_{vol}, see Eq. (14.34): the VD. For the development of that theory and results, see (Carmesin, 2021d, THM 21), and in a similar form, see (Carmesin, 2018f, section 2.7-2.13), (Carmesin, 2019b, section 2.11), Carmesin (2021d), (Carmesin, 2021b, Eq. 7), (Carmesin, 2023g, chapters 18-21).

(2) The energy density of volume u_{vol} is a consequence of the process of formation of volume that has been forming since the Big Bang until the present-day time t_0. If that process ranges from the Big Bang to another time $t_1 \neq t_0$, then that process provides the same density of volume.

(3) The energy density of volume u_{vol} is in precise accordance with the observed energy density $u_{\Lambda,obs}$ of the cosmological constant Λ or of dark energy, Planck-Collaboration (2020), Workman et al. (2022). This provides convincing evidence for the fact that the density of dark energy essentially is the density of volume in nature.

For the first time, a fundamental and very general theory clarifies the dark energy, and it provides functional relations for u_{vol} and u_Λ and the complete energy density of electromagnetic zero - point oscillations, see Eq. (14.34) and THMs (43, 40): the VD. For the development of that theory and results, see (Carmesin, 2021d, chapters 7,8), and in a similar form, see

(Carmesin, 2018f, section 2.7-2.13, Fig. 1.28), or see e. g. (Carmesin, 2019b, sections 2.11-2.13, Fig. 2.15), (Carmesin, 2023g, chapters 18-21).

(4) An even more precise and differentiated relation between u_{vol} and $u_{\Lambda,obs}$ is derived in chapter (16).

(5) Altogether, three quantities have been measured in the context of volume in nature or of vacuum:

(5.1) The energy density of volume u_{vol} has been measured with help of the accelerated expansion of the universe, see for instance Perlmutter et al. (1998), Riess et al. (2000), or also Planck-Collaboration (2020).

(5.2) In the volume in nature, there occur change tensors representing the (typically microscopic) fluctuations of the electromagnatic field, these have been measured with help of two parallel conducting plates with provide the Casimir force, see e. g. Casimir (1948), Klimchitskaya et al. (2009).

(5.3) The volume in nature exhibits a rate of expansion. It can be characterized with help of the Hubble constant H_0. H_0 is a function of the time t, or of the calendar date t starting at the Big Bang, or the cosmological redshift z_{em} or z. The function $H_0(z_{em})$, supplemented with data, is shown in Fig. (16.4).

We consider radiation that has been emitted in the late universe, and that has been observed at Earth. Such radiation has only traveled a relatively short time, compared to t_{H_0}. Consequently, such radiation has been emitted in the vicinity of Earth. As a consequence, the late universe value $H_0(z_{late})$ corresponds to a local universe value. The late universe value $H_0(z_{late})$ differs from the early universe value at the five sigma confidence level, see Fig. (14.1) or e. g. Riess et al. (2022).

(5.4) The observations in items (5.1), (5.2), and (5.3) are explained by the VD, see THMs (38, 40, 43). Thereby, no fit is executed, no hypothesis or supposition or assumption has been

proposed, and precise accordance with observation (within the errors of measurement) has been achieved. This provides a significant evidence in favor of the VD.

Moreover, the VD clarifies the relations among these three observations and observed quantities. In this sense, the VD solves the respective cosmological constant problem (THM 40) Thereby, the VD clarifies these three observed quantities together with the respective theories of QP, generalized and modified QFT, microscopic space, fluctuations of electromagnetic fields, GR, the cosmological constant Λ*, cosmology, outer space, and the energy density of volume* u_{vol}*.*

Additionally, the VD solves many fundamental problems, see Fig. (18.2). In addition, the VD predicts important fundamental quantities, see Fig. (18.3). Furthermore, the VD provides essential theories, see Fig. (18.1).

14.4 The essentials for formation of volume in nature

In this section, we provide a detailed investigation of the properties of volume in nature, that are related to that process of formation of volume in nature.

Theorem 39 The essentials of volume in nature

Volume in nature is related to the process of formation of volume in nature and to the energy density u_{vol} *of volume in nature, see THM (38):*

(1) Volume in nature can be described by **observable VPs***, see THM (4):* dV_R *or* dV_L*.*

(2) Such VPs exhibit change. It is described by **change tensors** $\varepsilon_{L,p}$*. In particular, rank two change tensors are* $\varepsilon_{L,ij}$*, see THM (4).*

(3) Change tensors $\varepsilon_{L,p}$ propagate according to the **DEQ of VD** *in THM (5), or generalized in THM (36)*[1].

(4) Change tensors $\varepsilon_{L,p}$ with a center of energy exhibit minimal energy VPs with derived universal **quanta** *or universal* **quantization**, *see THMs (18, 21, 22, 23, 24).*

(5) On the measurement of u_{vol}:

(5.1) The energy density u_{vol} can be **measured** *with help of the accelerated expansion of the universe, see for instance Perlmutter et al. (1998), Riess et al. (2000), or Planck-Collaboration (2020).*

(5.2) u_{vol} is positive. Consequently, u_{vol} is not the negative energy density of self - interaction. As a consequence, u_{vol} represents the positive **kinetic energy density** *of the quanta of the VPs, see THMs (11, 16, 25). This is in accordance with universal quantization in THMs (18, 25).*

(6) On the wavelength:

(6.1) In general, the changes are polychromatic.

(6.2) The LFV formed in the early universe has a **preferential wavelength** *and includes two modes, each with the energy density $u_{mode,1} = u_{mode,2} = \frac{\underline{\varepsilon}^2_{L,jj} \cdot c^2}{8\pi G}$, see Carmesin (2021d,a) and (Carmesin, 2023g, THM 41). Hereby, in the far distance approximation, $\dot{\underline{\varepsilon}}^2_{L,jj}$ is the same as $\dot{\varepsilon}^2_{L,jj}$, see (Carmesin, 2023g, Corollary 14 or Eq. (9.42)). The sum of these densities is equal to the energy density of volume that forms in an empty universe, $u_{vol} = \frac{\underline{\varepsilon}^2_{L,jj} \cdot c^2}{4\pi G}$.*

(6.3) That preferential wavelength explains **observed masses of elementary particles**, *see Carmesin (2021a,f, 2022e): the sum of the masses of the neutrinos, the mass of the Higgs boson, the masses of the Z - and W - bosons.*

[1] For instance, electromagnetic waves are described by asymmetric non - diagonal change tensors, and they propagate according to the DEQ of VD, see THM (7).

(7) The LFV that formed in the realistic heterogeneous universe **explains the Hubble tension** *and the local value of the Hubble constant, see THM (43).*

(8) As an advanced organizer, please note: The electromagnetic properties of volume in nature, including the Casimir force and the solution of the cosmological constant problem, will be treated in chapter (15) and THM (40).

Proof: It is included in the THM.

Chapter 15

VD solves cosm. const. problem

15.1 Kinetic energy of ZPOs

In general, in universal quantization (THM 18), the observable energy is a kinetic energy $p \cdot c$.

In order to solve the usual cosmological constant problem in Nobbenius (2006), we provide three results:

(1) We derive the energy density of volume (THM 38), $u_{vol} = 5.040\,(\pm 0.1395) \cdot 10^{-10}\,\frac{\mathrm{J}}{\mathrm{m}^3}$. And we show that u_{vol} is in precise accordance with the observed energy density of outer space $u_{\Lambda,obs} = 5.133\,(\pm 0.2432) \cdot 10^{-10}\,\frac{\mathrm{J}}{\mathrm{m}^3}$, see Planck-Collaboration (2020), Carmesin (2020e).

(2) We show that the electromagnetic waves (THM 7) have electromagnetic **z**ero-**p**oint **o**scillations, ZPOs (THMs 21), with a kinetic energy $E_{min,kin,\omega} = \frac{\hbar\omega}{2} = p_{min,kin,\omega} \cdot c$. These electromagnetic ZPOs are reflected at the pair of parallel conducting plates in Fig. (15.3). Thereby, their momenta $p_{min,kin,\omega}$ are transmitted to the plates, $\Delta p = 2p_{min,kin,\omega}$, and they cause a force upon the plates: the Casimir force $F_{Casimir}$ with a corresponding underpressure:

$$P_{Casimir}(R) = \frac{F_{Casimir}}{L^2} \frac{\hbar \cdot c \cdot \pi^2}{240 \cdot R^4} \tag{15.1}$$

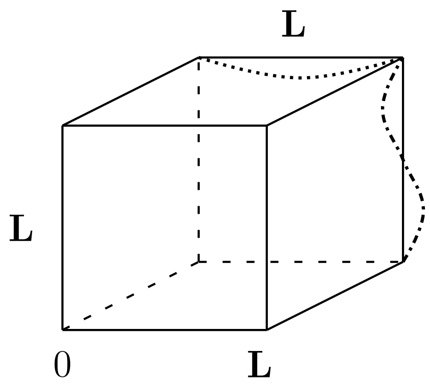

Figure 15.1: Kinetic energy of **z**ero-**p**oint **o**scillations, ZPOs, at circular frequencies $\omega = k \cdot c$ with $k \leq k_{max}$ in a cube with length L add up an energy $E(k_{max}, L)$. For instance, a mode with $\lambda_x = \lambda_1 = 2L$ (dotted) and a mode with $\lambda_y = \lambda_2 = L$ (dashdotted) are indicated.

That underpressure has been measured as a function of the distance R of the plates, see Klimchitskaya et al. (2009). Thereby, the amount of measured ZPOs is manipulated by the distance R of the plates. This amount of ZPOs can change (or manipulate) the measurable Casimir force $F_{Casimir}$. Thus, according to the Hacking (1983) criterion, the ZPOs of electromagnetic waves are real. For a derivation, including a critical comparison with other derivations, see sections (15.3.2, 15.3.4).

(3) We show that the kinetic energy of the electromagnetic ZPOs $E_{min,kin,\omega}$ is compensated exactly by a potential energy. It is the self interaction of quanta (25). We clarify that this exact compensation of energies provides the marginal stability of quanta.

15.1.1 Density of kinetic energy of ZPOs

The kinetic energies $E_{min,kin,D=1}$ of the ZPO modes (THMs 21, 22, 23, 24) in a cube with length L are added. Each mode with a circular frequency ω has the ZPE $\frac{\hbar\omega}{2}$, and the wave number $|k| = \omega/c = \sqrt{k_1^2 + k_2^2 + k_3^2} = \sqrt{\vec{k}^2}$. Hereby, each wave number

k_j describes a standing wave in the cube with a wavelength $\lambda_j = \frac{2L}{n_j} = \frac{2\pi}{k_j}$, with $n_j \in \{1, 2, 3, 4, 5, ...\}$ and $j \in \{1, 2, 3\}$. This Eq. is solved for n_j and triples $(n_1, n_2, n_3) := \vec{n}$ are abbreviated in terms of the vector notation, see Fig. (15.2):

$$n_j = \frac{L}{\pi} k_j \quad \text{and} \quad \vec{n} = \frac{L}{\pi} \vec{k} \quad \text{and} \quad n_{max} = \frac{L}{\pi} k_{max} \tag{15.2}$$

Each triple $\vec{n} = \frac{L}{\pi}\vec{k}$ represents the two polarization modes orthogonal to $\vec{k}$ and the energy $2 \cdot \frac{\hbar\omega(\vec{n})}{2}$.

Hence, $E_{min,kin}(k_{max}, L)$ is the sum of these energies for all triples, see Fig. (15.2):

$$E_{min,kin}(n_{max}, L) = 2 \sum_{\vec{n}}^{n_{max}} \frac{\hbar\omega}{2}(\vec{n}) \tag{15.3}$$

As each triple $\vec{n}$ has the volume one in the space of triples (n_1, n_2, n_3), the sum can be replaced by an integral over the positive octant of a sphere. Hereby, polar coordinates are used:

$$E_{min,kin}(n_{max}, L) = \frac{1}{8} \int_0^{n_{max}} \hbar\omega(\vec{n}) 4\pi n^2 dn \tag{15.4}$$

The integration variable n is substituted by $\frac{L}{\pi}k$, see Eq. (15.2). Additionally, $\omega = c \cdot k$ is used:

$$E_{min,kin}(k_{max}, L) = \frac{\pi}{2} \int_0^{k_{max}} \hbar\omega(\vec{k}) k^2 dk \cdot \left(\frac{L}{\pi}\right)^3 \tag{15.5}$$

$$\text{thus, } E_{min,kin}(k_{max}, L) = \frac{L^3}{2\pi^2} \int_0^{k_{max}} \hbar c k^3 dk = \frac{\hbar \cdot c \cdot k_{max}^4 L^3}{8\pi^2} \tag{15.6}$$

Consequently, the density $u_{ZPO,kin} = \frac{E_{min,kin}(k_{max}, L)}{L^3}$ of kinetic energy is as follows:

$$u_{ZPO,kin} = \frac{\hbar \cdot c \cdot k_{max}^4}{8\pi^2} \tag{15.7}$$

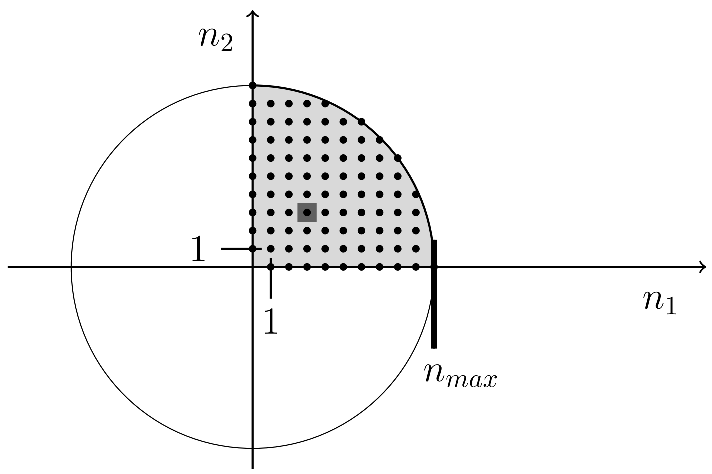

Figure 15.2: Modes (small dots) (n_1, n_2) in two - dimensional space: Area corresponding to one mode (dark gray) is equal to one. Area with $|\vec{n}| < n_{max}$ is in the positive quadrant (medium gray).

15.1.2 Maximal kinetic energy density of ZPOs

The kinetic energy density $u_{ZPO,kin}$ has the following upper limit: The lower physical limit of the wavelength is the circumference with the radius equal to the Planck length $2\pi L_P$, see THM (29). As a consequence, the upper physical limit of the wave number k is as follows, see THMs (21, 23, 24):

$$k_{max} = \frac{2\pi}{\lambda} = \frac{2\pi}{2\pi \cdot L_P} = \frac{2\pi}{\lambda} = \frac{1}{L_P} \tag{15.8}$$

Hereby, we used the fact that the maximal wave number k_{max} occurs at the smallest radius L_P of a fluctuation, whereby the ZPO has minimal energy at $\lambda = 2\pi r = 2\pi L_P$.

Thus, $L_P = 1.616 \cdot 10^{-35}\,\mathrm{m}$, see table (19.2), provides the following upper limit of the kinetic energy density $u_{ZPO,kin}$:

$$u_{ZPO,kin} = \frac{\hbar \cdot c}{8\pi^2 \cdot L_P^4} = 5.875 \cdot 10^{111}\,\frac{\mathrm{J}}{\mathrm{m}^3} \tag{15.9}$$

15.2 Foundation of the problem

Einstein (1917) proposed the cosmological constant Λ as a property of space that is the same for each cosmological redshift z. That constant corresponds to the following energy density of space, see THM (8) or Zeldovich (1968), Hobson et al. (2006):

$$u_\Lambda = \frac{\Lambda \cdot c^4}{8\pi \cdot G} \tag{15.10}$$

Also the kinetic energy density $u_{ZPO,kin}$ of the ZPOs in Eq. (15.9) is the same for each cosmological redshift z. Accordingly, Zeldovich (1968) proposed that the term that represents this energy density $u_{ZPO,kin}$ would be equal to u_Λ, see also (Nobbenius, 2006, p. 3), Cugnon (2012).

However, the observed value $u_{\Lambda,obs} = 5.133 \cdot 10^{-10} \, \frac{\mathrm{J}}{\mathrm{m}^3}$ of u_Λ is relatively small, see Eq. (14.35) or e. g. Planck-Collaboration (2020).

In contrast, the physical upper limit in Eq. (15.9),

$$u_{ZPO,kin,upper\ limit} = 5.875 \cdot 10^{111} \, \frac{\mathrm{J}}{\mathrm{m}^3}, \tag{15.11}$$

is relatively large and very huge, indeed. Similarly, the energy density $u_{ZPO,kin} = 24.4 \, \frac{\mathrm{J}}{\mathrm{m}^3}$ based on the value observed with help of the plates in Fig. (15.3) is also much larger than the value $u_{\Lambda,obs} = 5.133 \cdot 10^{-10} \, \frac{\mathrm{J}}{\mathrm{m}^3}$ observed in outer space or intergalactic space.

As a consequence, the kinetic energy density of the physically possible ZPOs exceeds the observed energy density $u_{\Lambda,obs}$ of outer space by the following factor:

$$factor = \frac{u_{ZPO,kin,upper}}{u_{\Lambda,obs}} = 1.14 \cdot 10^{121} \tag{15.12}$$

Consequently, that factor represents a huge discrepancy between the observation $u_{\Lambda,obs}$ and the result $u_{ZPO,kin,upper}$ of the present - day quantum field theory, QFT, see e. g. Zeldovich (1968), Peskin and Schroeder (1995), (Schwartz, 2014, section 4.2), Nobbenius (2006). This discrepancy is called cosmological constant problem, see e. g. Nobbenius (2006), Cugnon (2012), or (Schwartz, 2014, section 4.2).

15.3 Evidence for ZPOs

15.3.1 Two possilities

The cosmological constant problem in section (15.2) is based on the ZPOs of the electromagnetic field. Consequently, there remain two possibilities:

(1) Either the ZPOs of the electromagnetic field do not exist.

(2) Or the ZPOs of the electromagnetic field exist, and their energy density $u_{ZPO,kin}$ in Eq. (15.9) is compensated.

Next, we provide empirical evidence for the ZPOs of the electromagnetic field.

15.3.2 On the derivation of the Casimir force

In principle, ZPOs can be reflected at a conducting plate and exert a force upon that plate. This mechanism is analyzed with help of the system in Fig. (15.3).

Casimir (1948) proposed the force in Fig. (15.3). It is called Casimir force $\vec{F}_{Casimir}$.

The Casimir force has exceptional features: $\vec{F}_{Casimir}$ provides an underpressure $P_{Casimir} = -|\vec{F}_{Casimir}|/L^2$ between the conducting plates in Fig. (15.3). Thereby, $P_{Casimir}$ does neither depend on the temperature, nor on material constants of the conducting plates in Fig. (15.3), nor on the length L of the conducting plates in Fig. (15.3). Instead, $P_{Casimir}$ has the

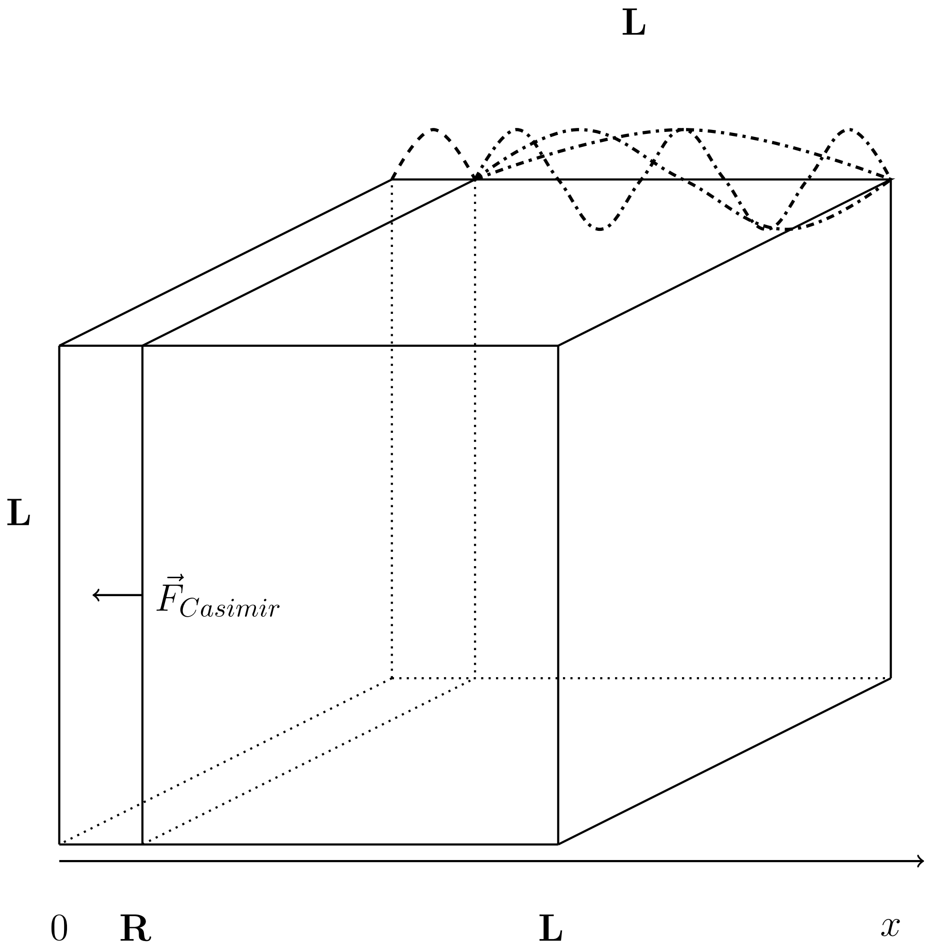

Figure 15.3: Two conducting plates at $x = 0$ and $x = R$ in the cube in Fig. (15.1): Between these plates, there are only modes with short wavelengths $\lambda \leq 2R$ (dashed). At $x > R$, there are also modes with long wavelengths $\lambda \leq 2(L - R)$ (dashdotted). The reflection of the additional modes at the plate causes the informative Casimir (1948) force $\vec{F}_{Casimir}$. It is valuable in nanotechnology, see Klimchitskaya et al. (2009), Klimchitskaya and Mostepanenko (2020), Schmidt et al. (2022), Gong et al. (2012). Essential are the energies $E_{min,kin}(k_{max}, R)$ in the left cuboid, $E_{min,kin}(k_{max}, L - R)$ in the right cuboid and $E_{min,kin}(k_{max}, L)$ in the full cube with length L, of all modes with $|\vec{k}| \leq k_{max}$ in the respective cuboid or cube.

following universal structure:

$$P_{Casimir} = -\frac{\hbar \cdot c \cdot \pi^2}{240 \cdot R^4} \tag{15.13}$$

There are two different derivations of the Casimir force:

(1) The momenta per time are added, that the ZPOs with wavelengths $\lambda \geq 2R$ transfer to a conducting plate, see Fig. (15.3) or e. g. (Ballentine, 1998, chapter 19.3, Eq. 19.45). That sum of momentum per time is the Casimir force in Eq. (15.36).

(2) Landau and Lifschitz (1981) analyzed the statistical properties of the electromagnetic field in material media, see paragraph 75, p. 314.

(2.1) For it, a tensor of correlations is integrated that includes all effects of quantum electrodynamics, see p. 315. In particular, the energy increment $dE(\omega)$ includes ZPOs with their energies $\frac{\hbar\omega}{2}$, see (Landau and Lifschitz, 1981, Eq. 77.13, p. 323, lines 10-28).

(2.2) The properties of the field are applied to the force at the two plates in Figs. (15.3) and (Landau and Lifschitz, 1981, Fig. 17, p. 338). A resulting general term for that force is presented in (Landau and Lifschitz, 1981, Eq. 81.10). That term is analyzed for three consecutive limiting cases:

(2.2.1) In the limit of small distance, see (Landau and Lifschitz, 1981, p. 342, lines 19-24) the quanta of modes between the plates have small wavelengths $\lambda < 2R$. Their energies are $E_{mode} = \hbar\omega = \hbar \cdot c \cdot k$ or $E_{mode} = \hbar \cdot c \cdot 2\pi/\lambda$ or $E_{mode} > h \cdot c/(2R)$ are large compared to thermal energies $E_{thermal} = k_B T$:

$$\frac{k_B \cdot T \cdot 2R}{h \cdot c} << 1 \quad \text{small distance limit} \tag{15.14}$$

For instance, a typical case of the Casimir force is at $T = 300K$

and $R = 10^{-6}$ m, the above fraction takes the following value:

$$\frac{k_B \cdot T \cdot 2R}{h \cdot c} = 0.042 << 1 \ \text{small distance limit} \qquad (15.15)$$

Thus, the limiting case of small distance is fulfilled in typical cases of the Casimir force.

(2.2.2) In the above limit of small distance, the considered modes have sufficiently long wavelengths $\lambda \geq 2R$, see (Landau and Lifschitz, 1981, p. 344, lines 10-27). This is realistic for the case of the Casimir force, as the modes with $\lambda \geq 2R$ in Fig. (15.3) are added, in order to derive the Casimir force, in accordance with part (1). As a consequence, oscillations are negligible and the electrostatic permittivity ε_0 suffices to describe the dynamics of the system. Thus, the combined limiting case of small, and sufficiently large, distance is fulfilled in typical cases of the Casimir force.

(2.2.3) In the limit of perfect conductors, the electrostatic permittivity ε_0 tends to infinity, see (Landau and Lifschitz, 1981, p. 344, lines 27-30). As a consequence, the pressure of the force derived in this 'Landau theory' is the same as the pressure of the Casimir force in the derivation in part (1), see (Landau and Lifschitz, 1981, Eq. 82.5) or Eq. (15.13):

$$P_{Casimir} = -\frac{\hbar \cdot c \cdot \pi^2}{240 \cdot R^4} \qquad (15.16)$$

Thus, the combined limiting case of small, and sufficiently large, distance at a pair of perfectly conducting plates is fulfilled in typical cases of the Casimir force.

(3) Comparison of the two derivations in parts (1) and (2):

(3.1) In the derivation in part (1), only the ZPOs are considered and added up. These are independent of the temperature and independent of material constants, such as the electric permittivity. And these provide the correct (and observed) Casimir force and underpressure, see Eq. (15.13).

(3.2) In the derivation in part (2), the field fluctuations include the ZPO, independent of temperature, and the field fluctuations include the polarization of charges in matter, depending on temperature, see (Landau and Lifschitz, 1981, Eq. 77.13).

(3.2.1) The three limiting cases in parts (2.2.1), (2.2.2) and (2.2.3) extract the ZPOs, and these limiting cases effectively exclude the polarization effects. And these limiting cases provide the correct (and observed) Casimir force of ZPOs in Eq. (15.16).

(3.2.2) As a consequence, the 'Landau theory' shows that the Casimir force is constituted by the ZPOs. Moreover, the 'Landau theory' shows that polarization of the material has no essential contribution to the Casimir force.

(3.2.3) In particular, the 'Landau theory' confirms that the source of the Casimir force is the set of ZPOs of the electromagnetic field. This is additionally confirmed by two facts:

(3.2.3.1) The Casimir force does not depend on temperature, but the polarization of the material does.

(3.2.3.2) The Casimir force does not depend on material parameters such as the electrostatic permittivity, but the material does.

(3.2.3.3) The source of the ZPOs in the 'Landau theory' is the usual present - day quantization procedure, that is also inherent to the 'Landau theory'. This is additionally confirmed by the remark about formation of electron-positron pairs, that are originally inherent in the 'Landau theory' and that are neglected artificially, see (Landau and Lifschitz, 1981, p. 316, lines 30-36). In contrast, in the VD, the quanta are implied by the VD, see THM (18); thus, the quanta are founded by the VD; hence, no quantization procedure is needed at all in the VD, so that no quantization procedure provides any problem, such as the cosmological constant problem in present - day physics as well as in the 'Landau theory'.

(3.2.4) In particular, conversely, the 'Landau theory' confirms that there is no other source of the Casimir force than the set of ZPOs of the electromagnetic field.

Unfortunately, some authors, see e. g. Jaffe (2005), realize that the 'Landau theory' of the Casimir force also describes polarization of a material, but they do not realize that the 'Landau theory' of the Casimir force also includes ZPOs, and that the 'Landau theory' of the Casimir force excludes the polarization of matter with help of three limiting case, in order to derive the Casimir force, so that the 'Landau theory' of the Casimir force confirms that the source of the Casimir force is exactly the set of ZPOs of the electromagnetic field.

15.3.3 Additional evidence for ZPOs

Additional evidence for the ZPOs of the electromagnetic field is provided by the anomalous magnetic moment of the electron, see item (1) in section (15.5.1)

Further evidence for the ZPOs of the electromagnetic field is provided by the Lamb shift, see item (3) in section (15.5.1)

Moreover, evidence for the ZPOs of the electromagnetic field is provided by the observations based on the QED - corrections, see item (2) in section (15.5.1), Greiner and Reinhardt (1995b), Carmesin (2021f).

In addition, ZPOs are observed in crystals, see e. g. Fornasini and Grisenti (2015).

15.3.4 Derivation of the Casimir force

The Casimir force occurs at the additional plate at R in Fig. (15.3). Without that plate, the ZPOs in the cube have the energy $E_{min,kin}(k_{max}, L)$ in Eq. (15.4). With that plate, the ZPOs in the right part have an energy $E_{min,kin}(k_{max}, L - R)$, and the ZPOs in the left part have an energy $E_{min,kin}(k_{max}, R)$.

The negative derivative of the kinetic energy difference is the Casimir force:

$$\Delta E_{min,kin} = E_{min,kin}(k_{max}, R) \\ +E_{min,kin}(k_{max}, L-R) - E_{min,kin}(k_{max}, L) \quad (15.17)$$

$$\vec{F}_{Casimir} = -\frac{\partial \Delta E_{min,kin}}{\partial \vec{R}} \quad (15.18)$$

Firstly, we analyze $E_{min,kin}(k_{max}, R)$: The wave function in the interval $[0, R]$ is as follows:

$$\Psi = \exp[-i\omega_n t + ik_x \cdot y + ik_z \cdot z) \cdot \sin(kn \cdot x) \quad (15.19)$$

$$\text{with } \; k_n = \frac{n\pi}{R} \; \text{ and } \; \omega_n = c \cdot \sqrt{k_y^2 + k_z^2 + k_n^2} \quad (15.20)$$

The kinetic energy is the sum in Eq. (15.3):

$$E_{min,kin}(n_{max}, R) = 2 \sum_{\vec{n}}^{n_{max}} \frac{\hbar\omega}{2}(\vec{n}) \quad (15.21)$$

We sum the y - and z - direction according to Fig. (15.2), similar to Eq. (15.4):

$$E_{min,kin}(n_{max}, R) = \frac{1}{4} \sum_{n_x}^{n_{x,max}} \int_0^{n_{max}} \hbar\omega_n 2\pi n dn \quad (15.22)$$

The integration variable n is substituted by $\frac{L}{\pi}k$, see Eq. (15.2):

$$E_{min,kin}(k_{max}, R) = \frac{\hbar}{4} \frac{L^2}{\pi^2} \sum_{n_x}^{n_{x,max}} \int_0^{k_{max}} \omega_n 2\pi k dk \quad (15.23)$$

We apply ω_n in Eq. (15.20):

$$E_{min,kin}(k_{max}, R) = \frac{\hbar}{2} \frac{L^2}{\pi} \sum_{n_x}^{n_{x,max}} \int_0^{k_{max}} [c^2 \cdot (k^2 + k_n^2)]^{1/2} k dk \quad (15.24)$$

In the limit $R >> \lambda = \frac{2\pi}{k_{max}}$, or $n_{x,max} >> \frac{2\pi}{R}$, we will evaluate the sum in the limit $n_{x,max}$ to infinity. In order to provide a converging sum, we apply an additional exponent $-s/2$ in the limit s to zero:

$$E_{min,kin}(k_{max}, R) = \lim_{s \to 0} \frac{\hbar c^{1-s} L^2}{2\pi} \sum_{n_x}^{n_{x,max}} \int_0^{k_{max}} (k^2 + k_n^2)^{\frac{1-s}{2}} k dk \tag{15.25}$$

We evaluate the integral, the result can be confirmed by application of the derivative:

$$E_{min,kin}(k_{max}, R) = \lim_{s \to 0} \frac{\hbar c^{1-s} L^2}{2\pi} \sum_{n_x}^{n_{x,max}} \left[\frac{(k^2 + k_n^2)^{\frac{3-s}{2}}}{3-s} \right]_0^{k_{max}} \tag{15.26}$$

Next, the boundary values are inserted. The short wavelength contribution at k_{max} does not contribute to the energy difference in Eq. (15.17), as the long wavelength contributions are the same at both sides of the conducting plate in Fig. (15.3). Correspondingly, the contribution at k_{max} is neglected in the following:

$$E_{min,kin}(k_{max}, R) \hat{=} -\lim_{s \to 0} \frac{1}{3-s} \frac{\hbar c^{1-s} L^2}{2\pi} \sum_{n_x}^{n_{x,max}} k_n^{3-s} \tag{15.27}$$

We use $k_n = \frac{n\pi}{R}$:

$$E_{min,kin}(k_{max}, R) \hat{=} -\lim_{s \to 0} \frac{\pi^{3-s}}{R^{3-s}} \frac{1}{3-s} \frac{\hbar c^{1-s} L^2}{2\pi} \sum_{n_x}^{n_{x,max}} n^{3-s} \tag{15.28}$$

In the limit $n_{x,max}$ to infinity, the sum is equal to the zeta function value $\zeta(s-3)$. We evaluate the limit. Moreover, we use $\zeta(-3) = 1/120$:

$$E_{min,kin}(k_{max}, R) = -\frac{\pi^3}{R^3} \frac{1}{3 \cdot 120} \frac{\hbar c \cdot L^2}{2\pi} \tag{15.29}$$

We simplify that expression. Consequently, in the limit $R >> \lambda = \frac{2\pi}{k_{max}}$, the energy $E_{min,kin}(k_{max}, R)$ is as follows:

$$E_{min,kin}(k_{max}, R) = -\frac{\hbar \cdot c \cdot \pi^2 \cdot L^2}{720 \cdot R^3} \tag{15.30}$$

Secondly, we analyze the difference of the second and third term in Eq. (15.17):

$$\delta E_{min,kin} = E_{min,kin}(k_{max}, L) - E_{min,kin}(k_{max}, L - R) \tag{15.31}$$

The absolute value of that difference is smaller than the following term:

$$|\delta E_{min,kin}| < |E_{min,kin}(k_{max}, L) - E_{min,kin}(k_{max}, 2L - 2R)| \tag{15.32}$$

The above difference represents the kinetic energy of the longest wavelength in $E_{min,kin}(k_{max}, 2L-2R)$. That energy is negligible. As a consequence, the full difference in Eq. (15.17) is equal to the energy in Eq. (15.30):

$$\Delta E_{min,kin} = -\frac{\hbar \cdot c \cdot \pi^2 \cdot L^2}{720 \cdot R^3} \tag{15.33}$$

A ZPO that propagates vertically to the conducting plate in Fig. (15.3) is reflected. Thereby, it transmits two times its original momentum $\vec{p}$ to the plate:

$$\Delta \vec{p} = 2 \cdot \vec{p} \tag{15.34}$$

As there are more ZPOs outside the left cuboid than inside, there is a permanent net rate of transfer of momentum. Thus, a force acts upon the conducting plate in Fig. (15.3). In an experiment with two such conducting plates, that force is called Casimir force $\vec{F}_{Casimir}$, and it causes an effective attractive interaction of the two plates:

$$\vec{F}_{Casimir} = \frac{\Delta \vec{p}}{\Delta t} \tag{15.35}$$

The Casimir force is equal to the negative derivative applied to $\Delta E_{min,kin}$ with respect to the distance R:

$$\vec{F}_{Casimir} = -\frac{\partial \Delta E_{min,kin}}{\partial \vec{R}} = -\vec{e}_x \cdot \frac{\hbar \cdot c \cdot \pi^2 \cdot L^2}{240 \cdot R^4} \qquad (15.36)$$

The ratio of the Casimir force and the area L^2 of the conducting plate in Fig. (15.3) is the force per area, or a pressure, $|P| = \frac{|F|}{A} = \frac{|F|}{L^2}$, see e. g. (Landau and Lifschitz, 1980, paragraph 13):

$$\frac{|\vec{F}_{Casimir}|}{L^2} = \frac{\hbar \cdot c \cdot \pi^2}{240 \cdot R^4} =: -P_{ZPO,kin} \qquad (15.37)$$

That ratio is identified with the underpressure or negative pressure $-P_{ZPO,kin}$.

15.3.5 Measurement of the Casimir force

In a review, (Klimchitskaya et al., 2009, Fig. 11) compare various measurements of the Casimir force. For instance, at a distance of $R = 500\,\text{nm}$, a negative pressure of $P_{ZPO} = -17\,\text{mPa}$ has been observed. The theory in Eq. (15.37) provides the following value:

$$P_{ZPO} = -\frac{\hbar \cdot c \cdot \pi^2}{240 \cdot 500^4\,\text{nm}^4} = -20\,\text{mPa} \qquad (15.38)$$

Thus, the maximal wave number is as follows:

$$k_{max} = \frac{\pi}{500\,\text{nm}} = 15.7 \cdot 10^6\,\frac{1}{\text{m}} \qquad (15.39)$$

Consequently, the modes that cause the measured negative pressure have the energy density in Eq. (15.9) is as follows:

$$u_{ZPO,kin} = \frac{\hbar \cdot c \cdot k_{max}^4}{8\pi^2} = 24.4\,\frac{\text{J}}{\text{m}^3} \qquad (15.40)$$

As a consequence, this density $u_{ZPO,kin}$ of kinetic energy of ZPOs has been observed with help of the negative pressure caused by the Casimir force.

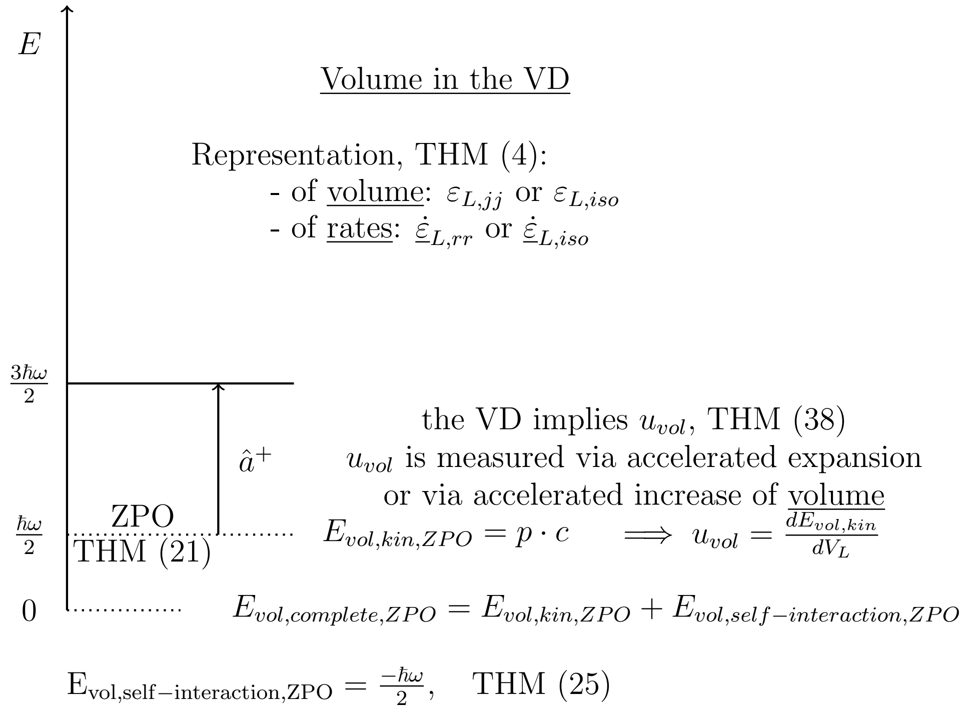

Figure 15.4: Spectrum (THM 35) and measurement of energy and energy density of volume.
Notes: The trace of $\varepsilon_{L,iso}$ or $\varepsilon_{L,jj}$ is nonzero, so these tensors represent volume and contribute to u_{vol}. Conversely, the trace of the change tensor ε_{ij} of electromagnetic waves (THM 7) is zero, so that this ε_{ij} does neither represent volume nor contribute to u_{vol}.

15.4 VD solves cosmological const. problem

In this section, we summarize our results, and we solve the cosmological constant problem, see Figs. (15.4, 15.5).

Theorem 40 The VD solves the cosmological constant problem:

The VD solves that problem by the following steps:

(1) The VD provides the kinetic energy density of the ZPOs in an empirically confirmed manner:

(1.1) The VD provides the **universal minimal kinetic en-**

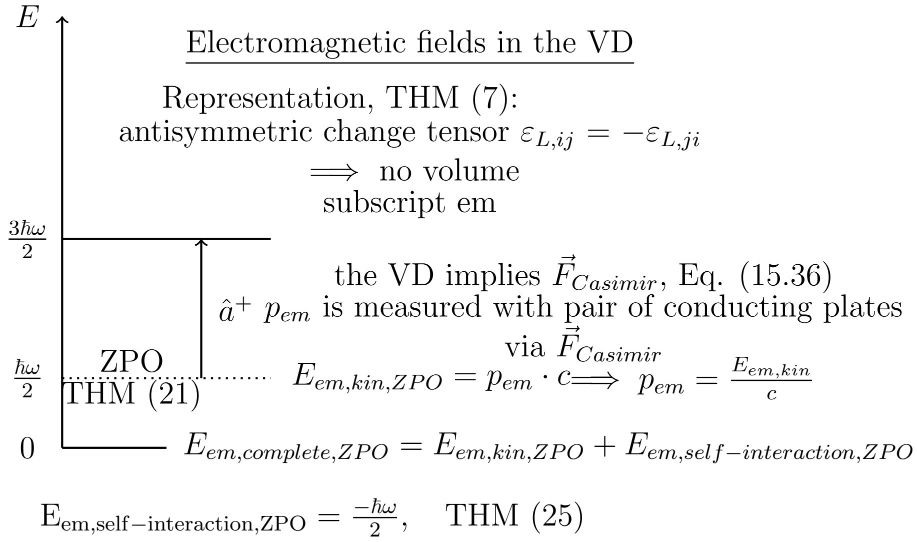

Figure 15.5: Spectrum (THM 35) and measurement of energy and energy density of volume. Notes: Electromagnetic fields (THM 7) are reflected at a conducting plate, so that they contribute to $\vec{F}_{Casimir}$. Conversely, u_{vol} does not contribute to $\vec{F}_{Casimir}$, as dV_L is not reflected by a conducting plate.

ergy *of a single mode of a* **z***ero -* **p***oint* **o***scillation, ZPO. It is* $E_{min,kin,1D} = \frac{\hbar\omega}{2} = p_{min,kin,1D} \cdot c$*, see THM (21).*

(1.2) Based on part (1), and according to the energy momentum relation $E = p \cdot c$ *in SR, the VD provides the momentum* $p_{min,kin,1D} = E_{min,kin,1D}/c$*.*

(1.3) Based on the momentum $p_{min,kin,1D}$ *in part (1), the VD provides the Casimir force* $\vec{F}_{Casimir}$ *in Eq. (15.36). It has been observed, with help of two conducting plates, see e. g. Klimchitskaya et al. (2009). Consequently, that observation confirms the ZPOs, the Casimir force in Eq. (15.36), the underpressure in Eq. (15.13), and the observation confirms the VD, so that it also confirms the energy density of the ZPOs in Eq. (15.7) derived by the VD:*

$$u_{ZPO,kin} = \frac{\hbar \cdot c \cdot k_{max}^4}{8\pi^2} \tag{15.41}$$

(1.4) According to translation invariance, see section (4.2.2), of course, the Casimir force can also be measured in outer space, with help of two conducting plates.

(2) The VD provides the energy density of outer space in an empirically confirmed manner:

(2.1) The VD provides the energy density of volume u_{vol} (THM 38), in accordance with the observed energy density $u_{\Lambda,obs}$ of outer space. This confirms that the VD provides the correct energy density $u_{\Lambda,obs}$.

(2.2) Essentially that energy density of volume u_{vol} has been measured with help of the accelerated expansion of the universe, see e. g. Perlmutter et al. (1998), Riess et al. (2000).

(2.3) The VD provides a more precise theory on that energy density of outer space, see THM (43) and Fig. (16.4).

(3) First complete solution and clarification of the cosmological constant problem:

(3.1) The empirically confirmed part (1) shows that the ZPOs of the electromagnetic field exist in nature.

(3.2) The empirically confirmed part (1) shows that the kinetic energy density of the ZPOs of the electromagnetic field exists in nature and is much larger than u_{vol}, see Eq. (15.9):

(3.2.1) In the case k_{max} according to the Planck length, L_P, the energy density is ca. 120 orders of magnitude larger than the observed value u_{vol}:

$$u_{ZPO,kin} = \frac{\hbar \cdot c}{8\pi^2 \cdot L_P^4} = 5.875 \cdot 10^{111} \, \frac{\mathrm{J}}{\mathrm{m}^3} \tag{15.42}$$

(3.2.2) In the case k_{max} according to ZPOs that have been observed with help of the casimir force, see section (15.3.5), the energy density is ca. 11 orders of magnitude larger than the

observed value u_{vol}:

$$u_{ZPO,kin} = \frac{\hbar \cdot c \cdot k_{max}^4}{8\pi^2} = 24.4 \, \frac{\mathrm{J}}{\mathrm{m}^3} \tag{15.43}$$

(3.3) The empirically confirmed part (2) shows that the energy density of volume is relatively small, see THM (38):

$$u_{vol} = \frac{c^2 H_0^2}{4\pi G} = 5.040 \, (\pm 0.1395) \cdot 10^{-10} \, \frac{\mathrm{J}}{\mathrm{m}^3} \tag{15.44}$$

(3.4) The above empirically confirmed parts (3.1), (3.2) and (3.3) imply that the kinetic energy density of ZPOs are compensated by an equally large energy density. This solves and clarifies the cosmological constant problem. Next, in part (4), the compensating energy density is derived in addition.

(4) Second complete solution and clarification of the cosmological constant problem:

(4.1) The energy density of an electromagnetic field with an electric filed $\vec{E}$ and a magnetic field $\vec{B}$ are as follows, see e. g. Landau and Lifschitz (1971) or Carmesin et al. (2020):

$$|u_{em}| = \frac{E}{V} = \frac{\vec{B}^2}{2\mu_0\mu_r} + \frac{\vec{E}^2}{2\varepsilon_0\varepsilon_r} \tag{15.45}$$

The frame can be chosen so that the magnetic field vanishes, see e. g. (Landau and Lifschitz, 1971, p. 64, lines 10-11, or paragraph 25). For instance, in the own frame of a charge, the magnetic field caused by the charge is zero. In the following, we use a frame with zero magnetic field:

$$|u_{em}| = \frac{\vec{E}^2}{2\varepsilon_0\varepsilon_r} \quad \text{in an appropriate frame} \tag{15.46}$$

(4.2) As most electric interactions among charges in the universe are attractive, the predominant electromagnetic field in

the universe has a negative energy density, see THM (11) and Eq. (15.46) and section (7.4):

$$u_{em} = \frac{-\vec{E}^2}{2\varepsilon_0\varepsilon_r} \text{ in an appropriate frame} \tag{15.47}$$

(4.3) The field in Eq. (15.47) is a generalized field of the VD, see section (7.4).

(4.4) As a consequence, the energy density u_{em} of the field in Eq. (15.47) provides the self - interaction in THM (25), which compensates the energy density of the ZPOs. Thus, $E_{min,kin,1D} = \frac{\hbar\omega}{2}$ is compensated by a self - interaction of each ZPO mode.

(4.5) As a consequence of part (4.4), the VD provides the compensation of $u_{ZPO,kin}$ by the field energy density u_{em}. This solves and clarifies the cosmological constant problem.

Proof: The theorem includes its derivation. This completes the solution of the cosmological constant problem. q. e. d.

15.5 Additional considerations

In this section, we present interesting additional considerations.

For instance, the above analysis of electromagnetic waves can similarly be performed for other change tensors. Thereby, other measurement techniques are needed, of course.

15.5.1 VD provides correct anomalous magnetic moment of the electron as well as the lamb shift

(1) Magnetic moment: Dyck et al. (1987) precisely measured the magnetic moment of the electron:

$$\langle\vec{\mu}\rangle = \mu_B \cdot 2 \cdot (1 + QED - corrections) \cdot \langle\vec{s}\rangle \tag{15.48}$$

Hereby, the Bohr magneton is $\mu_B := \frac{e\cdot\hbar}{2m_{electron}\cdot c}$, see e. g. Tipler and Llewellyn (2008), and $\langle\vec{s}\rangle$ represents the angular momentum

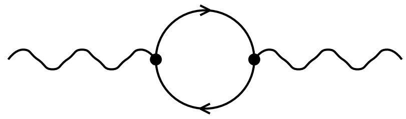

Figure 15.6: Vacuum polarization: At a photon (wiggly line), a virtual electron (upper half circle) positron (lower half circle) pair forms for a short time.

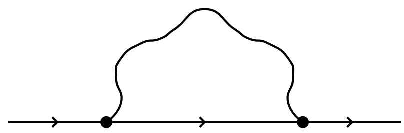

Figure 15.7: Self energy: At an electron (sold line), a virtual photon or electromagnetic field (wiggly line) forms for a short time.

of the electron. Thereby, the anomalous magnetic moment is caused by the QED - corrections, see e. g. (Schwartz, 2014, sections 16, 18) or Greiner and Reinhardt (1995a):

$$\langle \vec{\mu} \rangle_{anomalous} := \mu_B \cdot 2 \cdot (QED - corrections) \cdot \langle \vec{s} \rangle \qquad (15.49)$$

The basic physical process of the QED - corrections is provided by the VD, see item (2).

(2) QED - corrections: In general, QED - corrections are caused by virtual objects that form and are annihilated after a time corresponding to the uncertainty relation, see Figs. (15.6, 15.7) e. g. Greiner and Reinhardt (1995b). Such formation and annihilation can be described by ladder operators, see chapter (12). As a consequence, the basic physical process of the QED - corrections is provided by the VD.

(2.1) photons: The ladder operators of photons are directly provided by the VD, see THMs (7, 32, 33, 34, 35).

(2.2) electrons: The ladder operators of electrons are indirectly provided by the VD, see Carmesin (2021f): Firstly, volume portions bind to form in invariant mass[1]. Secondly, in

[1]The concept of an invariant mass is described in Tipler and Llewellyn (2008), for

their own frames, the bound VPs represent oscillators. Thirdly, the respective oscillations excite forced oscillations at the other bound oscillators. Fourthly, these forced oscillations emit the electric field and represent the elementary charge. The result is in precise accordance with observation.

Altogether, there are ladder operators of the VPs, and as a consequence, there are ladder operators of the electrons (or other elementary particles with an elementary charge).

(2.3) considered virtual objects: In the QED, the usual considered virtual objects are photons and electrons.

For instance, during the motion of an electron, a virtual photon can form for a short time, the process is called self - energy of the electron, see Fig. (15.7) or e. g. (Greiner and Reinhardt, 1995b, p. 349).

For instance, during the propagation of a photon, a virtual electron positron pair photon can form for a short time, the process is called vacuum polarization, see Fig. (15.6) or e. g. (Greiner and Reinhardt, 1995b, p. 310).

(3) Lamb shift: Lamb and Retherford (1947) observed a slight additional value in the spectrum of the hydrogen atom. That additional term is called Lamb shift.

It is explained on the basis of the QED-corrections in the above item (2). The basic physical process of the QED - corrections is provided by the VD, see item (2). As a consequence, the Lamb shift is also provided by the VD, in principle.

(4) Additional insight provided by the VD and its energy density: In the VD, the ladder operators are derived exactly and with the correct compensation of the density of kinetic energy and the density of the field energy. As a consequence, the cosmological constant problem is solved in the VD. Additionally, the VD provides the correct value of the Casimir force.

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instance.

In contrast, the ladder operators in present - day QFT are merely used as a useful tool, see (Peskin and Schroeder, 1995, p. xi). For instance, present - day QFT is based on the idea of second quantization, see e. g. (Schwartz, 2014, p. 7), Dirac (1927). Of course, second quantization is postulated and not derived. In contrast, universal quantization and its consequences in THMs (18, 25, 31, 32, 33, 34, 35, 43) are derived.

As a consequence, in present - day QFT and QED, there occurs the huge error of the cosmological constant problem, as second quantization does not provide the complete energy. Second quantization only provides the available energy $E_{av} = p \cdot c$, which can be measured with help of two parallel conducting plates, which provide the Casimir force in THM (40).

Moreover, in present - day QFT, the zero-point energy provides an infinity, see (Schwartz, 2014, section 4.2). In present - day QFT, this is treated by the restriction that only energy differences should be measured (the basic idea behind renormalization), see (Schwartz, 2014, section 4.2, p. 52, last two lines).

However, in cosmology, including the cosmological constant and the energy density of volume u_{vol}, absolute values of the measurable available energy are described and measured, see Figs. (15.4, 15.5). And this is provided by the VD, see THMs (38, 43, 40).

(5) The VD explains local and nonlocal phenomena as well as causality in a clarifying and fundamental manner: In the VD, the solutions of the DEQ of VD provide local and nonlocal phenomena, see chapters (9, 13).

In contrast, present - day QFT is merely a local theory, by definition, see e. g. (Schwartz, 2014, p. xvi), though present - day QFT also provides useful results.

(6) VD offers additional insight by providing gravity, quanta, curvature, electrodynamics and the expansion

of the universe from a single evident source: In the VD, the solutions of the DEQ of VD provide gravity, quanta, curvature, electrodynamics and the expansion of the universe in a coherent manner. Thereby, this is derived in a direct manner, see Fig. (1.4).

Hereby, the VD provides the correct energy density of volume (THM 38), and the VD provides the correct values of the H_0 tension, including H_0 as a function of the cosmological redshift z (THM 43).

In contrast, in present - day physics, very different postulates are used. These have been proposed at different times by different persons for different topics. See e. g. the postulates of quantum physics proposed by Hilbert et al. (1928), the EFE proposed by Einstein (1915), electrodynamics proposed by Maxwell (1865), rules of elementary particles proposed for instance by Higgs (1964) or Weinberg (1967).

In fact, these postulates are a basis for many discoveries and inventions that provide a huge technological progress, including a great benefit to mankind. In addition, the VD derives, explains, clarifies and generalizes these postulates, the VD solves related problems such as the cosmological constant problem, the questions of causality and nonlocality or the Hubble tension (THM 43), the VD provides and explains universal constants such as u_{vol} or the charge of the electron, see Carmesin (2021d).

15.5.2 Studies related to the cosm. const. problem

(1) Present - day QFT provides differences of physical quantities such as momentum or energy: (Schwartz, 2014, p. 52) proposes that only energy differences provided by present - day QFT are measurable. With that proposed additional present - day QFT-rule, the ZPE and u_{ZPO} are not measurable, as they represent absolute values. In the VD, this present - day QFT-rule is founded and confirmed. Moreover, the VD overcomes that limitation by the additional information provided by the real (according to the Hacking criterion, Hacking (1983)) volume portions, see chapter (12).

(2) Present - day QFT is a set of ideas and tools: (Peskin and Schroeder, 1995, p. xi) propose that present - day QFT is a set of ideas and tools. According to that proposal, the energy density u_{ZPO} provided by present - day QFT is only an idea. On the basis of the VD, this is now explained, and clarified, and the necessary modification is fundamentally derived with help of real VPs.

(3) Renormalization: In present - day QFT, there occur diverging quantities, such as the ZPE in the limit k to zero. These are corrected afterwords by a procedure named renormalization. (Schwartz, 2014, p. 300) summarizes: 'The core idea behind renormalization in present - day QFT is: Observables are finite and in-principle calculable functions of other observables.'

In contrast, in the VD, the compensation of energies occurs directly, for instance the energy density is zero, as the kinetic energy density u_{kin} and the gravitational energy density $u_{gr.f.}$ compensate each other, see THM (16). This applies even for the wave function, as gravity and quanta are both implied by the same and fundamental VD.

(4) Supersymmetry: In unbroken supersymmetry, the energy density of vacuum would vanish - however, supersymmetry is broken in the real world, see e. g. (Weinberg, 1989, p. 4).

(5) Supergravity: In supergravity, the cosm. const. problem is not solved as no physical principle provides the values of the energy density of vacuum, see (Weinberg, 1989, p. 6). Moreover, in supergravity, gravity is added artificially. In contrast, gravity, quanta and the observed value of the energy density $u_{\Lambda,obs} = 5.133 \cdot 10^{-10} \, \frac{\mathrm{J}}{\mathrm{m}^3}$ are derived properties of VD.

(6) Superstrings: In the field of superstrings, the cosm. const.

problem is not solved as there is no mechanism that could keep the effective cosmological constant sufficiently small, see e. g. (Weinberg, 1989, p. 6).

(7) Elementary particles: In modern theories of elementary particles, there occur ZPOs, for instance in quantum chromodynamics. As a consequence, the cosmological constant problem is not solved as there is no mechanism that could keep the effective cosmological constant sufficiently small, see e. g. (Weinberg, 1989, p. 6).

(8) Anthropic principle: The anthropic principles proposes to use implications of the fact that we live. This restricts u_{ZPO} only to values smaller than the tenfold energy density of matter u_m, see e. g. (Weinberg, 1989, p. 8). The energy density u_m is as follows, $u_m = u_{\Lambda,obs} \cdot \frac{\Omega_m}{\Omega_\Lambda} = 5.133 \cdot 10^{-10}\,\frac{\mathrm{J}}{\mathrm{m}^3} \cdot \frac{0.321}{0.679} = 2.43 \cdot 10^{-10}\,\frac{\mathrm{J}}{\mathrm{m}^3}$. Thus, the restriction is $u_{ZPO} < 24.3 \cdot 10^{-10}\,\frac{\mathrm{J}}{\mathrm{m}^3}$. However, this restriction is not sufficient.

(9) Adjustment mechanisms: Adjustment mechanisms have been proposed, see e. g. (Weinberg, 1989, p. 9). However, such mechanisms are hardly fundamental and do hardly provide a clarification.

(10) Changing gravity: Changes of gravity have been proposed, see e. g. (Weinberg, 1989, p. 11). However, the volume in nature directly implies gravity as it is, see THMs (9, 10). This fact excludes any arbitrary change of gravity.

(11) So-called quantum cosmology: A so-called quantum cosmology has been proposed, see e. g. (Weinberg, 1989, p. 16-20). However, many suppositions are included in that proposal, such as baby universes, wormholes and unexplained additional parameters. Moreover, the observed value $u_{\Lambda,obs} = 5.133 \cdot 10^{-10}\,\frac{\mathrm{J}}{\mathrm{m}^3}$ is not provided by quantum cosmology.

In contrast, the VD directly implies gravity, quanta and $u_\Lambda = 5.133 \cdot 10^{-10}\,\frac{\mathrm{J}}{\mathrm{m}^3}$.

Chapter 16

VD implies Hubble tension

Penzias and Wilson (1965) discovered the cosmic microwave background, CMB, a radiation that has been emitted in the early universe, 380 000 years after the Big Bang. Using the CMB, the Planck-Collaboration (2020) observed the value of the early universe value of the Hubble constant:

$$H_{0,obs,CMB} = 66.88(\pm 0.92)\,\frac{\text{km}}{\text{s}\cdot\text{Mpc}} \tag{16.1}$$

Hereby, the redshift is $z_{CMB} = 1090.3 \pm 0.41$, and the temperature - temperature correlation, TT, has been used, see (Planck-Collaboration, 2020, table 2, column 1).

Riess et al. (2022) used radiation emitted from supernovae of type Ia in near galaxies and obtained the following H_0-value:

$$H_{0,obs,near,Ia} = 73.04(\pm 1.01)\,\frac{\text{km}}{\text{s}\cdot\text{Mpc}} \tag{16.2}$$

Thereby, the averaged redshift is $\langle z \rangle = 0.055$, see (Riess et al., 2022, sections 5.1 and 5.2). Thus, Riess et al. (2022) observed a discrepancy between the H_0 - value based on the CMB and the H_0 - value based on near galaxies at the five σ confidence level. Such a discrepancy is called Hubble tension or H_0 tension, see Planck-Collaboration (2020).

This problem is explained and solved by the VD:

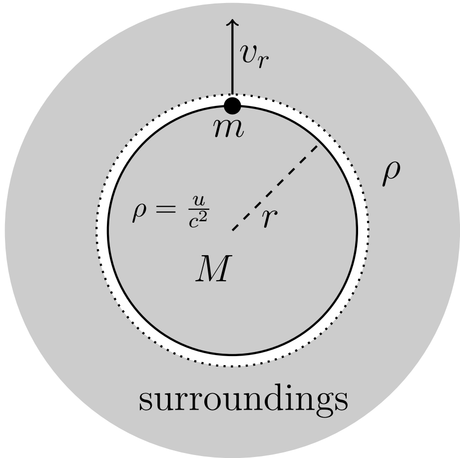


Figure 16.1: Ball with mass M and scale radius r embedded in a homogeneous surrounding and exhibited to a probe mass m with a radial velocity $v_r = \dot{r}$.

16.1 On the ΛCDM model

Insights about the age t_0 of the universe can be achieved by an analysis of the present-day values of the ΛCDM model. A present-day value of a quantity is marked by the subscript zero. For instance, the present-day value of the time is t_0, see Fig. (16.2):

$$t_{present-day} =: t_0 \quad \text{with} \quad t_{Big\ Bang} := 0 \tag{16.3}$$

Moreover, four densities are distinguished, so that each density has a characteristic scaling behavior as a function of the scale radius r in Fig. (16.1), whereby r is a function of the cosmological redshift z or of the time t:

ρ_r is the density of radiation,

ρ_m is the density of matter, including cold dark matter, CDM, see Planck-Collaboration (2020), Workman et al. (2022),

ρ_K is the density of a curvature parameter, it is zero according to observation, see Planck-Collaboration (2020), and as a result of a proof, see Carmesin (2023h,g),

ρ_Λ is the density of the cosmological constant Λ, it does not change as a function of the scale radius r.

At $\rho_K = 0$, the present-day value of the density is called critical density:

$$\rho_{present-day} =: \rho_{cr,0} \tag{16.4}$$

The ratios of the particular densities and the critical density are called density parameters:

$$\Omega_\Lambda := \frac{\rho_\Lambda}{\rho_{cr,0}} \quad \text{and} \quad \Omega_m := \frac{\rho_m}{\rho_{cr,0}} \quad \text{and} \quad \Omega_r := \frac{\rho_r}{\rho_{cr,0}} \tag{16.5}$$

According to the cosmological redshift, the Hubble parameter $H = \dot{r}/r$ is the following function of the densities in Eq. (16.5) and of the scale radius:

$$H^2 = \frac{8\pi G}{3} \cdot \rho_{cr,0} \cdot \left(\Omega_r \cdot \frac{r_0^4}{r^4} + \Omega_m \cdot \frac{r_0^3}{r^3} + \Omega_\Lambda \right) \tag{16.6}$$

In the ΛCDM model, the present-day value of the Hubble parameter H is regarded as a constant, named Hubble constant:

$$H(t_0) =: H_{0,\Lambda CDM} = \sqrt{\frac{8\pi G}{3}\rho_{cr,0}} \tag{16.7}$$

Its inverse is called Hubble time t_{H_0}:

$$t_{H_0} := 1/H_{0,\Lambda CDM} \tag{16.8}$$

The present-day time is equal to the Hubble time multiplied by the following integral I_0, see e. g. Carmesin (2019b):

$$I_0 := \int_0^1 \frac{x \cdot dx}{\sqrt{\Omega_r + \Omega_m \cdot x + \Omega_\Lambda \cdot x^4}} \approx 0.95 \tag{16.9}$$

It is clear, that the age of the universe is a calendar date. As a consequence, t_0 cannot be derived from universal constants of physics. Instead, t_0 must be measured. Consequently, the Hubble time t_{H_0} must be measured. As a further consequence, the Hubble constant must be measured. As a direct consequence,

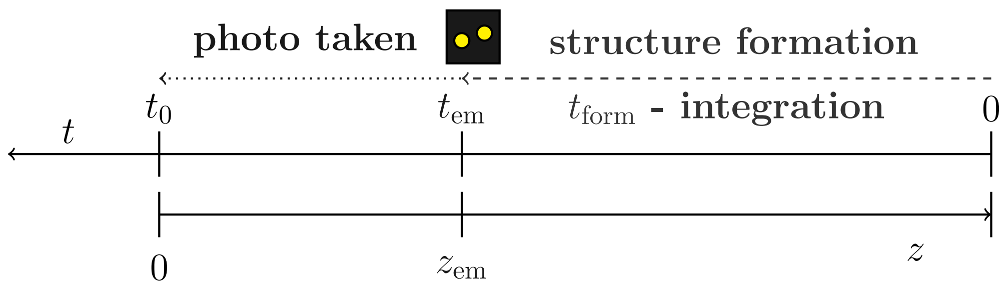


Figure 16.2: The time t after the Big Bang and the corresponding cosmological redshift z are illustrated. Thereby, the heterogeneity or structure in the universe has evolved since the Big Bang. That structure is observed with help of radiation or objects emitted at a time of emission t_{em}. For instance, such objects can be electromagnetic waves, neutrinos or gravitational waves. In this manner, a photograph or graphic representation of the heterogeneity at t_{em} can be taken.

the Hubble constant provides an opportunity to check the validity of the ΛCDM model:

The Hubble constant can be measured by using radiation or other physical objects that have been emitted at a time t or t_{em} or a corresponding cosmological redshift z or z_{em}, see (16.2). These are related as follows, whereby a scaled time $\tilde{t}$ is used, see e. g. Hobson et al. (2006), Carmesin (2019b):

$$\tilde{t} := \frac{t}{t_{H_0}} = \frac{1}{1+z} \quad \text{or} \quad \tilde{t}_{em} = \frac{1}{1+z_{em}} \tag{16.10}$$

As a consequence, in general, the observed values $H_{0,obs}$ of the Hubble constant form a function of the cosmological redshift:

$$H_{0,obs} = function(z) \tag{16.11}$$

If that function is a constant, then the ΛCDM model is confirmed. However, if that function is not a constant, then the ΛCDM model is falsified according to the hypothetico-deductive testing, see e. g. (Kircher et al., 2001, section 4.1.2), (Niiniluoto et al., 2004, p. 214 and section 1.5). As a matter of fact, with a level of confidence above five standard deviations, that function is not a constant, see e. g. Riess et al. (2022). This phenomenon has been called the local value of the Hubble constant. Before 2022, the available observations provided a level of confidence below five standard deviations, and the phenomenon has been named Hubble tension, accordingly.

16.2 Observation of the Hubble constant

In principle, the Hubble constant is a present - day value $H_0 = H(t_0)$.

However, an observation of H_0 requires radiation or objects that have been emitted at an earlier time, see Fig. (16.2). In this section, we summarize how such an observation can be achieved.

Idea: The values of the Hubble parameter $H(z)$ can be expressed as follows, see e. g. Carmesin (2019b, 2023g) or Hobson et al. (2006):

$$H(z) = H_0 \cdot \sqrt{\Omega_\Lambda + \Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4} \qquad (16.12)$$

This Eq. is solved for H_0:

$$H_0 = \frac{H(z)}{\sqrt{\Omega_\Lambda + \Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4}} \qquad (16.13)$$

If an observer uses radiation emitted at a redshift $z_{\rm em}$, then the state and the value of H_0 at $z = z_{\rm em}$ are observed:

$$H_0(z_{\rm em}) = \frac{H(z_{\rm em})}{\sqrt{\Omega_\Lambda + \Omega_{m,0}(1+z_{\rm em})^3 + \Omega_{r,0}(1+z_{\rm em})^4}} \qquad (16.14)$$

16.3 Observed age of the universe

In this section, we summarize the observed values of density parameters and H_0. In a first investigation, these values are based on the CMB. In a second investigation, these values are based on supernovae of type Ia in near galaxies. In both cases, the observations are used in order to derive the age of the universe.

Using the cosmic microwave background, CMB, emitted at $z_{CMB} = 1090.3 \pm 0.41$, the Planck-Collaboration (2020) achieved the following observed value:

$$H_{0,obs,CMB} = 66.88(\pm 0.92)\,\frac{\text{km}}{\text{s} \cdot \text{Mpc}} \tag{16.15}$$

Hereby, the unit Megaparsec is as follows, see Workman et al. (2022):

$$1\text{Mpc} = 3.085\,677\,581\,49 \cdot 10^{19}\,\text{km} \tag{16.16}$$

Thus,

$$H_{0,obs,CMB} = 2.167(\pm 0.03) \cdot 10^{-18}\,\frac{1}{\text{s}} \tag{16.17}$$

The observed density parameters are as follows, see Planck-Collaboration (2020) or Carmesin (2019b):

$$\Omega_\Lambda = 0.679(\pm 0.013) \tag{16.18}$$

$$\Omega_m = 0.321(\pm 0.013) \tag{16.19}$$

$$\Omega_r = 9.625 \cdot 10^{-5} \tag{16.20}$$

As a consequence, the integral in Eq. (16.9) is as follows:

$$I_0 = 0.9455 \tag{16.21}$$

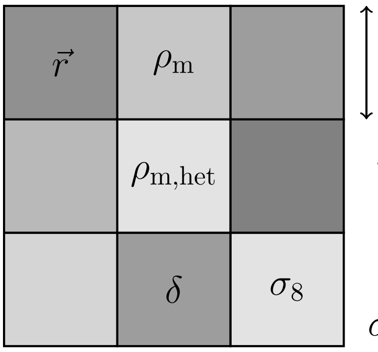
As a further consequence, the age of the universe is as follows:

$$t_{0,CMB} = 13.83(\pm 0.24) \cdot 10^9\,\text{years} \tag{16.22}$$

It is insightful to know that the density parameters Ω_Λ, Ω_m, Ω_r and Ω_K can also be derived directly from the VD, whereby the derived values are in precise accordance with observation, see Carmesin (2021a).

Based on the observation of supernovae of type Ia in near galaxies at an averaged cosmological redshift $\langle z \rangle = 0.055$, Riess et al. (2022) observed the following Hubble constant:

$$H_{0,obs,near,Ia} = 73.04(\pm 1.01)\,\frac{\text{km}}{\text{s} \cdot \text{Mpc}} \tag{16.23}$$



$$L_{\mathrm{box}} = 8\frac{\mathrm{Mpc}}{h} \quad \text{with } h = \frac{H}{100\,\frac{\mathrm{km/s}}{\mathrm{Mpc}}}$$

$$\rho_{\mathrm{m}}(t,\vec{r}) = \rho_{\mathrm{m,hom}}(t) + \rho_{\mathrm{m,het}}(t,\vec{r})$$

$$\rho_{\mathrm{m,het}}(t,\vec{r}) = \rho_{\mathrm{m,hom}}(t) \cdot \underbrace{\delta(t,\vec{r})}_{\text{overdensity}}$$

$$\sigma(t) = \sqrt{\langle \delta^2(t,\vec{r})\rangle_{\vec{r}}} \quad \text{and } \sigma(t_0) = \sigma_8$$

Figure 16.3: Local densities ρ_{m} are observed in boxes. The evaluation provides the following quantities:
the averaged or homogeneous density $\rho_{\mathrm{m,hom}}(t)$,
the local difference $\rho_{\mathrm{m,het}}(t,\vec{r}) = \rho_{\mathrm{m}} - \rho_{\mathrm{m,hom}}(t)$,
and the relative difference, the overdensity $\delta(t,\vec{r}) = \frac{\rho_{\mathrm{m,het}}(t,\vec{r})}{\rho_{\mathrm{m,hom}}(t)}$.

With it and with Eqs. (16.18, 16.19, 16.20, 16.21), the age of the universe is as follows:

$$t_{0,z=0.055} = 12.66(\pm 0.22) \cdot 10^9 \,\text{years} \tag{16.24}$$

The difference is as follows:

$$\Delta t_0 = t_{0,CMB} - t_{0,z=0.055} = 1.17(\pm 0.325) \cdot 10^9 \,\text{years} \tag{16.25}$$

The relative difference is as follows:

$$q = \frac{t_{0,CMB} - t_{0,z=0.055}}{t_{0,z=0.055}} = 9.2\% \tag{16.26}$$

Of course, there is only one age of the universe. What is the source of the apparently different ages of the universe obtained from the CMB and of radiation emitted in near galaxies? A very significant and unique property of the CMB is its very high homogeneity. In contrast, near galaxies have a very large heterogeneity. Accordingly, next, we analyze heterogeneity in the universe:

16.4 Spatial averages in cosmology

Idea: In order to measure a homogeneous density ρ_{hom}, it is necessary to execute an average in a volume L_{box}^3, see Fig. (16.3). In this section, we summarize methods of such averaging in cosmology, see Fig. (16.3) and see e. g. Peebles (1973), Kravtsov and Borgani (2012), Carmesin (2021d), Haude et al. (2022).

Boxes: As a convention, the spatial average is performed in a cubic volume $V_{win} = L_{box}^3$, related to 8 Mpc as follows, see e. g. Kravtsov and Borgani (2012), Carmesin (2021d)[1]:

$$L_{box} = 8h^{-1}\,\mathrm{Mpc} \;\; \text{with} \;\; h = H_0 \cdot \frac{1}{100}\frac{\mathrm{km}}{\mathrm{s}\cdot\mathrm{Mpc}} \tag{16.27}$$

Local density: A box has a location $\vec{r}$. A local density at a time t is measured for each box:

$$\text{local density} \;\; \rho_m(t,\vec{r}) \tag{16.28}$$

Global density: The globally averaged density is introduced:

$$\text{global density} \;\; \rho_{m,hom}(t) \tag{16.29}$$

Overdensity: The overdensity is introduced as follows:

$$\text{overdensity} \;\; \delta(t,\vec{r}) = \frac{\rho_m(t,\vec{r}) - \rho_{m,hom}(t)}{\rho_{m,hom}(t)} \tag{16.30}$$

The density $\rho_{m,het}(t,\vec{r})$ of the heterogeneity is the product of $\rho_{m,hom}(t)$ and the overdensity:

$$\rho_{m,het}(t,\vec{r}) = \rho_{m,hom}(t) \cdot \delta(t,\vec{r}) \tag{16.31}$$

We summarize the densities in the surroundings of the volume:

$$\rho = \rho_{m,hom}(t) + \rho_{m,het}(t,\vec{r}) + \rho_r + \rho_\Lambda \tag{16.32}$$

[1]Note that this scaling of the size L_{box} is a convention. In general, the scaling of L_{box} is not identical to the scale factor describing the expansion of space since the Big Bang. Sometimes, averages are performed in spheres, White et al. (1993).

16.4.1 Averages of fluctuations

In this section, we summarize spatial averaging of fluctuations in cosmology, see for instance Peebles (1973), Kravtsov and Borgani (2012), Carmesin (2021d).

A spatial average of a function $f(\vec{r})$ is applied within a volume V_{window} or V_{win} of a considered region (window) of averaging:

$$\langle f(\vec{r})\rangle_{V_{win}} = \frac{\int_{V_{win}} f(\vec{r})\, dr^3}{\int_{V_{win}} 1\, dr^3} \tag{16.33}$$

Fluctuations of a function $f(\vec{r})$ are usually described by the standard deviation $\sigma_{V_{win}}$:

$$\sigma^2_{V_{win}} = \langle [f(\vec{r}) - \langle f(\vec{r})\rangle_{V_{win}}]^2\rangle_{V_{win}} = \langle f^2(\vec{r})\rangle_{V_{win}} - \langle f(\vec{r})\rangle^2_{V_{win}} \tag{16.34}$$

As a convention, the spatial average is performed within a cubic volume $V_{win} = L^3_{box}$, related to 8 Mpc as follows, see Eq. (16.27) or e. g. Kravtsov and Borgani (2012), Carmesin (2021d)[2]. In that case, the standard deviation is named $\sigma_8(t)$ or $\delta(t)$:

$$\sigma_8(t) = \delta(t) = \sigma_{\delta,V_{win}}(t) \;\text{ with }\; \sigma_{8,0} = \sigma_8(t_0) \tag{16.35}$$

16.5 The field of a state of a mode

In VD, the field is the negative derivative of the potential, see THMs (9, 10):

$$\vec{G}_{gen} = -\frac{\partial}{\partial \vec{L}}\Phi_{gen} \tag{16.36}$$

In quantum physics, the field $\vec{G}_{gen}$ is represented by an eigenvalue generating self adjoint operator. For it, the imaginary unit i is multiplied:

$$\hat{\vec{G}}_{gen} = i\vec{G}_{gen} \tag{16.37}$$

[2]Note that this scaling of the size L_{box} is a convention. In general, the scaling of L_{box} is not identical to the scale factor describing the expansion of space since the Big Bang.

Moreover, we analyze states that approximate classical states. These are coherent states as follows:

$$|z_\mu\rangle = \exp\left(\frac{|z_\mu|^2}{2}\right)\sum_{n_\mu=0}^{\infty}\frac{z_\mu^{n_\mu}}{\sqrt{n_\mu!}}|n_\mu\rangle \tag{16.38}$$

In order to investigate the short range and long range effects of masses or other quanta or sources, we analyze fields and their matrix elements:

Proposition 5 The field of a state of a mode

The matrix element of the generalized field $\vec{G}_{gen}$ is as follows:

(1) In a general state $|z_\mu\rangle$ of a mode μ, it is as follows:

$$\langle z_\mu|\hat{\vec{G}}_{gen}|z_\mu\rangle = \sqrt{\frac{\hbar G c^2}{2\omega_\mu}} \cdot \int d\mu \vec{k}_\mu f_\mu \cdot \langle z_\mu|\hat{a}_\mu^+ + \hat{a}_\mu|z_\mu\rangle \tag{16.39}$$

(2) In an eigenstate of the number operator $|n_\mu\rangle$ of a mode μ, it is zero:

$$\langle n_\mu|\hat{\vec{G}}_{gen}|n_\mu\rangle = 0 \tag{16.40}$$

Proof:

Ad (1): The generalized field is derived from the generalized potential as follows:

$$\vec{G}_{gen} = -\frac{\partial}{\partial \vec{L}}\Phi_{gen} \tag{16.41}$$

That generalized potential is transformed to a linear combination of harmonic solutions (remind that $\hat{\varepsilon}_\mu$ represents an amplitude) of the DEQ of the VD as follows, see chapter (12), THM (32 part 2):

$$\Phi_{gen} = -c^2\varepsilon_{L,p} = -c^2\int d\mu \hat{\varepsilon}_\mu b_\mu f_\mu \tag{16.42}$$

With it, the field in Eq. (16.41) is as follows:

$$\vec{G}_{gen} = -ic^2 \int d\mu \vec{k}_\mu \hat{\varepsilon}_\mu b_\mu f_\mu \tag{16.43}$$

In that Eq., the operator variable $q_\mu = c/\sqrt{G}\hat{\varepsilon}_\mu b_\mu$ is used, see THM (33):

$$\vec{G}_{gen} = -ic\sqrt{G} \int d\mu \vec{k}_\mu q_\mu f_\mu \tag{16.44}$$

In that Eq., the operator variable q_μ is replaced by ladder operators, see THMs (34, 35),

$$q_\mu = \sqrt{\frac{\hbar}{2\omega_\mu}} \left(\hat{a}_\mu^+ + \hat{a}_\mu \right) \tag{16.45}$$

consequently,

$$\vec{G}_{gen} = \sqrt{\frac{\hbar G c^2}{2\omega_\mu}} \int d\mu \vec{k}_\mu \frac{f_\mu}{i} \cdot \left(\hat{a}_\mu^+ + \hat{a}_\mu \right) \tag{16.46}$$

As a consequence, the matrix elements are as follows:

$$\langle z_\mu | \hat{\vec{G}}_{gen} | z_\mu \rangle = \sqrt{\frac{\hbar G c^2}{2\omega_\mu}} \cdot \int d\mu \vec{k}_\mu f_\mu \cdot \langle z_\mu | \hat{a}_\mu^+ + \hat{a}_\mu | z_\mu \rangle \tag{16.47}$$

This completes the proof of part (1).

Ad (2): The corresponding matrix elements are analyzed. The same frequencies are considered, as states with different circular frequencies are mutually orthogonal:

$$\langle n_\mu | \hat{\vec{G}}_{gen} | n_\mu \rangle = \sqrt{\frac{\hbar G c^2}{2\omega_\mu}} \int d\mu \vec{k}_\mu f_\mu \cdot \langle n_\mu | \hat{a}_\mu^+ + \hat{a}_\mu | n_\mu \rangle \tag{16.48}$$

Next, the matrix elements in the integrand are analyzed:

$$\langle n_\mu | \hat{a}_\mu^+ + \hat{a}_\mu | n'_\mu \rangle \tag{16.49}$$

In a state with a fixed number $n_\mu = n'_\mu$, the matrix element is zero:

$$\langle n_\mu | \hat{a}_\mu^+ + \hat{a}_\mu | n_\mu \rangle = 0 = \langle n_\mu | \hat{\vec{G}}_{gen} | n_\mu \rangle \tag{16.50}$$

This completes the proof.

16.6 The field of a coherent state

In this section, we analyze expectation values and matrix elements of fields of the coherent states in Eq. (16.38):

Proposition 6 The field of a coherent state

In a coherent state in Eq. 16.38, the expectation value of the field is the following nonzero function:

$$0 \neq \langle z_\mu | \hat{\vec{G}}_{gen} | z_\mu \rangle = \sqrt{\frac{2\hbar G c^2}{\omega_\mu}} \int d\mu \vec{k}_\mu f_\mu \cdot Re(z_\mu) \cdot \langle z_\mu | z_\mu \rangle \quad (16.51)$$

Proof:

The lowering operator is applied to the coherent state in Eq. (16.38), see THM (34):

$$\hat{a}_\mu | z_\mu \rangle = \exp\left(\frac{|z_\mu|^2}{2}\right) \sum_{n_\mu=0}^{\infty} \frac{z_\mu^{n_\mu}}{\sqrt{n_\mu!}} \hat{a}_\mu | n_\mu \rangle \quad (16.52)$$

Thereby, the summand with $n_\mu = 0$ vanishes. Moreover, the ladder operator decreases the eigenvalue of the number operator by one and provides the factor $\sqrt{n_\mu}$. Furthermore, a factor z_μ is moved to the left:

$$\hat{a}_\mu | z_\mu \rangle = z_\mu \exp\left(\frac{|z_\mu|^2}{2}\right) \sum_{n_\mu=1}^{\infty} \frac{z_\mu^{n_\mu - 1}}{\sqrt{(n_\mu - 1)!}} | n_\mu - 1 \rangle \quad (16.53)$$

The condition $n_\mu = 1$ is replaced by the equivalent condition $n_\mu - 1 = 0$. Then $n_\mu - 1$ is substituted by another variable $\bar{n}_\mu$:

$$\hat{a}_\mu | z_\mu \rangle = z_\mu \left[\exp\left(\frac{|z_\mu|^2}{2}\right) \sum_{\bar{n}_\mu=0}^{\infty} \frac{z_\mu^{\bar{n}_\mu}}{\sqrt{\bar{n}_\mu!}} | \bar{n}_\mu \rangle \right] \quad (16.54)$$

In the above Eq., the rectangular bracket is identified with the coherent state. Thus, the following eigenvalue equation holds:

$$\hat{a}_\mu | z_\mu \rangle = z_\mu | z_\mu \rangle \quad (16.55)$$

The product $\langle z_\mu | a^+_\mu$ is the transposed and conjugate complex of $\hat{a}_\mu | z_\mu$. Thus,

$$\langle z_\mu | a^+_\mu = z^*_\mu \langle z_\mu | \tag{16.56}$$

Hence, the expectation value of the field is as follows, see Eq. 16.39:

$$\langle z_\mu | \hat{\vec{G}}_{gen} | z_\mu \rangle = \sqrt{\frac{\hbar G c^2}{2\omega_\mu}} \cdot \int d\mu \vec{k}_\mu f_\mu \cdot \langle z_\mu | z^*_\mu + z_\mu | z_\mu \rangle \tag{16.57}$$

The matrix element in the above equation $\langle z_\mu | z^*_\mu + z_\mu | z_\mu \rangle$ is equal to $\langle z_\mu | z_\mu \rangle \cdot (z^*_\mu + z_\mu)$. Hereby, the bracket is equal to twice the real part, $(z^*_\mu + z_\mu) = 2Re(z_\mu)$, so that the matrix element is equal to $2Re(z_\mu) \cdot \langle z_\mu | z_\mu \rangle$. Thus, the expectation value of the field is the nonzero function in Eq. (16.51). This completes the proof.

16.7 GFV and field of $\rho_{m,hom}$

In this section, the fields and the resulting GFV are analyzed for the case of a homogeneous density of matter.

Theorem 41 GFV and field of a homogeneous density of matter

A homogeneous density of matter causes nearly classical fields in the near vicinity of a matter object. In QP, classical fields are typically described by coherent states[3]. As a consequence, the VPs form as follows:

(1) In the very near vicinity of a mass m_j, there forms a field, see Eq. (16.51) in PROP (6):

$$0 \neq \langle z_\mu | \hat{\vec{G}}_{gen,j} | z_\mu \rangle \tag{16.58}$$

[3] Coherent states are described in PROP (6) or in (Ballentine, 1998, section 19.4), for instance.

(2) As the system is homogeneous, the average with respect to the masses m_i applied to these fields is zero, since the fields are vectors:

$$\frac{1}{N}\sum_{j=1}^{N}\langle z_\mu|\hat{\vec{G}}_{gen,j}|z_\mu\rangle = 0 \tag{16.59}$$

(3) As squares are positive, the average with respect to the masses m_i applied to the squared fields is nonzero:

$$\frac{1}{N}\sum_{j=1}^{N}\langle z_\mu|\hat{\vec{G}}_{gen,j}|z_\mu\rangle^2 \neq 0 \tag{16.60}$$

However, the above average of the squares represents a fluctuation, see section (16.4). Accordingly, it is caused by the heterogeneous density, and the effects are treated in section (16.9).

(4) As a consequence of parts (2) and (3), a homogeneous system of masses does not contribute to the rate at the probe volume dV_0 in Fig. (14.2).

More generally, this does also hold for a system of objects of radiation, as the used averages exhibit the same properties in the case of radiation.

Consequently, the energy density u_{vol} in a homogeneous universe is the same as the energy density of volume u_{vol} in an empty universe, see Carmesin (2023g).

(5) The density parameter of volume Ω_{vol} is as follows:

$$\Omega_{vol} = \rho_{vol}\cdot\frac{1}{\rho_{cr,0}} = \frac{H_0^2}{4\pi G}\cdot\frac{8\pi G}{3H_0^2} = \frac{2}{3} \tag{16.61}$$

Proof: The proof is included in the THM.

16.8 Fields caused by volume

In this section, we derive the fields that are caused by volume. With it, we derive the long range formation of volume in the probe volume dV_0 in Fig. (14.2):

Theorem 42 Fields and volume caused by volume

If quanta of volume cause quanta of volume, then the sources are eigenstates of the number operator. As a consequence, the following fields and long range effects occur:

(1) The expectation value of the field is zero, see PROP (5):

$$\langle n_\mu | \hat{\vec{G}}_{gen} | n_\mu \rangle = 0 \tag{16.62}$$

In general, many quanta j cause number states $|n_{\mu,j}\rangle$ and expectation values of fields with zero expectation value:

$$\langle n_{\mu,j} | \hat{\vec{G}}_{gen,j} | n_{\mu,j} \rangle = 0 \tag{16.63}$$

When the number states $|n_{\mu,j}\rangle$ propagate, they can overlap in a common region. Similarly as in the case of the homogeneous universe, we analyze the expectation values, and we form the average with respect to j:

$$\frac{1}{N} \sum_{j=1}^{N} \langle n_{\mu,j} | \hat{\vec{G}}_{gen,j} | n_{\mu,j} \rangle \tag{16.64}$$

The result is zero, as each summand is zero. However, in this average, the number states $|n_{\mu,j}\rangle$ do not cancel out, as they span mutually orthogonal subspaces of Hilbert space. Similarly, the operators do not cancel out, as they act on mutually orthogonal subspaces of Hilbert space. Altogether, nothing averages out in the case of volume that forms volume. It is instructive that this is different from the volume formed in a homogeneous universe.

In addition to our derivation, we can understand this lack of cancellation at a semiclassical level: The expectation values in Eq. (16.63) are zero, as there is no defined phase in a number state. As a consequence, there does not occur any destructive inference, so that there is no cancellation.

As a consequence of the lack of cancellation, there remain squared fields, for instance.

(2) The squared field exhibits the following nonzero expectation value:

$$\langle n_\mu|\vec{G}^2_{gen}|n'_\mu\rangle = \langle n_\mu|\hat{\vec{G}}_{gen}\hat{\vec{G}}^*_{gen}|n'_\mu\rangle = G\cdot\int d\mu\hbar\omega_\mu\cdot\left(\frac{1}{2}+n_\mu\right)\delta_{\mu,\mu'} \tag{16.65}$$

(3) As a consequence of parts (1) and (2), each quantum emits a field that does not cancel out, so that it causes LFV at the probe volume dV_0. This confirms the derivation of the LFV and of the energy density of volume in the process investigated in chapter (14).

Proof:

Ad (1): The derivation is included in the theorem.

Ad (2): The squared field is derived from the field in Eq. (16.46) by multiplication of the term with its conjugate complex:

$$\vec{G}^2_{gen} = \frac{\hbar G c^2}{2\omega_\mu}\int d\mu\int d\mu'\vec{k}_\mu\vec{k}_{\mu'}f_\mu f^*_{\mu'}\cdot\left(\hat{a}^+_\mu+\hat{a}_\mu\right)\cdot\left(\hat{a}^+_{\mu'}+\hat{a}_{\mu'}\right) \tag{16.66}$$

The corresponding matrix elements are as follows:

$$\langle n_\mu|\vec{G}^2_{gen}|n'_\mu\rangle = \frac{\hbar G c^2}{2\omega_\mu}\int d\mu\int d\mu'\vec{k}_\mu\vec{k}_{\mu'}f_\mu f^*_{\mu'}M_{\mu,\mu'} \tag{16.67}$$

$$\text{with }\; M_{\mu,\mu'} = \langle n_\mu|\left(\hat{a}^+_\mu+\hat{a}_\mu\right)\cdot\left(\hat{a}^+_{\mu'}+\hat{a}_{\mu'}\right)|n'_\mu\rangle \tag{16.68}$$

If the subscripts μ and μ' are different, then the above matrix elements are zero, as states with different circular frequencies are mutually orthogonal:

$$M_{\mu,\mu'} = \langle n_\mu|\left(\hat{a}^+_\mu+\hat{a}_\mu\right)^2|n_\mu\rangle\cdot\delta(\mu-\mu') \tag{16.69}$$

The above bracket $\langle n_\mu|\left(\hat{a}^+_\mu+\hat{a}_\mu\right)^2|n_\mu\rangle$ is equal to $\langle n_\mu|\hat{a}^+_\mu\hat{a}_\mu + \hat{a}_\mu\hat{a}^+_\mu|n_\mu\rangle$. This bracket is equal to $\langle n_\mu|2\hat{a}^+_\mu\hat{a}_\mu+1|n_\mu\rangle$. Hereby,

$\hat{a}^+_\mu \hat{a}_\mu$ is identified with the number operator. Thus, the above bracket is equal to $2n_\mu + 1$. With it, Eq. (16.67) is as follows:

$$\langle n_\mu | \vec{G}^2_{gen} | n'_\mu \rangle = \frac{\hbar G c^2}{2\omega_\mu} \int d\mu \int d\mu' \vec{k}_\mu \vec{k}_{\mu'} f_\mu f^*_{\mu'} (2n_\mu + 1) \cdot \delta(\mu - \mu') \tag{16.70}$$

In the above Eq., one integral is evaluated. Thereby, the relation $|f_\mu|^2 = 1$ is used:

$$\langle n_\mu | \vec{G}^2_{gen} | n'_\mu \rangle = \frac{\hbar G c^2}{2\omega_\mu} \int d\mu \vec{k}^2_\mu (2n_\mu + 1) \cdot \delta(\mu - \mu') \tag{16.71}$$

In the above Eq., $\vec{k}^2_\mu = \omega^2_\mu / c^2$ is applied:

$$\langle n_\mu | \vec{G}^2_{gen} | n'_\mu \rangle = G \int d\mu \hbar \omega_\mu \left(n_\mu + \frac{1}{2} \right) \cdot \delta(\mu - \mu') \tag{16.72}$$

This completes the proof of part (2).

Ad (3): The derivation is included in the theorem.

16.9 VPs and field caused by $\rho_{m,het}$

(1) Overdensity δ_j of a dynamic mass in a shell:

A dynamic mass dM_j in a shell with a dynamic mass $dM(R, dR)$ is considered. Thereby, the shell has a d_{GP} distance R from the probe volume dV_0 in Fig. (14.2), a thickness dR and a dynamic density $\rho_{m,hom}$. According to section (16.4), the overdensity of dM_j is as follows:

$$\delta_j = \frac{dM_j - dM(R, dR)}{dM(R, dR)} \quad \text{with} \quad dM(R, dR) = 4\pi R^2 dR \rho_{m,hom} \tag{16.73}$$

(2) Rate caused by the overdensity δ_j:

According to the $1/R^2$ law of the field in THMs (9, 10), the overdensity δ_j in part (1) causes the following field:

$$d\vec{G}_{gen,j} = -G\frac{dM_j - dM(R,dR)}{R^2}\vec{e}_R = -G\frac{\delta_j \cdot dM(R,dR)}{R^2}\vec{e}_R \tag{16.74}$$

The average $\langle ... \rangle_{s,R}$ with respect to the shell in part (1) at a radius R and with a thickness dR is derived for the squared field in THM (41 part 3):

$$\langle d\vec{G}^2_{gen,j}\rangle_{s,R} = \left(G\frac{dM(R,dR)}{R^2}\right)^2 \cdot \langle \delta_j^2 \rangle_{s,R} \tag{16.75}$$

$$\langle \delta_j^2 \rangle_{s,R} = \frac{\int_{shell} \delta_j^2 d^3x}{\int_{shell} 1 d^3x} \tag{16.76}$$

According to the law of LFV, the squared field causes the following squared rate:

$$d\vec{G}^2_{gen,j} = c^2 \dot{\underline{\varepsilon}}^2_{L,rr} \tag{16.77}$$

$$\left(G\frac{dM(R,dR)}{R^2}\right)^2 \cdot \langle \delta_j^2 \rangle_{s,R} = c^2 \langle \dot{\underline{\varepsilon}}^2_{L,rr} \rangle_{s,R} \tag{16.78}$$

In the above Eq., the root is applied, and $dM(R,dR)$ is replaced according to Eq. (16.73). Additionally, the propagation according to $dR = c \cdot dt$ is used:

$$4\pi G dt \rho_{m,hom} \cdot \sqrt{\langle \delta_j^2 \rangle_{s,R}} = \sqrt{\langle \dot{\underline{\varepsilon}}^2_{L,rr} \rangle_{s,R}} \tag{16.79}$$

The standard deviations in the above Eq. are abbreviated:

$$\sigma_m := \sqrt{\langle \delta_j^2 \rangle_{s,R}} \tag{16.80}$$

$$d\dot{\underline{\varepsilon}}_{het} := \sqrt{\langle \dot{\underline{\varepsilon}}^2_{L,rr} \rangle_{s,R}} \tag{16.81}$$

The rate caused by $\rho_{m,het}$ will be compared with the rate caused by $\rho_{m,hom}$. Accordingly, the density parameters are defined for the case of homogeneous densities:

$$\Omega_{vol} = \frac{\rho_{vol}}{\rho_{cr,0}} \tag{16.82}$$

$$\Omega_m = \frac{\rho_{m,hom}}{\rho_{cr,0}} \tag{16.83}$$

As a consequence, the densities are related as follows:

$$\rho_{m,hom} = \rho_{vol} \cdot \frac{\Omega_m}{\Omega_{vol}} \tag{16.84}$$

With it, and with the standard deviations in Eqs. (16.80, 16.81), the LFV - relation in Eq. (16.79) is as follows:

$$4\pi G dt \rho_{vol} \cdot \frac{\Omega_m}{\Omega_{vol}} \sigma_m = d\dot{\underline{\varepsilon}}_{het} \tag{16.85}$$

(3) Linear growth theory:

The standard deviation σ_m of the overdensity in the above equation increases as a function of time. In a good approximation, σ_m is proportional to the time t that has elapsed since the Big Bang, see e. g. Kravtsov and Borgani (2012) and Carmesin (2021d), $\sigma_m(t) \propto t$. Using the present-day value σ_8 and the above scaled time $\tilde{t}$, that proportionality is expressed as follows:

$$\sigma_m(\tilde{t}) = \sigma_8 \cdot \tilde{t} \tag{16.86}$$

With it, the rate in Eq. (16.85) is the following function of time:

$$4\pi G dt \rho_{vol} \cdot \frac{\Omega_m}{\Omega_{vol}} \sigma_8 \cdot \tilde{t} = d\dot{\underline{\varepsilon}}_{het} \tag{16.87}$$

In the above rate, the time increment is scaled $dt = d\tilde{t} \cdot t_{H_{0,het}}$:

$$[4\pi G \cdot t_{H_{0,het}} \cdot \rho_{vol}] \cdot \frac{\Omega_m \sigma_8}{\Omega_{vol}} \cdot \tilde{t} d\tilde{t} = d\dot{\underline{\varepsilon}}_{het} \tag{16.88}$$

(4) Comparison with the rate caused by $\rho_{m,hom}$:

In general, the Hubble time derived in the heterogeneous universe $t_{H_{0,het}}$ in Eq. (16.88) differs from the Hubble time derived in the homogeneous universe $t_{H_{0,hom}}$. The difference amounts to $q = 9.2\%$, see Eq. (16.26):

$$t_{H_{0,het}} = t_{H_{0,hom}} \cdot (1 - q) \tag{16.89}$$

With it, the rate in Eq. (16.88) is as follows:

$$(1-q)\cdot[4\pi G\cdot t_{H_{0,hom}}\cdot\rho_{vol}]\cdot\frac{\Omega_m\sigma_8}{\Omega_{vol}}\cdot\tilde{t}d\tilde{t}=d\dot{\underline{\varepsilon}}_{het}(q)\qquad(16.90)$$

The above rectangular bracket is identified with the rate in Eq. (14.23), it is the rate in a homogeneous system, see THM (41). It is abbreviated by $\dot{\underline{\varepsilon}}_{hom}$:

$$\dot{\underline{\varepsilon}}_{hom}:=\dot{\underline{\varepsilon}}_{L,rr,\,at\,dV_0,\,hom}=4\pi\cdot G\cdot\rho_{vol}\cdot t_{H_{0,hom}}\qquad(16.91)$$

With it, the rate in Eq. (16.90) is as follows:

$$(1-q)\cdot\dot{\underline{\varepsilon}}_{hom}\cdot\frac{\Omega_m\sigma_8}{\Omega_{vol}}\cdot\tilde{t}d\tilde{t}=d\dot{\underline{\varepsilon}}_{het}(q)\qquad(16.92)$$

The difference $q=0.92\%$ is relatively small. Accordingly, we treat it as a higher order approximation. At leading order, q is neglegted relativ to one in the difference $(1-q)$. As a consequence, in leading order in q, the rate in Eq. (16.92) is as follows:

$$\dot{\underline{\varepsilon}}_{hom}\cdot\frac{\Omega_m\sigma_8}{\Omega_{vol}}\cdot\tilde{t}d\tilde{t}=d\dot{\underline{\varepsilon}}_{het}\quad\text{at leading order}\qquad(16.93)$$

In the universe, the heterogeneity or structure at a time $\tilde{t}_{em}$ in Fig. (16.3) has grown from the Big Bang at $\tilde{t}=0$ until $\tilde{t}_{em}$. The resulting rate $\dot{\underline{\varepsilon}}_{het}(\tilde{t}_{em})$ is obtained from the rate in Eq. (16.93) by integration:

$$\int_0^{\tilde{t}_{em}}\dot{\underline{\varepsilon}}_{hom}\cdot\frac{\Omega_m\sigma_8}{\Omega_{vol}}\cdot\tilde{t}d\tilde{t}=\int_0^{\dot{\underline{\varepsilon}}_{het}(\tilde{t}_{em})}d\dot{\underline{\varepsilon}}_{het}\qquad(16.94)$$

The above integrals are evaluated:

$$\dot{\underline{\varepsilon}}_{hom}\cdot\frac{\Omega_m\cdot\sigma_8\cdot\tilde{t}_{em}^2}{2\Omega_{vol}}=\dot{\underline{\varepsilon}}_{het}(\tilde{t}_{em})\qquad(16.95)$$

The above fraction is abbreviated by the following *rate ratio* κ:

$$\boxed{\kappa(\tilde{t}_{em}):=\frac{\dot{\underline{\varepsilon}}_{het}(\tilde{t}_{em})}{\dot{\underline{\varepsilon}}_{hom}}=\frac{\Omega_m\cdot\sigma_8\cdot\tilde{t}_{em}^2}{2\Omega_{vol}}=\frac{\Omega_m\cdot\sigma_8}{2\Omega_{vol}\cdot(1+z_{em})^2}}\qquad(16.96)$$

(5) Effect of the sum of rates:

Both rates $\dot{\varepsilon}_{het}(\tilde{t}_{em})$ and $\dot{\varepsilon}_{hom}$ add up to the sum:

$$\dot{\varepsilon}_{sum} := \dot{\varepsilon}_{het} + \dot{\varepsilon}_{hom} = \dot{\varepsilon}_{hom} \cdot (1+\kappa) \tag{16.97}$$

Effects of that sum can be observed in the heterogeneous universe. These effects include the age of the heterogeneous universe $t_{H_{0,het}}$ and the density that is independent of the cosmological redshift $\rho_{\Lambda,het}$. Thereby, $\rho_{\Lambda,het} = \Omega_{\Lambda,het} \cdot \rho_{cr,0}$ can be determined with help of the Hubble rate in Eq. (16.6):

$$H^2 = \frac{8\pi G}{3} \cdot \rho_{cr,0} \cdot \left(\Omega_r \cdot \frac{r_0^4}{r^4} + \Omega_m \cdot \frac{r_0^3}{r^3} + \Omega_{\Lambda,het} \right) \tag{16.98}$$

For it, the Hubble rate is used at the present - day time $t = t_0$, so that $r = r_0$. Hereby, the value of $H(t_0) = H_0$ is marked by the subscript *het*. As a consequence, the squared rate in Eq. (16.98) is as follows:

$$H_{0,het}^2 = \frac{8\pi G}{3} \cdot \rho_{cr,0} \cdot (\Omega_r + \Omega_m + \Omega_{\Lambda,het}) \tag{16.99}$$

The density that is the same for each cosmological redshift is described by the density parameter $\Omega_{\Lambda,het}$, in the heterogeneous universe, and it is described by the density parameter Ω_{vol}, in the homogeneous universe. The *density ratio* of these density parameters can be described with help of the rate ratio κ in Eq. (16.96), and with help of a new exponent ξ, without loss of generality - of course, the value of ξ has to be derived:

$$\frac{\Omega_{\Lambda,het}}{\Omega_{vol}} = (1+\kappa)^{\xi} \tag{16.100}$$

During the time evolution of the universe, the fraction of radiation decreases. At the redshift $z_{eq} = 3411 \pm 48$, matter dominates radiation. The corresponding scaled time is as follows, see Planck-Collaboration (2020):

$$\frac{t}{t_{H_0}} = \tilde{t} = \frac{1}{1+z}, \quad \text{thus,} \quad \tilde{t}_{eq} = \frac{1}{1+z_{eq}} = 2.9 \cdot 10^{-4} \tag{16.101}$$

The corresponding standard deviation of the heterogeneity is the following function, see Eq. (16.86), of the present - day value $\sigma_8 = 0.8118 \pm 0.0089$ of that standard deviation, see Planck-Collaboration (2020) Eq. (16.86):

$$\sigma_m(\tilde{t}) = \sigma_8 \cdot \tilde{t}, \quad \text{thus,} \tag{16.102}$$

$$\sigma_m(\tilde{t}_{eq}) = 0.811 \cdot 2.9 \cdot 10^{-4} = 2.38 \cdot 10^{-4} \tag{16.103}$$

Consequently, the heterogeneity is essential at times at which the radiation is negligible. As a consequence, the squared Hubble rate in Eq. (16.99) is as follows:

$$H^2_{0,het} = \frac{8\pi G}{3} \cdot \rho_{cr,0} \cdot \left(\Omega_m + \Omega_{vol} \cdot (1+\kappa)^\xi\right) \quad \text{at} \quad \Omega_r << \Omega_m \tag{16.104}$$

Alternatively, the density of matter and radiation can be summarized $\Omega_{\bar{m}} = \Omega_m + \Omega_r$, see e. g. Carmesin (2024c). In the above Eq. (16.104), the product $\frac{8\pi G}{3} \cdot \rho_{cr,0}$ is identified with the Hubble constant of the ΛCDM model in Eq. (16.7). Consequently, the Hubble constant in the heterogeneous universe in Eq. (16.104) is as follows:

$$\boxed{H_{0,het} = H_{0,\Lambda CDM} \cdot \sqrt{\Omega_m + \Omega_{vol} \cdot (1+\kappa)^\xi}} \tag{16.105}$$

The relation of the density parameters in Eq. (16.100) is expanded by the critical density and by $4\pi G t_{H_{0,het}}$:

$$\frac{\Omega_{\Lambda,het}}{\Omega_{vol}} = (1+\kappa)^\xi = \frac{\rho_{\Lambda,het} \cdot 4\pi G t_{H_{0,het}}}{\rho_{vol} \cdot 4\pi G t_{H_{0,het}}} \tag{16.106}$$

The above fraction is equivalently expanded by $\frac{t_{H_{0,hom}}}{t_{H_{0,het}}}$:

$$(1+\kappa)^\xi = \frac{\rho_{\Lambda,het} \cdot 4\pi G t_{H_{0,het}}}{\rho_{vol} \cdot 4\pi G t_{H_{0,hom}}} \cdot \frac{t_{H_{0,hom}}}{t_{H_{0,het}}} \tag{16.107}$$

The above denominator is identified with the homogeneous rate in Eq. (16.91):

$$\dot{\underline{\varepsilon}}_{hom} = 4\pi \cdot G \cdot \rho_{vol} \cdot t_{H_{0,hom}} \tag{16.108}$$

In the heterogeneous system, the homogeneous rate is replaced by the sum $\dot{\underline{\varepsilon}}_{sum}$ in Eq. (16.97), and the density ρ_{vol} is replaced by $\rho_{\Lambda,het}$, and the Hubble time $t_{H_{0,hom}}$ is replaced by $t_{H_{0,het}}$. Consequently, the rate in Eq. (16.108) takes the following form:

$$\dot{\underline{\varepsilon}}_{sum} = 4\pi \cdot G \cdot \rho_{\Lambda,het} \cdot t_{H_{0,het}} \tag{16.109}$$

The above term is identified with the numerator in Eq. (16.107). As a consequence, the denominator and the numerator in Eq. (16.107) are identified with respective rates:

$$(1+\kappa)^{\xi} = \frac{\dot{\underline{\varepsilon}}_{sum}}{\dot{\underline{\varepsilon}}_{hom}} \cdot \frac{t_{H_{0,hom}}}{t_{H_{0,het}}} \tag{16.110}$$

The sum $\dot{\underline{\varepsilon}}_{sum}$ in Eq. (16.97) is equal to $\dot{\underline{\varepsilon}}_{hom} \cdot (1+\kappa)$. Consequently, the above Eq. is equivalent to the following Eq.:

$$(1+\kappa)^{\xi} = (1+\kappa) \cdot \frac{t_{H_{0,hom}}}{t_{H_{0,het}}} \tag{16.111}$$

In the above Eq., the Hubble times are equivalently replaced by the corresponding Hubble parameters. Thereby, $H_{0,\Lambda CDM}$ describes the homogeneous universe:

$$(1+\kappa)^{\xi} = (1+\kappa) \cdot \frac{H_{0,het}}{H_{0,\Lambda CDM}} \tag{16.112}$$

In the above Eq., the Hubble rate in Eq. (16.105) is applied:

$$\boxed{(1+\kappa)^{\xi} = (1+\kappa) \cdot \sqrt{\Omega_m + \Omega_{vol} \cdot (1+\kappa)^{\xi}}} \tag{16.113}$$

(6) Derivation of the exponent ξ:

In order to solve Eq. (16.113), we apply the square. Additionally, we abbreviate $1+\kappa(z_{\mathrm{em}})$ by y, and we abbreviate y^{ξ} by w:

$$y^2(\Omega_m + \Omega_{vol} \cdot w) = w^2 \quad \text{or} \tag{16.114}$$

$$0 = w^2 - w \cdot \Omega_{vol} \cdot y^2 - \Omega_m \cdot y^2 \text{ with} \tag{16.115}$$

$$y = 1 + \kappa(z_{\rm em}) \text{ and } w = y^\xi \tag{16.116}$$

The above quadratic equation has the following two solutions:

$$w_\pm = \frac{\Omega_{vol} \cdot y^2}{2} \cdot \left(1 \pm \sqrt{1 + \frac{4\Omega_m}{\Omega_{vol}^2 \cdot y^2}}\right) \tag{16.117}$$

The solution w_- is negative. Thus, it does not provide a real exponent, see Eq. (16.116). Accordingly, we solve the above equation for the exponent ξ by using the solution w_+ (see Eq. 16.116):

$$\xi = \frac{\ln(w_+)}{\ln(y)} \text{ and } y = 1 + \kappa(z_{\rm em}) \text{ with} \tag{16.118}$$

$$w_+ = \frac{\Omega_{vol} \cdot y^2}{2} \cdot \left(1 + \sqrt{1 + \frac{4\Omega_m}{\Omega_{vol}^2 \cdot y^2}}\right) \tag{16.119}$$

(7) Relation of $H_0(z_{CMB})$ and $H_{0,\Lambda CDM}$:

At z_{CMB}, the universe was almost homogeneous.

Consequently, the density parameter Ω_{vol} of the homogeneous universe in Eq. (16.61) is applied:
$\Omega_{vol} = \frac{2}{3}$, consequently, $\Omega_m = 1 - \Omega_{vol} = \frac{1}{3}$. Moreover,
$\sigma_8 = 0.8118 \pm 0.0089$, (Planck-Collaboration, 2020, table 2),
$z_{\rm CMB} = 1090.3 \pm 0.41$, (Planck-Collaboration, 2020, table 2),
$H_0(z_{CMB}) = 66.88\,(\pm 0.92)\,\frac{\rm km}{\rm s \cdot Mpc}$, (Planck-Collaboration, 2020, table 2).

Thus, the ratio $\kappa(z_{\rm CMB})$ in Eq. (16.96) has the following value:

$$\kappa(z_{\rm CMB}) = \frac{0.8118 \cdot 1/3}{2 \cdot 2/3 \cdot (1 + 1090.3)^2} = 1.7 \cdot 10^{-7} \tag{16.120}$$

With it, we evaluate ξ, see Eqs. (16.118, 16.119):

$$\xi_{\mathrm{CMB}} = 1.5 \tag{16.121}$$

Consequently, the Hubble constant $H_{0,het}(z_{CMB})$ is derived with help of $\kappa(z_{\mathrm{CMB}}) = \mathcal{O}(10^{-7})$, see Eqs. (16.120, 16.105):

$$H_{0,het}(z_{CMB}) = H_{0,\Lambda CDM} \cdot (1 + \mathcal{O}(10^{-7})) \approx H_{0,\Lambda CDM} \tag{16.122}$$

This relation is informative and helpful, as it shows that the Hubble constant observed with the CMB, $H_{0,het}(z_{CMB})$ is almost the same as the Hubble constant $H_{0,\Lambda CDM}$ in the ΛCDM model.

(8) Hubble constant $H_{0,het}(z_{late})$ at $z_{late} = 0.055$ or $z_{near} = 0.055$:

At the redshift $z = 0.055$, the universe is relatively heterogeneous. Consequently, the Hubble constant is derived with help of Eq. (16.105):

$$H_{0,het} = H_{0,\Lambda CDM} \cdot \sqrt{\Omega_m + \Omega_{vol} \cdot (1+\kappa)^{\xi}} \tag{16.123}$$

The other terms in Eq. (16.123) are as follows:
$\Omega_{vol} = \frac{2}{3}$, $\Omega_m = 1 - \Omega_{vol} = \frac{1}{3}$,
$\sigma_8 = 0.8118 \pm 0.0089$, (Planck-Collaboration, 2020, table 2).
As a consequence, the ratio $\kappa(z_{\mathrm{late}})$ is as follows:

$$\kappa(z_{\mathrm{late}}) = \frac{0.8118 \cdot 1/3}{2 \cdot 2/3 \cdot (1+0.055)^2} = 0.182 \pm 0.002 \tag{16.124}$$

With it, ξ is evaluated, see Eqs. (16.118, 16.119):

$$\xi_{\mathrm{late}} = 1.5317 \tag{16.125}$$

Consequently, the theoretical value of the Hubble constant is as follows:

$$H_{0,het}(z_{\mathrm{near}}) = H_{0,\Lambda CDM}\sqrt{\frac{1}{3} + \frac{2}{3} \cdot 1.182^{1.5317}} \tag{16.126}$$

$$= 73.11 \pm 1.08 \, \frac{\mathrm{km}}{\mathrm{s} \cdot \mathrm{Mpc}} \tag{16.127}$$

Thus, our theoretical result is in precise accordance (within errors of measurement) with the currently most precise observation in Eq. (16.2):

$$H_{0,obs}(z_{late}) = 73.04\,(\pm 1.01)\,\frac{\text{km}}{\text{s}\cdot\text{Mpc}} \quad \text{at } \langle z\rangle = 0.055 \qquad (16.128)$$

This is the baseline result in (Riess et al., 2022, sections 5.1 and 5.2), obtained at the 'near field' or 'low redshift' $\langle z\rangle = 0.055$.

The relative difference between theory and measurement is very low at 0.096 %:

$$\Delta_{obs,theo} = \frac{H_{0,obs}(z_{late}) - H_{0,het}(z_{\text{late}})}{H_{0,obs}(z_{late})} \qquad (16.129)$$

$$= \frac{73.11 - 73.04}{73.04} = 0.096\% \qquad (16.130)$$

We summarize our results:

Theorem 43 Time evolution of the Hubble constant
(1) The Hubble constant H_0 is an essential parameter of outer space, cosmology, general relativity and the VD.

However, observations show that H_0 is not constant, see the Planck-Collaboration (2020) and Riess et al. (2022). This discrepancy is derived, solved and clarified by the VD:
(2) The following conditions apply:
(2.1) Space is globally flat, $\Omega_K = 0$.
(2.2) At a redshift z with essential heterogeneity, the radiation is very small and negligible: $\Omega_r << 1$.
(2.3) The variation of H_0 provides a variation in the age of the universe of $q = 9.2\%$. The following results are derived at leading order in q.
(2.4) In the ΛCDM model, the Hubble constant is the following function of the critical density $\rho_{cr,0}$:

$$H_{0,\Lambda CDM} = \sqrt{\frac{8\pi G}{3}\rho_{cr,0}} \qquad (16.131)$$

(2.5) In the homogeneous universe, the following density parameters have been derived, see THM (38):

$$\Omega_{vol} = \frac{2}{3} \text{ and } \Omega_m = 1 - \Omega_{vol} = \frac{1}{3} \tag{16.132}$$

(3) In the heterogeneous universe, the derived Hubble constant is the following function of the redshift:

$$H_{0,het}(z_{em}) = H_{0,\Lambda CDM} \cdot \sqrt{\Omega_m + \Omega_{vol} \cdot (1 + \kappa(z_{em}))^{\xi}} \tag{16.133}$$

Hereby, the density parameter of the cosmological constant is identified:

$$\Omega_\Lambda(z_{em}) = \Omega_{vol} \cdot (1 + \kappa(z_{em}))^{\xi} \tag{16.134}$$

(3.1) Thereby, the rate ratio κ is the following function of the amplitude of matter fluctuations σ_8 and of the redshift z_{em} of the emission of the considered object, see Fig. (16.3):

$$\kappa(\tilde{t}_{em}) := \frac{\dot{\underline{\varepsilon}}_{het}(\tilde{t}_{em})}{\dot{\underline{\varepsilon}}_{hom}} = \frac{\Omega_m \cdot \sigma_8}{2\Omega_{vol} \cdot (1 + z_{em})^2} \tag{16.135}$$

$$\text{with } \frac{t_{em}}{t_{H_0}} \tilde{t}_{em} = \frac{1}{1 + z_{em}} \tag{16.136}$$

(3.2) Moreover, the exponent ξ is the following function of the redshift z_{em}:

$$\xi = \frac{\ln(w_+)}{\ln(y)} \text{ and } y = 1 + \kappa(z_{\text{em}}) \text{ with} \tag{16.137}$$

$$w_+ = \frac{\Omega_{vol} \cdot y^2}{2} \cdot \left(1 + \sqrt{1 + \frac{4\Omega_m}{\Omega_{vol}^2 \cdot y^2}}\right) \tag{16.138}$$

(3.3) The derived function $H_{0,het}(z_{em})$ in Eq. (16.133) predicts future measurements, see Fig. (16.4).
(3.4) The derived function $H_{0,het}(z_{em})$ in Eq. (16.133) is in precise accordance with various observations $H_{0,obs}$ of the Hubble constant at different redshift values z_{em}, see Fig. (16.4).

Thereby, we do not execute any fit or introduce any hypothesis or proposal. Accordingly, thus result provides great evidence for the VD.
(3.5) In particular, at the redshift $z_{late} = 0.055$ *or* $z_{near} = 0.055$ *our derived value*

$$H_{0,het}(z_{\text{late}}) = 73.11 \pm 1.08 \, \frac{\text{km}}{\text{s} \cdot \text{Mpc}} \tag{16.139}$$

is in precise accordance with the observed value

$$H_{0,obs}(z_{late}) = 73.04 \, (\pm 1.01) \, \frac{\text{km}}{\text{s} \cdot \text{Mpc}} \quad \text{at } \langle z \rangle = 0.055 \tag{16.140}$$

Thus, the relative difference is smaller than 0.1 %:

$$\Delta_{obs,theo} = \frac{H_{0,obs}(z_{late}) - H_{0,het}(z_{\text{late}})}{H_{0,obs}(z_{late})} = 0.096\% \tag{16.141}$$

Thereby, we do not execute any fit or introduce any hypothesis or proposal. Accordingly, this result provides great evidence for the VD.
(3.6) In particular, at the redshift $z_{CMB} = 1090.3$ *our derived value is almost the same as the observed value:*

$$\begin{aligned} H_{0,het}(z_{CMB}) &= H_{0,obs}(z_{CMB}) & (16.142) \\ &= H_{0,\Lambda CDM} \cdot (1 + \mathcal{O}(10^{-7})) & (16.143) \\ &\approx H_{0,\Lambda CDM} & (16.144) \end{aligned}$$

(3.7) The items (3.4), (3.5) and (3.6) show that the VD derives and explains the observed discrepancy of H_0 *values, the so-called Hubble tension:*
(3.7.1) The heterogeneity causes an additional squared gravitational field.
(3.7.2) The additional squared gravitational field provides an additional change $\dot{\varepsilon}_{L,rr}$ *at a probe volume* dV_0 *in Fig. (14.2).*
(3.7.3) This additional change provides an increased rate of formation of volume, so that the present state of the universe is

achieved in a decreased time, resulting in a decreased age $t_{H_{0,het}}$ *of the universe, see section (16.3).*
(3.7.4) This decreased age $t_{H_{0,het}}$ *corresponds to an increased Hubble constant* $H_{0,het}$*.*
(3.7.5) This increased Hubble constant $H_{0,het}$ *corresponds to an additional density parameter* $\Delta\Omega = \Omega_{vol} \cdot (1+\kappa)^\xi - \Omega_{vol}$ *in Eq. (16.135).*
(3.8) The age of the universe in the homogeneous universe is characterized by $t_{H_{0,\Lambda CDM}} = 13.8 \cdot 10^9$ years. *The age of the universe in the heterogeneous universe is characterized by,* $t_{H_{0,het}} = 12.66 \cdot 10^9$ years.
(4) For the first time, a fundamental and very general theory clarifies the source of the Hubble tension. Moreover, that theory provides the Hubble constant $H_0(z_{em})$ *as a function of the redshift* z_{em}*. Furthermore, that function* $H_0(z_{em})$ *includes predictions for future measurements. That general and fundamental theory is the VD, and it does not execute any fit (indeed, the used cosmological parameters are derived by the VD) or introduced hypothesis.*

In a first and direct version, the VD solves the H_0 *tension directly, for instance see (Carmesin, 2021d, chapter 7), (Carmesin, 2023g, chapters 20). Hereby, in (Carmesin, 2023g, chapter 20), as well as in Carmesin (2024c), and in the present book, the change of the Hubble time* t_{H_0} *by heterogeneity has been included. In a second and direct version, the change of* t_{H_0} *has been neglected in a relatively good approximation in (Carmesin, 2021d, chapter 7).*

In version three, the VD analyzes the early universe at the Planck scale in (Carmesin, 2018f, sections 2.7-2.13, Fig. 1.28), Carmesin (2018e), Carmesin (2019b), Carmesin (2019e), or Carmesin (2021g), Carmesin (2021e). Hereby, the essential equivalence of this derivation with direct derivations has been shown in (Carmesin, 2021d, chapters 7, 8), (Carmesin, 2023g, chapters 20-21).

Theorem 44 Reality of cosmological constant density

(1) The energy density or dynamic density of the cosmological constant Ω_Λ *is real according to the Hacking (1983) criterion:*

(1.1) The heterogeneity $\sigma(t)$ *in the universe is caused by the spatial distribution of real masses. It causes gravity, curvature and locally formed volume, LFV (THMs 9, 10, 13).*

As a consequence, the heterogeneity is real, according to the Hacking (1983) criterion, since the heterogeneity σ *is the cause of the measurable effect of an increased light - travel distance* d_{LT}.

(1.2) The local relation between cause and effect in item (1.1) takes place at larger distances as well as at global distances of the scale of the universe. In this manner, the global density of the cosmological constant will become real in the following:

The LFV in item (1.1) contributes to globally formed volume, GFV (THM 15). Consequently, heterogeneity σ *and LFV contribute to the density of the cosmological constant, see Eqs. (16.133, 16.131, 16.135, 16.137, 16.138):*

$$\Omega_\Lambda(z_{em}) = \Omega_{vol} \cdot (1 + \kappa(z_{em}))^\xi \tag{16.145}$$

(1.3) The density of the cosmological constant can be described by the density parameter $\Omega_\Lambda(z_{em})$, *and the density influences the measurable Hubble constant, see Eq. (16.133):*

$$\begin{aligned} H_{0,het}(z_{em}) &= H_{0,\Lambda CDM}\sqrt{\Omega_m + \Omega_{vol}(1 + \kappa(z_{em}))^\xi} \\ &= H_{0,\Lambda CDM}\sqrt{\Omega_m + \Omega_\Lambda(z_{em})} \end{aligned} \tag{16.146}$$

As a consequence, the density of the cosmological constant is real, according to the Hacking (1983) criterion, since $\Omega_\Lambda(z_{em})$ *is the cause of the measurable effect of the Hubble constant* $H_{0,het}(z_{em})$.

(1.4) There is an additional derivation of the reality of $\Omega_\Lambda(z_{em})$: *Hereby, the Hubble constant in Eq. (16.146) influences the real*

Hubble time $t_{H_{0,het}}$ and the real age of the universe t_0, see e. g. Carmesin (2019b), as $t_{H_{0,het}} = 1/H_{0,het}$. Thereby, the real age of the universe can additionally be confirmed with help of independent measurements, such as observations of star clusters or of radioactive nuclids, see e. g. Karttunen et al. (2007). As a consequence, the density of the cosmological constant $\Omega_\Lambda(z_{em})$ is real, according to the Hacking (1983) criterion, since $\Omega_\Lambda(z_{em})$ is the cause of the measurable effect of the real age of the universe t_0.

(2) The density of volume Ω_{vol} is a very important, useful and enlightening ideal value, with the corresponding real density of the cosmological constant $\Omega_\Lambda(z_{em})$, according to the Hacking (1983) criterion:

(2.1) The density of the cosmological constant $\Omega_\Lambda(z_{em})$ is real, as you can change the positions of matter. With it, you can change the LFV. The LFV changes the GFV. The GFV modifies the density of the cosmological constant $\Omega_\Lambda(z_{em})$. That density changes the Hubble constant, the Hubble time and the real age of the universe. As a consequence, the cosmological constant $\Omega_\Lambda(z_{em})$ is real, according to the Hacking (1983) criterion, see item (1) above.

(2.2) In contrast, you cannot manipulate the density of volume Ω_{vol}, as it is the derived (THM 38) and constant (THM 8) ideal value corresponding to the real and variable cosmological constant $\Omega_\Lambda(z_{em})$, that can be manipulated by replacing masses.

(2.3) The ΛCDM value of the Hubble constant $H_{0,\Lambda CDM}$ represents the calendar date $t_{H_{0,\Lambda CDM}}$. Consequently, $H_{0,\Lambda CDM}$ is a measured value, see Planck-Collaboration (2020). It corresponds to the homogeneous universe. Hence it corresponds to the density of volume Ω_{vol}. Thus, it corresponds to the early universe. Thence, in a very good approximation, it corresponds to the redshift of the CMB $z_{CMB} = 1090.3(\pm 0.41)$, see Planck-

Collaboration (2020).

(3) In the late universe, at $z_{late} = 0.055$*, the density of the cosmological constant* $\Omega_\Lambda(z_{em})$ *is as follows (THM 43):*

$$\Omega_\Lambda(z_{em}) = \Omega_{vol} \cdot (1 + \kappa(z_{late}))^\xi \tag{16.147}$$

$$\Omega_\Lambda(z_{late}) = \frac{2}{3} \cdot (1 + 0.182(\pm 0.002))^{1.5317} = 0.861(\pm 0.002) \tag{16.148}$$

Consequently, in the late universe, the difference between the density of the real cosmological constant $\Omega_\Lambda(z_{late})$ *and its ideal value* Ω_{vol} *is as follows:*

$$\Omega_\Lambda(z_{late}) - \Omega_{vol} = 0.861(\pm 0.002) - \frac{2}{3} = 0.194(\pm 0.002) \tag{16.149}$$

That difference is equal to the difference of the real cosmological constant $\Omega_\Lambda(z_{late})$ *and the real cosmological constant* $\Omega_\Lambda(z_{early})$ *in the early universe:*

$$\Omega_\Lambda(z_{late}) - \Omega_\Lambda(z_{early}) = 0.194(\pm 0.002) \tag{16.150}$$

(4) The reality of the cosmological constant density $\Omega_\Lambda(z_{late})$ *(described by its density parameter) in THM (43) provides the importance of the corresponding ideal value of the density of volume* Ω_{vol} *in THM (38).*

(4.1) Similarly, the reality of the real gas, see van der Waals (1873), provides the importance of the ideal gas, see e. g. Landau and Lifschitz (1980).

(4.2) For instance, the reality of the area of the real unit circle $A_{real} \approx \pi$ *in real curved space or in real VD, provides the importance of the ideal area of the unit circle in flat space,* $A_{ideal} = \pi$*.*

(4.3) Moreover, the real observed global flatness $\Omega_{K,obs} \approx 0$*, see Planck-Collaboration (2020), provides the importance of the derived and ideal global flatness, see Carmesin (2023h,g) of space* $\Omega_{K,derived} = 0$*.*

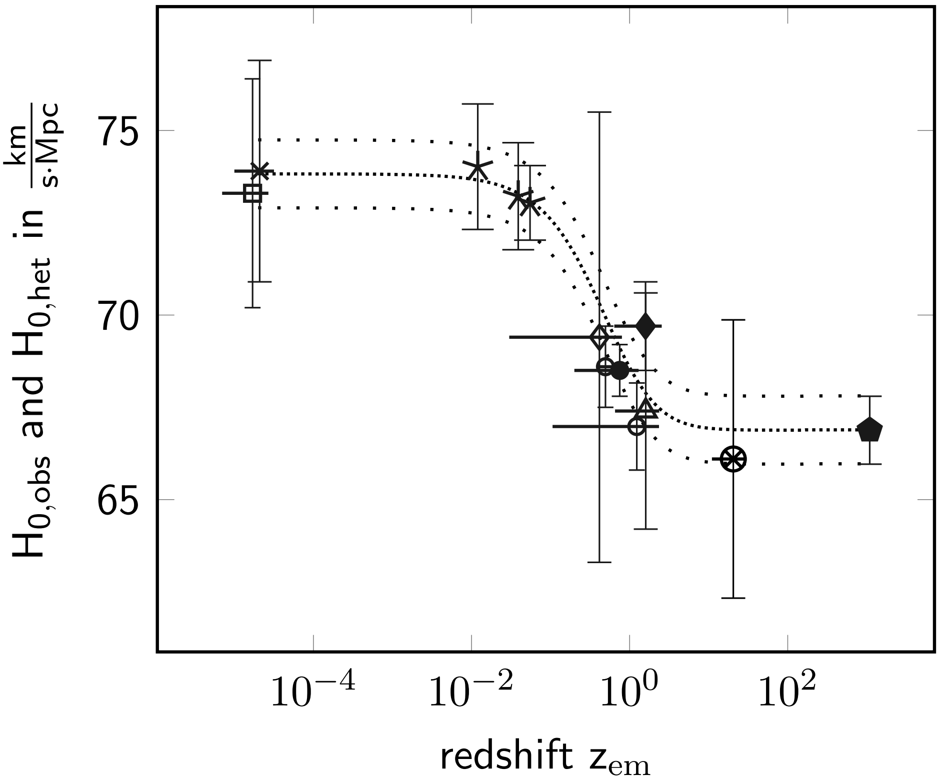


Figure 16.4: Observed values $H_{0,obs}$ and derived values $H_{0,het}$ of H_0 as a function of z_{em}.

Probes: ×, megamaser, Pesce et al. (2020). ⋆, distance ladder with SN type Ia, Riess et al. (2022), Galbany et al. (2023), (Uddin et al., 2024, most precise evaluation). full ⋄, starburst galaxies, Cao et al. (2021). o, baryonic acoustic oscillations, BAO, Philcox et al. (2020), Addison et al. (2018)). •, weak gravitational lensing and galaxy clustering, Abbott et al. (2020)). Δ, strong gravitational lensing, Birrer et al. (2020). ⋄, gravitational wave, Escamilla-Rivera and Najera (2022). ⊗, old galaxies or stars, Cimatti and Moresco (2023), (Tab. 1). Square, surface brightness, Blakeslee et al. (2021). Pentagon, CMB, Planck-Collaboration (2020).

Derived from the VD: densely dotted (loosely dotted: range of theoretical values, resulting from error of measurement of the CMB value). Measured and derived values are in accordance.

Chapter 17

On the graviton

In the standard model of elementary particles, the propagation of gravity should be instantiated by an elementary particle, the hypothetical graviton, see e. g. Workman et al. (2022), Blokhintsev and Galperin (1934).

The VD does instantiate gravity in an exact manner and in terms of volume portions VPs and their change tensors, see THMs (9, 10). Thereby, the change tensors exhibit properties of quanta, see chapters (9, 12, 16). As a consequence, the VPs and their change tensors provide essential properties of the graviton.

The graviton should have six essential properties: It should propagate gravity, it should be a quantum, and it should have spin two, see e. g. (Workman et al., 2022, p. 25, 109, paragraph 21). Moreover, it should have zero rest mass, it should provide curvature, and it should cause the expansion of space (Workman et al., 2022, p. 1142). Next we show how these properties are provided by volume portions:

(1) The VPs and their change tensors provide the propagation of the gravitational interaction, see THMs (5, 36, 9, 10).

(2) The VPs and many of the change tensors of VPs are quantized, see chapters (9, 21, 22, 23, 24), 12, 16).

(3) The VPs and many of their change tensors represent tensors of rank two. As a consequence, in general, these tensors exhibit

quadrupolar symmetry, see e. g. Landau and Lifschitz (1971). As a further consequence, these change tensors have periodicity $\varphi_{per} = \pi$, see PROP (3). Consequently, these tensors have spin two, see PROP (4) or Carmesin (2024h).

(4) VPs have zero rest mass, see section (4.2.2) & THM (3).

(5) The VPs and some of their change tensors describe the formation of the local curvature of space and time, see THMs (9, 10).

(6) The VPs and some of their change tensors describe densities that are essential for the expansion of space since the Big Bang, see chapters (14, 16). Moreover, in this context, the VPs provide the density of volume, the solution of the cosmological constant problem and the solution of the Hubble tension, see THMs (38, 43, 40).

Furthermore, the VPs provide an instructive and momentous property: The VPs and some of their change tensors provide wave functions of quantum physics, quantum postulates as well as founded tools of quantum field theory, see chapters (9, 10, 12). This property is far beyond the usually expected properties of the graviton.

Altogether, the VD provides the properties of the graviton. Moreover, the VD represents a deeper and more momentous concept of minimal energy volume portions: These VPs are quanta and these VPs are the foundation of quanta in nature, of universal quantization, and of the whole field of quantum physics. VPs provide gravity, electromagnetic waves including photons (THMs 7, 18), curvature of space and time as well as quanta in an indivisible and impartial manner!

Chapter 18

Discussion

18.1 Overview

18.1.1 Volume in nature founds physics

Research showed that volume in nature is fundamental[1]. It founds physics in the following steps, see Fig. (1.4):

(1) Present - day evident properties are identified.

(2) Volume dynamics, VD, is derived and implies:

(3) Gravity

(4) Curvature and GR

(5) Quanta and QP

(6) Parts of the standard model of elementary particles, SMEP

(6.1) Electrodynamics & elementary charge, Carmesin (2021f).

(6.2) Electroweak interaction & couplings, Carmesin (2022e).

[1]See e. g. Carmesin (2017c, 2018g,f,e, 2019b, 2020d), or Carmesin (2021d,a,f, 2022d,e,f), Carmesin (2023g,h,j), Carmesin (2022a,c), Carmesin (2023i), Carmesin (2012, 2017b,a), Sprenger and Carmesin (2018); Carmesin (2018c,d,b), Carmesin (2019a,e, 2020b); Heeren et al. (2020); Schöneberg and Carmesin (2020), Carmesin (2021b,c); Schöneberg and Carmesin (2021); Lieber and Carmesin (2021); Sawitzki and Carmesin (2021), Carmesin and Schöneberg (2022); Carmesin (2022b, 2021g), Carmesin (2023b,c,f), Carmesin (2023d, 2024a), Carmesin (2024d,h,g,c), Carmesin (2024i,f,b), Carmesin and Carmesin (2018); Carmesin and Brüning (2018), Carmesin (2019f,d), Carmesin (2020a,c), Carmesin (2021e), Carmesin (2023a,e) Carmesin (2024e,j) .

18.1.2 Clarifications and solutions by VD

The VD implies essential physical theories. Moreover, the VD solves many fundamental problems in present - day physics. Furthermore, the VD clarifies many open questions in physics in a valuable and revealing manner. Additionally, the VD discovers many deep and essential relations among the important present - day physical theories. In particular, the VD unifies and generalizes the essential theories of physics[2].

18.1.3 Aim and method

The aim of this book is to present the VD on the basis of evident properties, to derive the essential physical theories from the VD, and to solve important present - day fundamental problems in physics, see Fig. (1.4).

For it, the hypothetico deductive method is applied[3]. Thereby, it is useful and illuminative that we use only evident properties of present - day physics in the hypothetical part, see section (4.2.2). As a consequence, the implications are very reliable. The most essential properties are the fact that volume in nature has no rest mass $m_{0,vol} = 0$, and that electromagnetic waves exhibit linear superposition.

In the deductive part, more than forty helpful and enlightening theorems are derived on the basis of the present - day evident properties, see Fig. (1.4). The THMs provide measurable results.

18.1.4 Essential results and insights

(1) VD implies new fundamental physical methods and facts:

[2] See e. g. Perlmutter et al. (1998), Riess et al. (2000, 2022), Smoot (2007), Planck-Collaboration (2020), Carmesin (2017c, 2018f,e, 2019b). Carmesin (2021d,a,f), Carmesin (2022d,e, 2023g).

[3] See e. g. Popper (1935), Kircher et al. (2001), Niiniluoto et al. (2004).

(1.1) Insightful and utile facts and methods are derived, for instance, the invariance of c (THM 1), as well as the new and useful simultaneously measurable distance measures (THM 2). These provide new relations, functional descriptions and deep insights of causes and respective effects. These are essential in order to derive new physical results, and to avoid the necessity of preventable hypothetical methods. Consequently, these new methods are very reliable, exact, useful, valuable, fundamental, enlightening, generally applicable and fruitful.

(1.2) Basic results are implied: the existence of many volume portions, VPs (THM 3), the representation of VPs and their change tensors (THM 4), the volume dynamics, VD.

(2) VD implies propagation.

(2.1) The propagation of VPs and of change tensors are derived and expressed in the form of the differential equation ,DEQ of VD, see THMs (5, 36). The VPs and change tensors can form stationary solutions of the DEQ of VD, see THMs (6, 28). This DEQ includes electromagnetic waves, for instance, see THM (7).

(2.2) In general, the propagation of volume is (and has been) permanently causing the formation of volume since the Big Bang, see e. g. THMs (14, 15) and Carmesin (2019b, 2021d, 2023g).

(2.2.1) Thereby, the space becomes globally flat. This has been derived, see Carmesin (2023h,g), and observed, see Planck-Collaboration (2020).

(2.2.2) In nature, the rate of increase of time can be modified by two conditions: a curvature of space and a relative velocity $\vec{v}$. In the globally flat space, the average of curvature is zero, and the average of relative velocities $\vec{v}_j$ of objects j is zero, as the space is isotropic, see section (4.2.2). As a consequence, in the globally flat space, there is no deviation from the constant rate of increase of time. Thus, global flatness provides a valuable measurable constant rate of time. For instance, that time is used in cosmology and in the time evolution of the universe, see e. g. Hobson et al. (2006), Karttunen et al. (2007). Moreover, local deviations from a constant rate of increase of time can be measured with help of the reference clear frame: the globally flat space with its uniform global time[4].

(3) VD implies gravity and curvature.

(3.1) Newton (1687) developed classical mechanics as well as Newtonian gravity. Both can be obtained from the VD as an approximation, see section (19.4).

(3.2) Moreover, the VD implies and achieves gravity as well as curvature of space and time in an exact manner, see THMs (8, 9, 10): These are caused by the change tensor of an unidirectional relative additional volume. In addition, this explains the exact gravitational potential in curved space and time by the relative additional volume multiplied by $-c^2$. This clarifies how and why volume in nature causes gravity in an exact manner. Furthermore, this concept is generalized to further interactions, see section (7.4).

(3.3) Furthermore, there occur a gravitational energy density and a kinetic energy density. It is very useful, instructive and momentous, that a gravitational self - energy and a kinetic energy compensate each other, see THMs (11, 16, 25).

(3.4) Present - day physics describes the expansion of space since the Big Bang by a global expansion of space, see e. g. Hobson et al. (2006). However, in nature, the volume does not expand, but the amount of volume in nature increases. This process is described by the VD: The VD implies, explains and clarifies the local and global formation of volume since the Big Bang, see THMs (12, 13, 14, 15, 17). In particular, it describes

[4] See e. g. Carmesin (2018e, 2019b, 2021a, 2023g).

the local and global dynamics of VPs, space and time in an informative and unifying manner.

(3.5) In addition, the VD implies, explains and clarifies that the exact law of gravity has a universal gravitational constant G, that does not vary, see THMs (9, 10). This has been questioned by a so-called MOND - proposal[5], see Milgrom (1983). But THMs (9, 10) show that there is no physical possibility for a varying gravitational constant G. Moreover, the universal constant of gravitation G has been confirmed by observation, see Banik et al. (2024).

(4) VD implies quantum physics.

(4.1) The VD implies and explains essential universal properties of quanta and causality, see THMs (18, 19, 20, 21, 22, 23, 24), 22, 23, 25, 37): universal quantization, nonlocality, causality, minimal fluctuations, quanta of VPs, zero-point oscillations, ZPOs, and self - interaction of quanta.

Hereby, that self - interaction clarifies the essential marginal energetic stability of quanta.

(4.2) Based on universal quantization in item (4.1), the VD implies: At each ω, each minimal energy VP $dE_{VP,min}$ is quantized, see THMs (21, 22, 23). However, the energy density of volume u_{vol} is not quantized, see THMs (21, 22, 23, 24). Similarly, atoms are quantized, but the density of atoms in a gas is not quantized.

(4.3) Based on universal quantization in item (4.1), the VD implies, explains, clarifies and generalizes the quantum postulates, see THMs (26, 27, 28, 29, 31).

(4.4) Each object causes locally formed volume at a rate $\dot{\underline{\varepsilon}}_{L,rr}$. That rate provides the full information inherent to the wave

[5]The MOND propsal has been suggested as an alternative to dark matter. Note that the VD also implies the process of formation of dark matter, see e. g. Carmesin (2018g, 2019b).

function of that object, see THMs (26, 27, 31). Consequently, that rate $\dot{\underline{\varepsilon}}_{L,rr}$ provides and founds the wave function in QP. Moreover, that rate contributes to the expansion of space since the Big Bang. Furthermore, that rate explains the density of volume and the Hubble tension, see THMs (8, 38, 40, 43).

(4.5) Based on universal quantization in part (4.1), the VD implies, explains, clarifies and deepens the understanding of quantum correlations and spectra, see THMs (32, 33, 34, 35). The results derived here are very valuable, as they provide typical tools of present - day quantum field theory, QFT, such as ladder operators or number operators. These results are useful and instructive, as they are beyond present - day QFT. For instance, these results are founded by the VD, whereas present - day QFT is a set of tools and ideas, see e. g. Peskin and Schroeder (1995). Moreover, present - day QFT is local, see e. g. Schwartz (2014), whereas these results include nonlocal phenomena, such as observed in Aspect et al. (1982). Furthermore, these results do not cause a diverging zero - point energy, ZPE, in contrast to present - day QFT, see e. g. Nobbenius (2006) and chapter (13).

(5) On the properties of volume in nature:

(5.1) The VD implies, explains and clarifies the energy density u_{vol} of volume, in empty space, in a homogeneous universe and in a heterogeneous universe. This energy density u_{vol} of volume in nature can be measured macroscopically by observation of the rate of expansion of space since the Big Bang[6].

(5.2) Moreover, the VD implies the ZPOs of volume in nature. These ZPOs can be measured microscopically with help of the Casimir force that attracts two very narrow electrically conducting parallel plates, see Casimir (1948), Klimchitskaya et al. (2009) and THM (40).

[6]See e. g. Einstein (1917), Lemaître (1927), Perlmutter et al. (1998), Riess et al. (2000).

(5.3) Furthermore, the VD implies the energetic compensation of the kinetic energy of the ZPOs by the self - interaction of quanta in item (4.1). This self - interaction is a consequence of three indivisible and impartible consequences of the VD: gravity, curvature and quanta!

As a consequence, the VD solves the cosmological constant problem, see THMs (38, 39, 40, 41).

(5.4) Additionally, the VD solves and clarifies the problem of the Hubble tension and the local value of the Hubble constant, see THMs (38, 41, 42, 43). These results also clarify[7] the dark energy and the energy density of the cosmological constant Λ: In the time evolution of our universe, there occurs an increase of heterogeneity, and that heterogeneity causes an additional rate of formation of volume, which causes the Hubble tension as well as the local value of the Hubble constant.

(5.5) The expected properties, see (Workman et al., 2022, p. 1142), of the hypothetical graviton are implied by the VD, see chapter (17).

(5.6) The VD implies, includes and provides the wave functions, which provide the quantum postulates, energy densities, quantum properties, nonlocality, causality, self - energy, as well as the generalized and modified quantum field theory, see THMs (11, 16, 18, 19, 20, 25, 26, 27, 31, 32, 33, 34, 35).

(5.7) For the first time, the VD fundamentally implies, derives, clarifies and explains the properties of dark energy, without executing any fit or introducing any hypothesis: Thereby, the real VPs and their change tensors provide the observed energy density (THMs 38, 43, 44), the observed Hubble rate H (THM 14), the observed Hubble constant $H_0(z_{em})$ (THM 43, Fig. 16.4), the observed Casimir force of the electromagnetic ZPOs (THM 40), the observed zero complete energy density of the electro-

[7] See e. g. Einstein (1917), Zeldovich (1968), Huterer and Turner (1999), Carmesin (2023g).

magnetic ZPOs (THM 40), the observed volume (THM 38), the observed gravity and its transmission (THM 9), the observed curvature (THM 10), and the spin of the graviton (chapter 17).

As the dark energy is fundamentally derived and explained by the VPs, the dark energy is also related to the additional properties of VPs: This includes matter formed from VPs via in phase transitions, see Carmesin (2021a). Moreover, this includes fundamental interactions emerging in formed elementary particles by internal oscillations, see tables (18.3, 18.1, 18.2) and Carmesin (2021f, 2022e).

(6) The VD implies, explains and clarifies the mechanism of nonlocality, see THM (37).

(7) The above items (3), (4), (5) as well as (6) clearly show that the VD implies gravity, general relativity, quantum physics, cosmology, as well as electromagnetic and electroweak interaction in a fundamental, insightful, utile and inherently indivisible as well as impartible manner.

Consequently, the VD explains gravity, general relativity, quantum physics, cosmology, as well as electromagnetic and electroweak interaction. As a consequence, the VD is the basic or underlying dynamics of gravity, general relativity, quantum physics, cosmology, as well as electromagnetic and electroweak interaction. Thereby, the VD clarifies the indivisible and impartible relation between gravity, general relativity, quantum physics, cosmology, as well as electromagnetic and electroweak interaction. In particular, the VD unifies gravity, general relativity, quantum physics, cosmology as well as electromagnetic and electroweak interaction.

The VD provides results in the present - day essential fields of physics summarized above: gravity, general relativity, quantum physics, cosmology, as well as fundamental interactions. As a consequence, the VD is the underlying theory of the present - day fields of physics. In particular, the DEQ of VD, see THMs

(5, 36, 7), can be regarded as a key equation of the world. Such an equation is sometimes called fundamental equation, see e. g. Hilbert (1915); Hilbert et al. (1928).

The VD achieves this underlying dynamics, as the DEQ of VD implies gravity, general relativity and stationary waves, see THMs (9, 10, 6)). It is this implied triple that implies universal quantization, which implies the full field of quantum physics, including solutions of problems (see e. g. 40) and generalizations (see e. g. 26, 37, 38, 41, 43).

(8) More fundamental results of the VD have been derived, see for instance sections (18.3, 18.4).

18.1.5 Universal quanta

In present - day QP, the description is based on two steps: Firstly, a system is described in a relatively classical manner. Secondly, a quantization procedure is applied.

In contrast, in the VD, quanta are derived in a universal manner (THM 18). On that basis, the postulates of QP are derived (THMs 27, 31). With it, universal nonlocality is derived (THMs 19, 37, 20). Moreover, the zero-point oscillation, ZPO, and the zero-point energy, ZPE, are derived (THM 21). Furthermore, the self-interaction of quanta is derived (THM 25). It is interpreted as an energy that causes a marginal binding of quanta, a binding at the complete energy zero.

Additionally, the quantum correlations and spectra are derived (THM 32, 33, 34, 35). The VD provides the same ladder operators as present - day QFT. However, the complete energy is zero in the VD, whereas it is nonzero in present - day QFT. Consequently, that zero complete energy derived in the VD represents an essential modification of present - day QFT, that is achieved by the VD.

Moreover, these results show that the ladder operators do not provide evidence for the methods used in present - day

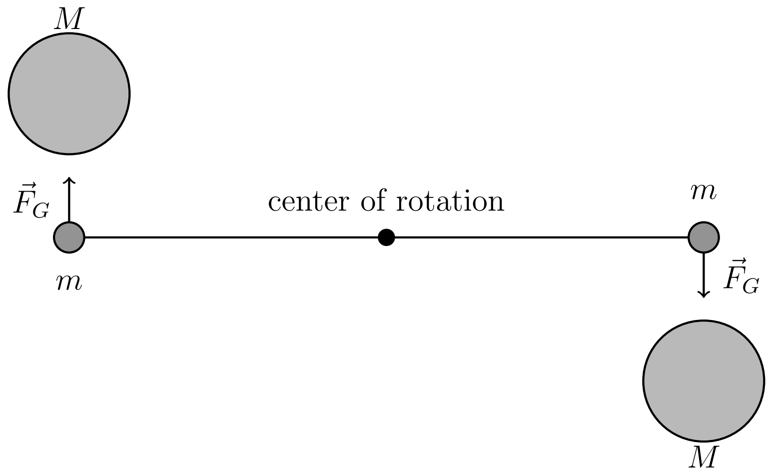


Figure 18.1: Cavendish (1798) measured the gravitational force F_G that a mass M causes at a mass m. (Of course, m causes the opposite force at M.)

QFT, as the same ladder operators are provided by the VD. But present - day QFT causes the cosmological constant problem, whereas the VD solves that problem. Similarly, present - day QFT requires a postulated quantization procedure, whereas quanta form in a natural manner in the VD.

Correspondingly, the 'universal quantization' (see THM 18 or Carmesin (2023j), Carmesin (2023g)) in the VD can be named 'universal quanta' or 'emerging universal quanta', more clearly.

18.1.6 Reality of volume portions

Hacking (1983) proposes the following criterion for the reality: A putative entity is real, if that entity can be used to manipulate other entities whose existence has already been established, see e. g. (Niiniluoto et al., 2004, p. 560). We apply this criterion to VPs:

The mass M in Fig. (18.1) can manipulate a nearby mass m. Such a mass M is usually called field generating mass, or it might be named curvature generating mass in GR. Thus, in general, a field generating mass is real.

Moreover, a field generating mass M manipulates and increases the radial light - travel distance d_{LT} in its vicinity, see Fig. (5.1) or e. g. Will (2014).

Furthermore, the increased radial light - travel distance d_{LT} in the vicinity of a field generating mass M manipulates the orbit of a planet of the mass M. For instance, the increased radial light - travel distance d_{LT} of Sun manipulates the orbit of Mercury: The perihelion of Mercury moves in a retarded manner, see e. g. Einstein (1915), Hobson et al. (2006). Accordingly, that increased radial light - travel distance d_{LT} is real.

As a mathematical fact, the real increased radial light - travel distance d_{LT} implies an increased amount of volume in its vicinity, the additional and measurable volume δV, see THMs (4, 2). Consequently, the additional volume δV is as real as the real increased radial light - travel distance d_{LT}.

Additionally, a volume portion δV has the energy $\delta E = u_{vol} \cdot \delta V$, whereby u_{vol} has been derived (THMs 38, 8, 43), and, in essence, it has been measured, see e. g. Perlmutter et al. (1998), Riess et al. (2000), Smoot (2007), Planck-Collaboration (2020). For comparison, the mass or energy of the electron, which is often used as an example for a real entity, see Hacking (1983), has been measured, but it has not yet been derived in a fundamental manner. However, the charge has been derived by the VD, see Carmesin (2021d).

In addition, the real additional volume δV, or volume - portions thereof, have rest mass zero, see (Workman et al., 2022, p. 1142). As a consequence, the VPs propagate at the velocity c, see THM (3).

Accordingly, the real volume - portion is provided by such an additional volume δV or by parts thereof.

18.2 Derived Theory

In this section, we summarize the VD, and essential present - day physical theories that are derived by the dynamics of real volume in nature, the volume dynamics, VD, analyzed here, see table (18.3).

derived theory	reference
↓ volume dynamics ↓	THMs (3, 2, 4, 5, 26, 36)
exact gravity	THM (9)
curvature and GR	THMs (10, 30)
formation of quanta	THM (18)
quantum postulates	THM (27, 31)
modified & generalized QFT	THMs (32, 33, 34, 35)
density of volume	THMs (8, 39, 40), Carmesin (2018f)
H_0 tension	THM (43), Carmesin (2021d)
electrodynamics	THM (7)
elementary charge	Carmesin (2021f)
electroweak couplings and theory	Carmesin (2022e)
masses of some elementary particles	Carmesin (2021a)
theory of cosmological parameters	Carmesin (2021a)
causality	THM (20)
era of cosmic 'inflation' or unfolding	Carmesin (2021a)

Table 18.1: Derived theory: The VD is derived from evident properties. Essential present - day theories are derived on the basis of the fundamental and general dynamics of real volume in nature: the VD.

18.2.1 Real and completing change tensor

The change tensor $\varepsilon_{L,ij}$ has two essential groups of elements:

Firstly, the diagonal elements $\varepsilon_{L,jj}$ and the isotropic version

$\varepsilon_{L,iso}$ represent relative additional volume. Thereby, additional volume is in principle the same as the volume that is already present. Moreover, that volume is real, see section (18.1.6).

Secondly, the non - diagonal and antisymmetric components $\varepsilon_{L,ij} = -\varepsilon_{L,ji}$ represent electromagnetic waves (THM 7).

Electromagnetic waves are real: They can be used in telecommunication. Hereby, an emitted electromagnetic wave manipulates the output at the receiver. Consequently, electromagnetic waves are real, according to the criterion proposed by Hacking (1983), see section (18.1.6).

Altogether, the change tensor represents two essential real entities in a derived and coherent manner: volume and electromagnetic waves.

Moreover, these two real entities are the two measured and real entities that are the same for each cosmological redshift. Consequently, these two real entities are described by the cosmological constant Λ, see THMs (8, 38, 40, 43) and chapter (15). The entities described by the cosmological constant have been named dark energy, see Huterer and Turner (1999). Thus, these two real entities of the dark energy are described by the change tensor in a derived, fundamental, precise, predictive, problem solving, quanta - forming and coherent or unified manner.

Furthermore, the change tensors of volume mediate and explain the nonlocality (THMs 19, 37). In contrast, the Einstein (1948) locality principle excludes mediation, so that it is incomplete with respect to nonlocality. As a consequence, the change tensors of volume overcome that incompleteness.

Additionally, Einstein (1907) proposed that a superluminal or nonlocal effect at $w_{eff} > c$ would cause a causality violation. In contrast, the change tensors of volume provide universal causality combined with nonlocality (THMs 19, 37, 20).

18.2.2 How VPs form & found light, gravity & quanta

Here, we summarize the results with respect to the book title, see Fig. (1.4):

(1) VPs exist and propagate.

(2) VPs are modified via change tensors $\varepsilon_{L,ij}$ (THM 4).

(3) VPs and their changes propagate according to the DEQ of VPs (THMs 5, 7, 36).

(4) Items (1-3) imply: $\varepsilon_{L,rr}$ provides gravity in terms of the exact gravitational potential: $\Phi_L = -c^2 \cdot \varepsilon_{L,rr}$ (THMs 9, 10).

(5) Items (1-4) imply: $\varepsilon_{L,rr}$ provides curvature in terms of the position factor $\varepsilon_E = 1 - \varepsilon_{L,rr}$ (THMs 9, 10).

(6) Items (1-3) imply: Non - diagonal antisymmetric elements $\varepsilon_{L,ij,antisymmetric}$ provide electromagnetic waves (THM 7).

(7) Items (3, 6) imply: Minimal energy portions of electromagnetic waves are quanta with a universal constant $K = E_{min}(\omega)/\omega$ and with marginal stability. These quanta are called photons and K is called $\hbar$.

(8) Items (1-7) imply: There occur systems of quanta with a nonzero rest mass m, see e. g. Carmesin (2021a,f, 2022e). Higgs (1964) proposed a similar formation of mass from volume.

(9) Items (3, 7) imply: $\dot{\varepsilon}_{L,jj}$ provide a wave function $\Psi = t_n \cdot \dot{\varepsilon}_{L,rr}$ (THMs 5, 26, 27, 31), the postulates of QP, the PLA, PGI, EFE, and GR (chapters 10, 18). VPs provide generalized charges and wave functions (section 7.4, THM 26).

(10) The normalized diagonal elements $\underline{\varepsilon}_{L,jj}$ provide LFV, GFV, the expansion of space since the Big Bang, the energy densities u_{vol} and u_Λ and the Hubble constant as a function of the redshift $H_0(z_{em})$ (THMs 13, 15, 38, 14, 43).

(11) The non - diagonal symmetric elements $\varepsilon_{L,ij,symmetric}$ provide gravitational waves, for instance, see PROP (2).

(12) In general, there occur linear combinations of change tensors and systems of quanta providing the elementary charge e and the couplings of the electroweak interaction, see Carmesin (2021a,f, 2022e).

18.3 Solved problems

The VD provides solutions to many fundamental problems of physics. These are summarized in table (18.2). Examples are as follows:

18.3.1 Solution of the EPR paradox

The VD solves the EPR paradox as follows: Einstein et al. (1935) showed that QP does not simultaneously provide the following two properties, see also Levi (2015):

(1) locality

(2) reality or observability or measurability

Einstein (1907) proposed that nonlocality would imply causality violation. Accordingly, Einstein (1948) refused nonlocality.

However, that proposed causality violation is not fully founded, see section (2.2.3) and THM (20).

Indeed, the VD solves the EPR paradox by providing nonlocality (THMs 19, 37) as well as measurability and observability (section 18.5) and causality (THM 20).

18.3.2 VD founds local Aharonov - Bohm effect I

In an Aharonov - Bohm experiment, see Kasunic (2019), Fig. (18.2), electrons propagate through a double slit. Then they propagate in a region without a magnetic flux density $\vec{B}$, with a nearby magnetic flux density $\vec{B}$. That magnetic flux density $\vec{B}$ causes a shift Δy of the interference maxima, though the

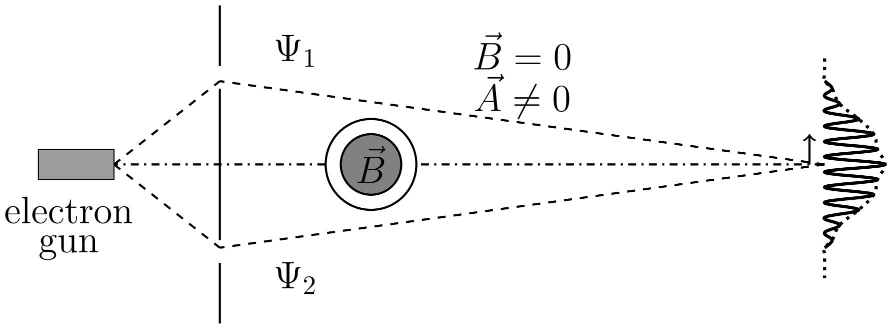


Figure 18.2: Aharonov - Bohm effect: In the experiment, the magnetic flux density $\vec{B}$ is restricted to the marked region (dark grey). Though there is no magnetic flux density $\vec{B}$ at the wave function, the magnetic flux density $\vec{B}$ causes a shift Δy (arrow) of the locations of the interference maxima. Hereby, the envelop (dotted) is not shifted. This is an apparent nonlocal phenomenon.

electrons do not propagate in the region of the magnetic flux density.

Problem: The interference maxima are shifted by $\vec{B}$, though $\vec{B}$ is at a distance. This is an apparent nonlocal effect.

How real VPs solve nonlocality problem: The above nonlocality problem occurs at the electromagnetic interaction acting upon wave functions Ψ. In the VD, both are described by real VPs, $\varepsilon_{L,ij,antismmetric}$ (THM 7) and $\dot{\varepsilon}_{l,jj} \propto \Psi$ (THMs 26, 27, 31 and section 18.1.6), respectively. In the VD, the electromagnetic interaction is described by the real vector potential A^j (THM 7 and section 18.1.6). Such an application of a vector potential to a wave function can be analyzed with the principle of gauge invariance, PGI. That principle is used in (Landau and Lifschitz, 1971, paragraph 18). Moreover, we found the PGI on the basis of the wave function, which is based on the VD, see e. g. Carmesin (2022e).

18.3.3 VD founds the PGI

In the VD, the electromagnetism is founded by the real vector potential A^j (THM 7 and section 18.1.6). In general, the momentum p^j of an object with a charge q can be changed by A^j. In VD and in QP, the state of that object is described by a wave function $\Psi \propto \dot{\varepsilon}_{L,jj}$, and the momentum is proportional to the spatial derivative, $p^j = -i\hbar\partial^j$ (THMs 18, 31).

The Aharonov - Bohm effect provides valuable indications about the analysis of a very useful invariant I: In the Aharonov - Bohm effect, the classical momentum is not changed by $\vec{A}$, since the envelope in Fig. (18.2) is not changed by $\vec{A}$. But the interference pattern in QP is changed by $\vec{A}$. So, there is a local phase $\Theta(x)$. Accordingly, we describe the momentum operator by an extended derivative D^j, so that both observations are provided:

(1) The expectation value $\langle D^j \rangle$, describing the classical momentum $\langle P^j \rangle = -i\hbar \cdot \langle D^j \rangle$, is an invariant I with respect to a change of A^j.

(2) This local phase $\Theta(x)$, describing the interference pattern, is changed locally at $\vec{A}$:

$$A^{j\prime} := A^j - \frac{\partial \Theta}{\partial x^j} \tag{18.1}$$

$$\psi'(x) := \psi(x) \cdot e^{\frac{i}{\hbar} \cdot \frac{q}{c} \cdot \Theta(x)} \tag{18.2}$$

$$D^j(A^j) := \partial^j + \frac{i}{\hbar}\frac{q}{c} A^j \tag{18.3}$$

Correspondingly, local expectation values are introduced:

$$\int_{|x-\tilde{x}|<\varepsilon} \psi'^*(\tilde{x}) D^j(A^{j\prime}) \psi'(\tilde{x}) d^3\tilde{x} =: \langle D^j(A^{j\prime}) \rangle'_{at\ x} \tag{18.4}$$

$$\int_{|x-\tilde{x}|<\varepsilon} \psi^*(\tilde{x}) D^j(A^j) \psi(\tilde{x}) d^3\tilde{x} =: \langle D^j(A^j) \rangle_{at\ x} \tag{18.5}$$

These values are invariant with respect to a change of A^j:

$$\langle D^j(A^{j\prime})\rangle_{at\,x} = \langle D^j(A^j)\rangle_{at\,x} =: I \tag{18.6}$$

Proof:

$$D^j(A^{j\prime})\psi' = \left(\partial^j + \frac{i}{\hbar}\frac{q}{c}A^j - \frac{i}{\hbar}\frac{q}{c}\frac{\partial\Theta}{\partial x^j}\right)\cdot e^{\frac{i}{\hbar}\frac{q}{c}\Theta(x)}\psi, \text{ thus} \tag{18.7}$$

$$D^j(A^{j\prime})\psi' = \left(\frac{\partial\psi}{\partial x^j} + \frac{i}{\hbar}\frac{q}{c}\frac{\partial\Theta}{\partial x^j}\psi + \frac{i}{\hbar}\frac{q}{c}A^j\psi - \frac{i}{\hbar}\frac{q}{c}\frac{\partial\Theta}{\partial x^j}\psi\right)\cdot e^{\frac{i}{\hbar}\frac{q}{c}\Theta(x)} \tag{18.8}$$

$$\text{thence, } D^j(A^{j\prime})\psi' = \left(\frac{\partial\psi}{\partial x^j} + \frac{i}{\hbar}\frac{q}{c}A^j\psi\right)\cdot e^{\frac{i}{\hbar}\frac{q}{c}\Theta(x)} \quad |\cdot e^{-\frac{i}{\hbar}\frac{q}{c}\Theta(x)} \tag{18.9}$$

$$\text{hence, } e^{-\frac{i}{\hbar}\frac{q}{c}\Theta(x)}D^j(A^{j\prime})\psi' = D^j(A^j)\psi \quad |\cdot\psi^* \text{ from left} \tag{18.10}$$

$$\text{so, } \psi^* e^{-\frac{i}{\hbar}\frac{q}{c}\Theta(x)}D^j(A^{j\prime})\psi' = \psi'^* D^j(A^{j\prime})\psi' = \psi^* D^j(A^j)\psi \tag{18.11}$$

$$\text{comsequently, } \langle D^j(A^{j\prime})\rangle'_{at\,x} = \langle D^j(A^j)\rangle_{at\,x} \text{ q.e.d.} \tag{18.12}$$

Clarification: $\langle D^j(A^j)\rangle_{at\,x}$ includes a change by A^j and another change by $\Psi \propto e^{\frac{i}{\hbar}\frac{q}{c}\Theta(x)}$, so that both changes compensate each other. Thus, the VD exhibits the real A^j, the real $\Psi \propto \dot{\varepsilon}_{L,ij,antisymmetric}$, and the resulting invariant $\langle D^j(A^j)\rangle_{at\,x}$. That invariant is very useful, as it provides the solution of the above nonlocality problem, as well as many fundamental interactions:

Generalization: The PGI can be used in order to derive the terms of fundamental interactions for electric charges, the Maxwell equations, see e. g. Carmesin (2021f), Landau and Lifschitz (1971). More generally, the PGI provides terms of interaction for the electroweak interaction and other interactions via generalized charges, sometimes called couplings, see e. g. Carmesin (2022e), Weinberg (1996), Workman et al. (2022).

18.3.4 VD founds local Aharonov - Bohm effect II

Based on the VD, the electromagnetic interaction is described by the vector potential (THM 7). This solves the nonlocality problem in section (18.3.2):

Solution: The VD provides a foundation of the PGI. The PGI extends the initial momentum $\vec{p}_0$ to the momentum $\vec{p} = \vec{p}_0 - \frac{e}{c}\vec{A}$ at $\vec{A}$, see Eq. (18.3) or (Landau and Lifschitz, 1971, Eq. 16.7). Thus, the momentum is $\langle p^j \rangle = -i\hbar \cdot \langle D^j \rangle = -i\hbar \cdot \langle \partial^j \rangle - i\hbar \cdot \frac{i}{\hbar}\frac{q}{c} A^j$ $= p_0^j - \frac{e}{c} \cdot A^j$, as the electron has $q = -e$. Consequently, the de Broglie wave length is $\lambda = \frac{h}{|\vec{p}_0 - \frac{e}{c}\vec{A}|}$. The changed wave length at $\vec{A}$ causes the changed phase $\Theta(x)$ and the shifted interference maxima, for details see Kasunic (2019). Altogether, the VD founds a local solution of the nonlocality problem in section (18.3.2):

Interpretation: The apparent nonlocality problem (S. 18.3.2) of the Aharonov - Bohm effect is solved via the mediation via the VPs and its change $\varepsilon_{L,ij,antisymmetric}$, which represents the vector potential $\vec{A}$ (THM 7). With it, $\vec{A}$ causes the observed shift Δy, see Kasunic (2019) in a local manner, according to Einstein's locality principle.

Reality: According to the Hacking criterion, Hacking (1983), the vector potential $\vec{A}$ (THM 7) is real in classical Maxwell theory and in observed interference patterns in QP. In contrast, the magnetic flux density $\vec{B}$ is real in classical Maxwell theory, but it does not provide the complete reality in observed interference patterns in QP, as $\vec{B}$ does not change the interference pattern in the Aharonov - Bohm effect according to the observed interference pattern.

A potential provides more information, predictions and clarifications than the corresponding fields: A field can be derived as a derivative a respective potential. But, the potential cannot be derived from the field, as the integration constant is a missing information. Thus, the field has less information. The Aharonov - Bohm effect shows clearly, how this becomes essential. As a consequence, a potential has more information and predictive and explanatory power than a respective field.

A potential can solve some apparent nonlocality problems: A potential has a local value, and this value contains the integrated respective fields. Thereby, the integration can be considered as a nonlocal collection of values. As a consequence, a potential can solve some apparent nonlocality problems. An example is the vector potential $\vec{A}$ in the Aharonov - Bohm experiment, see Fig. (18.2).

The VD provides and founds potentials: The VD provides the real VPs providing the exact gravitational potential $\Phi_L = -c^2 \varepsilon_{L,jj}$ (THM 9, 10) and the exact electromagnetic vector potential (THM 7). Moreover, the VD and VPs provide the exact wave function $\Psi = \dot{\varepsilon}_{L,jj}$ (THM 26, 27, 31) and quanta (18). As a consequence, the VD provides the insightful, highly predictive and useful foundation of light, gravity, curvature, GR and quanta, including QP and modified and generalized QFT. In particular, the potential in the VD is an advantage with respect to field theories, such as QFT, moreover, the clarification and reality of the potential by the volume in nature (and in outer space) is an additional and insightful advantage of VD.

18.4 Predictions

In this section, we summarize physical quantities predicted by the new theory, the VD, see table (18.3). Many of the predicted results confirm the VD, such as the predicted function $H_0(z)$ (Fig. 16.4) or the value of the elementary charge, see Carmesin (2021f), or the derived cosmological parameters, see Carmesin (2021a). Thereby, physical quantities within the light horizon R_{lh} are causally separated - thus, states in the R_{lh} form a causally closed system. Further values of physical quantities obtained by the VD are presented in Carmesin (2021d,a,f, 2022e).

quantity	prediction	reference
dimensional horizon D_{hori}	301	Carmesin (2019b)
present - day stable dimension	3	Carmesin (2019b)
critical energy/dynamical densities	$\tilde{\rho}_{cr.,D=3} \approx 0.37$	Carmesin (2023g)
era of cosmic 'inflation'	Figs. 1.1 and 2.4 in	Carmesin (2021a)
time evolution of H_0	Fig. 16.4	THM 43
spectrum of primordial ZPOs	$E = 43.9\ \mu\text{eV}$	Carmesin (2023g)
spectrum of quanta of VPs	section 7	Carmesin (2021a)
the 6 cosmological parameters	table 14.1	Carmesin (2021a)
sum of neutrino masses	$\Omega_\nu = 3.9556 \cdot 10^{-5}$	Carmesin (2021a)
elementary charge	$\tilde{e} = 0.085\,424\,548$	Carmesin (2021f)
mass of Higgs boson	$m_H = 125.541\ \frac{\text{GeV}}{\text{c}^2}$	Carmesin (2021a)
<u>weak interaction</u> weak angle W-boson mass Z-boson mass VEV	- $\Theta_W = 29.118^o$ $M_W = 81.409\ \frac{\text{GeV}}{\text{c}^2}$ $M_Z = 91.717\ \frac{\text{GeV}}{\text{c}^2}$ $VEV = 247.6\ \frac{\text{GeV}}{\text{c}^2}$	Carmesin (2022e) Carmesin (2022e) Carmesin (2022e) Carmesin (2022e) Carmesin (2022e)

Table 18.3: Predictions: Essential results derived by the VD are summarized. These provide precise accordance with observation, within errors of measurement. Thereby, no fit is executed, and no proposal or hypothesis is introduced. The era of cosmic 'inflation' is explained, see e. g. Carmesin (2017c, 2021b, 2023g). The VD provides the vacuum expectation value, VEV. Thus, the VD bridges elementary particle physics & cosmology.

18.5 Measurements and tests

In this section, we show that volume in nature is measurable and observable. Thereby, we use the SI units, see e. g. Newell et al. (2018), whereby other units are alternatively applicable, such as Planck - units or natural units in table (19.2) or such as Gaussian units, see e. g. Jackson (1975).

(1) The amount of volume dV_L or dV_R is measurable in everyday life in m^3, with help of the d_{LT} or d_{GP}.

(2) The energy density u_{vol} of volume is measurable in outer space in $\frac{\mathrm{J}}{\mathrm{m}^3}$. Moreover, u_{vol} is derived by the VD, see THMs (38, 41, 43).

(3) The additional volume $\delta V = dV_L - dV_R$ is measurable in m^3, with help of the d_{LT} or d_{GP}, see THM (2). Furthermore, δV is derived by the VD, see THMs (4, 38, 41, 43).

(4) The normalized rate of formation of volume is measurable in terms of the Hubble parameter and of the Hubble constant H_0 in $\frac{1}{\mathrm{s}}$, see THM (2, 14, 15, 13). Additionally, $H_0(z)$ is derived by the VD, see THM (43) and Fig. (16.4.

(5) The global curvature of volume in nature (or of space) is measurable in terms of the density parameter Ω_K, see e. g. Planck-Collaboration (2020), Carmesin (2019b, 2023g). In addition, $\Omega_K = 0$ is derived by the VD, see Carmesin (2023h,g).

(6) The normalized rate of formation of volume in item (4) combined with the global flatness $\Omega_K = 0$ in item (5) provides a uniform global time in a measurable manner. As a consequence, that uniform global time can be verified with help of other clocks, such as radioactive decay, see e. g. Carmesin et al. (2020), in a measurable manner.

(7) The time evolution of volume in nature is measurable with help of the combined measurement of distance and redshift, see e. g. Hubble (1929), Perlmutter et al. (1998), Riess et al.

(2022). Moreover, the VD predicts that time evolution as well as the necessary cosmological parameters, see e. g. Carmesin (2021a).

(8) The zero-point oscillations, ZPOs, of the change tensors describing electromagnetic waves in THM (7) can be measured with help of the Casimir force in THM (40) in chapter (15). Furthermore, the VD provides these ZPOs in precise accordance with observation. Thereby, the VD shows that the kinetic energy of these ZPOs is compensated by the potential energy that marginally stabilizes the quanta of these ZPOs. Hereby, the VD solves the cosmological constant problem, see THMs (25, 40) and section (7.4).

(9) Gravitational phenomena are measurable in everyday life. In addition, the VD provides these phenomena, as it provides gravity in an exact manner, see THMs (9, 10).

(10) Curvature of spacetime phenomena are measurable. Additionally, the VD provides these phenomena, as it provides curvature of spacetime, see THM (10) and section (10.6).

(11) Quantum phenomena are measurable. Furthermore, the VD provides these phenomena, as it provides QP, see THMs (18, 31, 33, 34, 35, 40).

(12) Electromagnetic phenomena are measurable. Moreover, the VD provides these phenomena, as it provides electromagnetic waves, see THM (7) and the electric charge, see e. g. Carmesin (2021f).

solved problem	reference
era of cosmic 'inflation' or unfolding	Carmesin (2023g)
Big Bang singularity	Carmesin (2019b)
flatness problem	Carmesin (2023g,h)
dark matter problem	Carmesin (2019b)
dark energy problem	Carmesin (2019b, 2023g)
horizon problem	Carmesin (2019b)
EPR paradox	THMs 19, 37
nonlocality & delayed choice experiments	THMs 19, 37
causality problem	THM 20
cosmological constant problem	THM 40
graviton problem	C. 17
H_0 tension problem	THM 43
universality of quantization	THM 18
marginal stability of quanta	THMs 25, 6, 28
derivation of postulates of QP	THM 31
derivation of correlations in QP	THMs 32, 33, 34, 35
derivation of entanglement of quanta	Carmesin (2021f)
derivation of elementary charge	Carmesin (2021f)
masses of elementary particles	Carmesin (2021a, 2022e)
derivation of weak couplings	Carmesin (2022e)
derivation of cosmological parameters	Carmesin (2021a)
fine-tuning problem	Carmesin (2019b)
Aharonov-Bohm nonlocality	section (18.3.4)

Table 18.2: Fundamental problems solved by the VD.

Chapter 19

Glossary

19.1 Natural units

quantity	observed value	reference
G	$6.674\,30(15) \cdot 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}$	Workman et al. (2022)
c	$299\,792\,458\,\frac{\mathrm{m}}{\mathrm{s}}$, exact	Workman et al. (2022)
h	$6.626\,070\,15\ \cdot 10^{-34}$ Js, exact	Newell et al. (2018)
k_B	$1.380\,649\ \cdot 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$, exact	Newell et al. (2018)
ε_0	$8.854\,187\,817\ \cdot 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}}$	Workman et al. (2022)

Table 19.1: Universal constants, ((Newell et al., 2018, table 3), (Workman et al., 2022, tables 1.1, 2.1)).

Planck (1899) introduced natural units. We mark quantities in natural units by a tilde. Natural units can be expressed by G, c and $\hbar$. Even the electric field constant ε_0, the elementary charge e and electroweak couplin constants are derived from the VD, see Carmesin (2021f, 2022e). k_B relates statistics to physics.

physical entity	Symbol	Term	in SI-Units
Planck length	L_P	$\sqrt{\frac{\hbar G}{c^3}}$	$1.616 \cdot 10^{-35}$ m
Planck time	t_P	$\frac{L_P}{c}$	$5.391 \cdot 10^{-44}$ s
Planck energy	E_P	$\sqrt{\frac{\hbar \cdot c^5}{G}}$	$1.956 \cdot 10^{9}$ J
Planck mass	M_P	$\sqrt{\frac{\hbar \cdot c}{G}}$	$2.176 \cdot 10^{-8}$ kg
Planck volume	$V_{D,P}$	L_P^D	
Planck volume, ball	$\bar{V}_{D,P}$	$V_D \cdot L_P^D$	
Planck density	ρ_P	$\frac{c^5}{G^2 \hbar}$	$5.155 \cdot 10^{96} \frac{\mathrm{kg}}{\mathrm{m}^3}$
Planck density, ball	$\bar{\rho}_P$	$\frac{3c^5}{4\pi G^2 \hbar}$	$1.2307 \cdot 10^{96} \frac{\mathrm{kg}}{\mathrm{m}^3}$
Planck density, ball	$\bar{\rho}_{D,P}$	$\frac{M_P}{\bar{V}_{D,P}}$	
Planck temperature	T_P	$T_P = \frac{E_P}{k_B}$	$1.417 \cdot 10^{32}$ K
scaled volume	$\tilde{V}_D$	$\frac{V_D}{\bar{V}_{D,P}}$	
scaled energy	$\tilde{E}$		$E = \tilde{E} \cdot E_P$
scaled density	$\tilde{\rho}_D$	$\frac{\tilde{M}}{\tilde{r}^D} = \frac{\tilde{E}}{\tilde{r}^D}$	$\rho_D = \tilde{\rho}_D \cdot \bar{\rho}_{D,P}$
scaled length	$\tilde{x}$		$x = \tilde{x} \cdot L_P$
Planck charge	q_P	$M_P \sqrt{G 4\pi\varepsilon_0}$	11,71 e

Table 19.2: Planck - units or natural units.

19.2 General notation

$\vec{a}$: acceleration

dA: area

$\vec{A}$: vector potential (THM 7)

A_D: area of D-dimensional ball with radius 1, its volume is $V_D = A_D \cdot D$ (THM 9)

$\hat{a}$ and $\hat{a}^+$: ladder operators, with number operator $\hat{N}$ (THM 34)

E_{av}: available energy (THM 18). Criterion: An energy is called available, iff (if and only if) it can be transformed to another form of energy. Example: The energy $\hbar\omega$ of a photon can be absorbed by an electron. Hereby, the electron takes up that energy. Criterion: In SR, an object with zero rest mass $m_0 = 0$ has the kinetic energy $E_{kin} = p \cdot c$. In SR, an object with nonzero rest mass $m_0 > 0$ has the kinetic energy $E_{kin} = \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2$. Example: A photon has the kinetic energy $E_{kin} = p \cdot c = \hbar\omega$.

E_0: rest energy. An object with nonzero rest mass $m_0 > 0$ has the rest energy $E_0 = m_0c^2$.

e: elementary charge

$\vec{e}$ direction vector, it has length one

$\varepsilon_E(r)$: position factor (THM 10)

$\hat{\varepsilon}_{L,\mu,\nu}$: amplitude (THM 21)

f: frequency

Fourier transformation with functions $f_\mu(\vec{L})$ as well as $b_\mu(\tau)$ (THM 32)

g_{ij}: metric tensor (THM 4)

$\vec{G}^*$: gravitational field (THM 9)

$\vec{G}_{gen}$: generalized field (THM 9)

GR: General relativity describes physics in which curvature of space and time occurs. In this book, respective results are derived from the VD.

γ: Lorentz factor

$\hbar = \frac{h}{2\pi}$: reduced Planck constant

$H = \frac{\dot{r}}{r}$: Hubble parameter (THMs 12, 14), with Hubble constant $H_0 = H(t_0)$ and Hubble time $t_{H_0} = 1/H_0$

$\mathcal{H}$: Hilbert space (THM 31)

$\vec{k}$: wave vector

k: absolute value of $\vec{k}$, wave number

k: factor of increase, in the context of GFV

k: curvature parameter (THMs 12, 14)

λ: wavelength

Λ: cosmological constant with the corresponding energy density $u_\Lambda = \frac{\Lambda}{8\pi G}$ (THMs 41, 38, 43)

localizable object: It has a center of energy.

m, M: mass

m_0: rest mass

ω: circular frequency

$\vec{p}$: momentum

P: pressure

p_{grav}: angle of gravitational parallax (THM 2)

$\hat{p}$ or $\hat{p}_\mu$ or q_μ or $\hat{\vec{p}}_\mu$: eigenvalue generating operator (THMs 18, 33)

Ψ: wave function (THMs 26, 27, 31), with the normalization factor t_n

$Q_{gen,\mu,\nu}$: generalized charge (section 7.4)

φ and ϑ: polar angles

Φ: potential

ρ: density or dynamical density (THMs 12, 14),with critical density ρ_{cr} and densitiy parameters $\Omega_j = \frac{\rho_j}{\rho_{cr}}$ (section 16.1).

r, R radial coordinatesee Fig. (1.2)

R_S: Schwarzschild radius

SR: Special relativity describes physics at high velocity and in inertial frames. SR can be derived from the invariance of the velocity of light, we derive this fact in THM (1). That THM implies the time dilation in section (2.2.2) and the energy momentum relation in section (2.2.1), see e. g. Carmesin (2020e) - these two results of SR are used in this book. In this book, respective results are derived from the VD.

s: signal (THM 1)

S: action (section 10.6)

σ: standard deviation. In QP, ΔA is the uncertainty of A.

t: time

T: periodic time

U: voltage

u: energy density

u_{vol}: energy density of volume

u: velocity, in the context of SR

Units: Basically, SI units are used. If natural units or Planck units are used, these the quantities are marked by a tilde.

v: velocity, in the context of SR

V: in the context of volume

Var: variance: $Var^2 = \sigma$

w: velocity

z: redshift, see Fig. (1.1)

ω: circular frequency

ZPE: zero-point energy (THMs 21, 22, 23, 24,35)

ZPO: zero-point oscillation (THMs 21, 22, 23, 24,35)

19.3 Notation on volume in nature

increments: are marked by a d or δ or Δ

d_{GP}: gravitational parallax distance (THM 2).

d_{LT} or dL: light - travel distance (THM 2).

dx_j: incremental coordinates in a local orthogonal coordinate system

$dx_{L,j}$: incremental coordinates in a local orthogonal coordinate system with the d_{LT} as a measure

$d\xi_j$: incremental coordinates in a local orthogonal coordinate system scaled by $\sqrt{g_{jj}}$. $d\xi_j = \frac{dx_{L,j}}{\sqrt{g_{jj}}}$

$\mathcal{H}$: Hilbert space

$\vec{L}$: Additional volume can cause curved spacetime. At each location, a tangent space is used, see e. g. Hobson et al. (2006). In it, a vector $\vec{L}$ is used.

τ or $d\tau$: time in curved spacetime

dV_L: complete volume, e. g. $4\pi R^2 dL$ in a shell

additional volume: $\delta V = dV_L - dV_R$

formed volume: $\underline{\delta}V = \delta V(\tau + \underline{\delta}\tau) - \delta V(\tau)$

normalized rate of formed volume: $\underline{\dot{\varepsilon}}_L = \frac{\delta V}{dV_L \delta\tau}$

relative additional volume: $\varepsilon_L = \frac{\delta V}{dV_L}$

rate of globally formed volume, GFV: $\frac{\dot{V}}{V}$

present-day probe volume: dV_0

19.4 Derivation of classical mechanics

In this section, we derive Newton's second axiom, the main equation of classical mechanics, with help of the postion factor $\varepsilon_E(r) = \sqrt{1 - \frac{R_S}{r}}$, with $R_S = \frac{2GM}{c^2}$ in THM (10 part 5). With it, we analyze the free fall of a mass m_0 that starts at the velocity $v \to 0$ at $r \to \infty$. Thus, the energy is m_0c^2. During the process of free fall, the energy is constant. Moreover, the energy is multiplied by the position factor $\varepsilon_E(r)$, describing the position. Furthermore, the energy is multiplied by the Lorentz factor, describing the velocity, see also Landau and Lifschitz (1971):

$$m_0c^2 = E_0 = E_0 \cdot \varepsilon_E(r) \cdot \gamma(v) \tag{19.1}$$

The above Eq. is divided by E_0, and the two factors are expressed by the respective functions of r and v:

$$1 = \varepsilon_E(r) \cdot \gamma(v) = \sqrt{1 - \frac{v^2}{c^2}}^{-1} \cdot \sqrt{1 - \frac{R_S}{r}} \tag{19.2}$$

Classical mechanics is achieved in the case of small velocity $\frac{v}{c} << 1$ and small Schwarzschild radius $\frac{R_S}{r} << 1$. In a respective approximation, both ratios are used in linear order only:

$$1 \doteq \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) \cdot \left(1 - \frac{1}{2}\frac{R_S}{r}\right) \doteq 1 + \frac{1}{2}\frac{v^2}{c^2} - \frac{1}{2}\frac{R_S}{r} \tag{19.3}$$

The velocity is expressed by $\dot{r}$. Moreover, $\frac{1}{2}\frac{R_S}{r}$ is added:

$$\frac{1}{2}\frac{R_S}{r} \doteq 1 + \frac{1}{2}\frac{\dot{r}^2}{c^2} \tag{19.4}$$

Moreover, the time derivative is applied to the above equation:

$$-\frac{1}{2}\frac{R_S}{r^2}\cdot\dot{r}\doteq\frac{1}{2}\frac{2\dot{r}\ddot{r}}{c^2} \tag{19.5}$$

The above Eq. is multiplied by $m_0c^2/\dot{r}$, and $R_S = \frac{2GM}{c^2}$ is inserted:

$$-\frac{G\cdot M\cdot m_0}{r^2}\doteq m_0\cdot\ddot{r} \tag{19.6}$$

In the above equation, the left hand side is identified with the gravitational force, and $\ddot{r}$ is identified with the acceleration. The equation is supplemented by the correct vectors:

$$\vec{F}\doteq m_0\cdot\vec{a} \tag{19.7}$$

This equation represents Newton's second axiom. Altogether, we derived Newton's second axiom for the case of a falling mass.

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abbreviation	meaning
C.	chapter
DEF	definition
DEQ	differential equation
FLE	Friedmann Lemaître equation
GFV	globally formed volume
GR	general relativity
GSEQ	generalized SEQ
LFV	locally formed volume
PLA	principle of least action
PROP	proposition
QFT	quantum field theory
QP	quantum physics
SEQ	Schrödinger equation
SM	Schwarzschild metric
SR	special relativity
THM	theorem
VD	volume dynamics
VP	volume portion

Table 19.3: Abbreviations.

Bibliography

Aad, G. et al. (2012). Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Phys. Lett. B*, 716:1.

Abbott, B. P. e. a. (2016). Observation of gravitational waves from a binary black hole merger. *Phys. Rev. Lett.*, 116:1–16.

Abbott, T. M. C. et al. (2020). Dark energy survey year 1 results: Cosmological contraints from galaxy clustering and weak lensing. *Phys. Rev. D*, 102:1–34.

Adams, R. A. and Essex, C. (2010). *Calculus*. Rearson, Edison, Wesley, Ontario.

Addison, G. E., Watts, D. J., Bennett, C. L., Halperin, M., Hinshaw, G., and Weiland, J. L. (2018). Elucidating ΛCDM: Impact of Baryon Acoustic Oscillation Measurements on the Hubble Constant Discrepancy. *ApJ*, 853:1–12.

Ash, R. B. (2008). *Basic Probability Theory*. Dover, Mineola N Y.

Aspect, A., Grangier, P., and Roger, G. (1982). Experimental Realization of the Einstein-Podolski-Rosen-Bohm Gedankenexperiment: A new Violation of Bell's Inequalities. *Phys. Rev. Lett.*, 49:91–94.

Ballentine, L. E. (1998). *Quantum Mechanics*. World Scientific Publishing, London and Singapore.

Banik, I. et al. (2024). Strong constraints on the gravitational law from Gaia DR3 wide binaries. *MNRAS*, 527:4573–4615.

Barbero, A. P. L., Hernandez-Figueroa, H. E., and Recami, E. (2010). Propagation speed of evanescent modes. *Phys. Rev. E*, 62:8628–8635.

Bennett, C. L. et al. (2013). Nine-year Wilkinson microwave anisotropy probe (WMAP) Observations: final maps and results. *The Astrophysical Journal Supplement Series*, 208:1–54.

Birrer, S. et al. (2020). TDCOSMO: IV. Hierarchical time-delay cosmography - joint inference of the Hubble constant and galaxy density profiles. *Astr. and Astroph.*, 643:1–40.

Blakeslee, J. P. et al. (2021). The Hubble Constant from Infrared Surface Brightness Fluctuation Distances. *The Astrophysical Journal*, 911(65):1–12.

Blokhintsev, D. I. and Galperin, F. M. (1934). Neutrino hypothesis and conservation of energy. *Pod Znamenem Marxisma*, 6:147–157.

Burisch, C., Carmesin, H.-O., Emse, A., Konrad, U., Küblbeck, J., Pardall, C.-J., Piehler, M., and Pröhl, I. K. (2022). *Universum Physik Sekundarstufe II Gesamtband Qualifikationsphase.* Cornelsen Verlag, Berlin.

Cao, S., Ryan, J., and Ratra, B. (2021). Cosmological constraints from H II starburst galaxy, quasar angular size, and other measurements. *MNRAS*, 000:1–13.

Carmesin, H.-O. (2002). Himmelsmechanik im Unterricht. In Nordmeier, V., editor, *Tagungs-CD Fachdidaktik Physik ISBN 3-936427-11-9*, Berlin. Deutsche Physikalische Gesellschaft, DPG.

Carmesin, H.-O. (2006). Entdeckungen im Physikunterricht durch Beobachtungen des Himmels. In Nordmeier, V. and Oberländer, A., editors, *Tagungs-CD Fachdidaktik Physik, ISBN 978-386541-190-7*, Berlin. Deutsche Physikalische Gesellschaft, Deutsche Physikalische Gesellschaft.

Carmesin, H.-O. (2009). Projekttag Energie in Klasse 7: Von Fotovoltaik bis Biodiesel. In Nordmeier, V. and Grötzebauch, H., editors, *Tagungs-CD Fachdidaktik Physik*, Berlin. Deutsche Physikalische Gesellschaft, ISBN: 978-3-86541-371-0.

Carmesin, H.-O. (2012). Schüler entdecken die Einstein-Geometrie mit dem Beschleunigungssensor. *PhyDid B Internetzeitschrift*, ISSN 2191-379X.

Carmesin, H.-O. (2017a). Quantum Gravity: Discoveries about the Early Universe Including Big Bang, Big Bounce and a Critical Discussion of these. German Astronomical Society Conference at University Göttingen.

Carmesin, H.-O. (2017b). Schülerinnen und Schüler entdecken Gravitationswellen mithilfe von Computerexperimenten. *PhyDid B*, pages 11–18.

Carmesin, H.-O. (2017c). *Vom Big Bang bis heute mit Gravitation: Model for the Dynamics of Space.* Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2018a). A Model for the Dynamics of Space - Expedition to the Early Universe. *PhyDid B*, pages 1–9.

Carmesin, H.-O. (2018b). A Model for the Dynamics of Space - Expedition to the Early Universe. *PhyDid B, FU Berlin, hal-02077596*, pages 1–9.

Carmesin, H.-O. (2018c). Einstein in der Schule (Teil 1) Unterrichtskonzepte zur allgemeinen Relativitätstheorie. *Astronomie und Raumfahrt im Unterricht*, 55(3/4):55–59.

Carmesin, H.-O. (2018d). Einstein in der Schule (Teil 2) Unterrichtskonzepte zur allgemeinen Relativitätstheorie. *Astronomie und Raumfahrt im Unterricht*, 55(6):33–36.

Carmesin, H.-O. (2018e). *Entstehung der Raumzeit durch Quantengravitation - Theory for the Emergence of Space, Dark Matter, Dark Energy and Space-Time*. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2018f). *Entstehung dunkler Energie durch Quantengravitation - Universal Model for the Dynamics of Space, Dark Matter and Dark Energy*. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2018g). *Entstehung dunkler Materie durch Gravitation - Model for the Dynamics of Space and the Emergence of Dark Matter*. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2019a). A Novel Equivalence Principle for Quantum Gravity. *PhyDid B Internet Journal*, pages 1–9.

Carmesin, H.-O. (2019b). Die Grundschwingungen des Universums - The Cosmic Unification - With 8 Fundamental Solutions based on G, c and h. In Carmesin, H.-O., editor, *Universe: Unified from Microcosm to Macrocosm - Volume 1*, pages 1–226. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2019c). *Die Grundschwingungen des Universums - The Cosmic Unification - With 8 Fundamental Solutions based on G, c and h - With Answers to 42 Frequently Asked Questions*. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2019d). Equivalence Principle of Quantum Gravity. *Verhandl. DPG (VI)*, 54(2):74.

Carmesin, H.-O. (2019e). Kontroverse Konstante. *Physik Journal*, 18(7):14.

Carmesin, H.-O. (2019f). The Elementary Particle of Dark Matter. *Verhandl. DPG (VI)*, 54(3):42.

Carmesin, H.-O. (2020a). Density Parameter Ω_Λ of the Dark Energy is a function of g, c and h, and it Evolves with Time. *Verhandl. DPG*, DPG Tagung Bonn GR18.1.

Carmesin, H.-O. (2020b). Explanation of the Rapid Enlargement of Distances in the Early Universe. *PhyDid B*, pages 9–17.

Carmesin, H.-O. (2020c). Explanation of the Rapid Enlargement of Distances in the Early Universe. *Verhandl. DPG*, DPG Tagung Bonn DD4 Astronomie Location P-HS 5.

Carmesin, H.-O. (2020d). The Universe Developing from Zero-Point Energy: Discovered by Making Photos, Experiments and Calculations. In Carmesin, H.-O., editor, *Universe: Unified from Microcosm to Macrocosm - Volume 3*, pages 1–279. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2020e). Wir entdecken die Geschichte des Universums mit eigenen Fotos und Experimenten. In Carmesin, H.-O., editor, *Universe: Unified from Microcosm to Macrocosm - Volume 2*. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2021a). Cosmological and Elementary Particles Explained by Quantum Gravity. In Carmesin, H.-O., editor, *Universe: Unified from Microcosm to Macrocosm - Volume 5*, pages 1–198. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2021b). Lernende erkunden die Raumzeit. *Der Mathematikunterricht*, 67(2):47–56.

Carmesin, H.-O. (2021c). Physical Explanation of the H_0-Tension. *International Journal of Engineering and Science Invention*, 10(8)II:34–38.

Carmesin, H.-O. (2021d). Quanta of Spacetime Explain Observations, Dark Energy, Graviton and Nonlocality. In Carmesin, H.-O., editor, *Universe: Unified from Microcosm to Macrocosm - Volume 4*, pages 1–2743. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2021e). Solution of the H_0 Tension. *Verhandl. DPG*, DPG Tagung SMuK GR13.1.

Carmesin, H.-O. (2021f). The Elementary Charge Explained by Quantum Gravity. In Carmesin, H.-O., editor, *Universe: Unified from Microcosm to Macrocosm - Volume 6*, pages 1–149. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2021g). The Origin of the Energy. *PhyDid B, FU Berlin*, pages 29–34.

Carmesin, H.-O. (2022a). Explanation of Quantum Physics by Gravity and Relativity. *PhyDid B Internet Journal*, pages 425–438.

Carmesin, H.-O. (2022b). Geometric Derivation of the Spectrum of Vacuum. *International Journal of Engineering and Science Invention*, 11(4)I:1–11.

Carmesin, H.-O. (2022c). How Excitations of Vacuum Form Matter. *PhyDid B Internet Journal*, pages 447–458.

Carmesin, H.-O. (2022d). Quantum Physics Explained by Gravity and Relativity. In Carmesin, H.-O., editor, *Universe: Unified from Microcosm to Macrocosm - Volume 7*, pages 1–148. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2022e). The Electroweak Interaction Explained by and Derived from Gravity and Relativity. In Carmesin, H.-O., editor, *Universe: Unified from Microcosm to Macrocosm - Volume 8*, pages 1–168. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2022f). Unification of Spacetime, Gravity and Quanta. In Carmesin, H.-O., editor, *Universe: Unified from Microcosm to Macrocosm - Volume 9*, pages 1–258. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2023a). A Physically Founded and Exact Model of Dark Energy. *Verhandlungen, DPG-Tagung Dresden*, Reihe VI, Band 58, 2/2003, ISSN 2751-0522:GR 10.2–4.

Carmesin, H.-O. (2023b). A Unifying Derivation of Dark Energy. *SciPost Physics*, preprint.

Carmesin, H.-O. (2023c). A Unifying Derivation of Dark Energy. *SciPost Physics*, Preprint.

Carmesin, H.-O. (2023d). Derivation of Dark Energy and of the Source of the Hubble Tension. *MNRAS*, submitted:1–73.

Carmesin, H.-O. (2023e). Exact Unification of Spacetime, Gravity and Quanta. *Verhandlungen, DPG-Tagung Hannover*, Reihe VI, Band 58, 1/2023, ISSN 2751-0522:QI 20.8, DD 20.6–7, DD 26.3, DD 52.1.

Carmesin, H.-O. (2023f). Fundamental Solution to the Cosmological Constant Problem. *Results in Physics*, submitted:1–47.

Carmesin, H.-O. (2023g). Geometric and Exact Unification of Spacetime, Gravity and Quanta. In Carmesin, H.-O., editor, *Universe: Unified from Microcosm to Macrocosm - Volume 10*, pages 1–320. Verlag Dr. Köster, Berlin.

Carmesin, H.-O. (2023h). Students Derive an Exact Solution of the Flatness Problem. *PhyDid B, FU Berlin*, pages 23–30.

Carmesin, H.-O. (2023i). Students discover the Schwarzschild metric at a free fall tower. *PhyDid B, FU Berlin*, pages 31–38.

Carmesin, H.-O. (2023j). Students Exactly Derive Quantization and its Universality. *PhyDid B, FU Berlin*, pages 39–44.

Carmesin, H.-O. (2024a). Fundamental Solution to the Cosmological Constant Problem. *Annals of Physics*, submitted and DOI: 10.13140/RG.2.2.19094.32323:1–72.

Carmesin, H.-O. (2024b). On the Dynamics of Volume in Nature, Gravity, Relativiy and Quanta. *Science Post*, Preprint.

Carmesin, H.-O. (2024c). Students Analyze the Impact of the H_0 Tension on the Worldview. *PhyDid B, FU Berlin.*

Carmesin, H.-O. (2024d). Students Learn the Fundamental Exact Unification of Gravity, Relativity, Quanta and Elementary Charge. *PhyDid B, FU Berlin.*

Carmesin, H.-O. (2024e). Students Learn the Fundamental Exact Unification of Gravity, Relativity, Quanta and Elementary Charge. *Verhandlungen, DPG-Tagung Greifswald*, Reihe VI, Band 59, 1/2024, ISSN 2751-0522:DD 13.4, 13.5, 30.2, 30.3, 38.1–7, 38.9.

Carmesin, H.-O. (2024f). Students Learn to Derive Nonlocality form Fundamental Physics. *PhyDid B, FU Berlin.*

Carmesin, H.-O. (2024g). Students Learn to Derive the Energy Density of Volume. *PhyDid B, FU Berlin.*

Carmesin, H.-O. (2024h). Students Learn to Derive Universal Properties of Gravitons. *PhyDid B, FU Berlin.*

Carmesin, H.-O. (2024i). Students Learn to Solve the Cosmological Constant Problem. *PhyDid B, FU Berlin.*

Carmesin, H.-O. (2024j). The Macroscopic Dynamics of the Big Bang is Derived from the Microscopic Dynamics. *Verhandlungen, DPG-Tagung Berlin*, Reihe VI, Band 59, 5/2024, ISSN 2751-0522:DY 31.8, 31.7.

Carmesin, H.-O. and Brüning, P. (2018). A Monte Carlo Simulation of Cosmic Inflation. *Verhandl. DPG*, page DD27(3).

Carmesin, H.-O. and Carmesin, M. (2018). Explanation of Cosmic Inflation by Gravitation. *Verhandl. DPG*, page GR16(7).

Carmesin, H.-O., Emse, A., Conrad, U., and Pröhl, I. K. (2021). *Universum Physik Sekundarstufe II Nordrhein-Westfalen Einführungsphase*. Cornelsen Verlag, Berlin.

Carmesin, H.-O., Emse, A., Piehler, M., Pröhl, I. K., Salzmann, W., and Witte, L. (2020). *Universum Physik Sekundarstufe II Niedersachsen Qualifikationsphase*. Cornelsen Verlag, Berlin.

Carmesin, H.-O., Kahle, J., Konrad, U., Trumme, T., Witte, L., and Hagedorn, A. (2015). *Universum Physik 7 und 8*. Cornelsen, Berlin.

Carmesin, H.-O., Martens, K., and Rösler, K. (2012). Fotovoltaik im Unterricht - Dreifacher Wirkungsgrad - eine Revolution in der Solartechnik? *MNU*, 65/6:340–348.

Carmesin, H.-O. and Schöneberg, P. (2022). Droplet Model Used to Analyze the Early Universe. *International Journal of Engineering and Science Invention*, 11(2)I:58–66.

Casimir, H. (1948). On the attraction between two perfectly conducting plates. *Proceedings of the Section of Sciences, Koninklijke Nederlandsche Akademie van Wetenschappen*, 51:793–795.

Cavendish, H. (1798). Experiments to determine the density of the earth. *Phil. Trans. R. Soc. Lond.*, 88:469–516.

Chatrchyan, S. et al. (2012). Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. *Phys. Lett. B*, 716:30.

Choudhuri, A. R. (2010). *Astrophysics for Physicists*. Cambridge University Press, Cambridge.

Cimatti, A. and Moresco, M. (2023). Revisiting oldest stars as cosmological probes: new constraints on the hubble constant. *Astrophysical Journal*, 953:149:1–9.

Collaboration, P. (2020). Planck 2018 results. VI. Cosmological parameters. *Astronomy and Astrophysics*, pages 1–73.

Condon, J. J. and Mathews, A. M. (2018). ΛCDM Cosmology for Astronomers. *Publications of the Astronomical Soc. of the Pacific*, 130:1–17.

Cugnon, J. (2012). The Casimir Effect and the Vacuum Energy: Duality in the Physical Interpretation. *Few-Body Systems*, 53:181–188.

D'Ariano, G. M. (2018). Causality re-established. *Phil. Trans. R. Soc. A*, 376:20170313.

Davisson, C. and Germer, L. H. (1927). Diffraction of electrons by a crystal of nickel. *Phys. Rev.*, 30:705–740.

de Sitter, W. (1913). Ein astronomischer Beweis für die Konstanz der Lichgeschwindigkeit. *Pysik. Zeitschr.*, 14:429.

Dirac, P. A. M. (1927). The quantum theory of the emission and absorption of radiation. *Proc. Roy. Soc.*, 114:243–265.

Dirac, P. A. M. (1928). The quantum theory of the electron. *Proc. Roy. Soc.*, 117:610–624.

Dyck, v. R. S., Schwinberg, P. B. J., and Dehmelt, H. G. (1987). Anomalous magnetic moment of the electron. *Phys. Rev. lett.*, 59:26.

Dyson, F. W., Eddington, A. S., and Davidson, C. (1920). A determination of the deflection of light by the sun's gravitational field, from observations made at the total eclipse of may 29, 1919. *Philosophical Transactions of the Royal Society of London*, A220:291–333.

Einstein, A. (1905a). Über einen die Erzeugung und Verwandlung des Lichts betreffenden heuristischen Gesichtspunkt. *Annalen der Physik*, 17:132–148.

Einstein, A. (1905b). Zur Elektrodynamik bewegter Körper. *Annalen der Physik*, 17:891–921.

Einstein, A. (1907). Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen. *Jahrbuch der Radioaktivität und Elektronik*, 4:411–462.

Einstein, A. (1915). Die Feldgleichungen der Gravitation. *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften*, pages 844–847.

Einstein, A. (1917). Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie. *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften*, pages 142–152.

Einstein, A. (1948). Quanten-Mechanik und Wirklichkeit - Quantum Mechanics and Reality. *Dialectica*, 2(3-4):320–324.

Einstein, A., Podolski, B., and Rosen, N. (1935). Can the quantum-mechanical description of physical reality be considered complete? *Phys. Rev.*, 47:777–780.

Escamilla-Rivera, C. and Najera, A. (2022). Dynamical Dark Energy Models in the Light of Gravitational-Wave Transient Catalogues. *JCAP*, 2022(3):60–85.

Feynman, R. P. (1965). *The Character of Physical Law*. MIT Press, Boston.

Feynman, R. P. (1985). *QED - The Strange Theory of Light and Matter*. Princeton University Press, Princton.

Flanders, H. (1989). *Differential Forms*. Dover, New York.

Fornasini, P. and Grisenti, R. (2015). On EXAFS Debye - Waller factor and recent advances. *Journal of Sychrotron Radiation*, 22:1242–1257.

Friedmann, A. (1922). Über die Krümmung des Raumes. *Z. f. Physik*, 10:377–386.

Galbany, L. et al. (2023). An updated measurement of the Hubble constant from near-infrared observations of type Ia supernovae. *Astr. and Astrophys.*, 679:1 – 26.

Garrisi, F. et al. (2019). Experimental test of the collapse time of a delocalized photon state. *Scientific Reports*, 9:11897–11904.

Gong, T. et al. (2012). Recent progress in engineering the Casimir effect - applications to nanophotonics, nanomechanics, and chemistry. *Nanophotonics*, 10:523–536.

Goodstein, D. (1997). The Big Crunch. *EOS, Transactions, American Geophysical Union*, 78:329–334.

Greiner, W. (1979). *Theoretische Physik 4: Quantenmechanik*. Verlag Harri Deutsch, Frankfurt am Main.

Greiner, W. and Reinhardt, J. (1995a). *Theoretische Physik 7: Quantenelektrodynamik*. Verlag Harri Deutsch, Frankfurt am Main, 2. edition.

Greiner, W. and Reinhardt, J. (1995b). *Theoretische Physik 7: Quantenfeldtheorie*. Verlag Harri Deutsch, Frankfurt am Main, 2. edition.

Griffiths, D. (2013). *Introduction to Electrodynamics*. Pearson, London, 4 edition.

Griffiths, D. J. (1994). *Introduction to Quantum Mechanics*. Prentice Hall, Upper Saddle River.

Hacking, I. (1983). *Representing and Intervening.* Cambridge University Press, Cambridge.

Hart, C. A. (1912). *Plane and Solid Geometry.* American Book Company, New York - Cincinatti - Chicago.

Haude, S., Salehi, S., Vidal, S., Maturi, M., and Bartelmann, M. (2022). Model-independent determination of the cosmic growth factor. *SciPost Astronomy*, 2:1–22.

Heeren, L., Sawitzki, P., and Carmesin, H.-O. (2020). Comprehensive Derivation of a Density Limit of the Evolution of Space. *PhyDid B, FU Berlin*, pages 39–42.

Heisenberg, W. (1925). Quantum-Theoretical Re-Interpretation of Kinematic and Mechanical Relations. *Z. f. Phys.*, 33:879–893.

Heisenberg, W. (1927). Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. *Z. f. Phys.*, 43:172–198.

Heisenberg, W. (1958). *Physics and Philosophy.* Harper and Brothers, New York.

Helmcke, B. J., Carmesin, H.-O., Sprenger, L., and Brüning, P. (2018). Three methods for the observation of the Big Bang with our school telescope. *PhyDid B*, pages 55–60.

Higgs, P. W. (1964). Broken Symmetries, Massless Particles and Gauge Fields. *Phys. Lett.*, pages 132–133.

Hilbert, D. (1915). Die Grundlagen der Physik. *Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Math-Physik. Klasse, November.*, pages 1–30.

Hilbert, D., Nordheim, L., and Neumann, J. v. (1928). Über die Grundlagen der Quantenmechanik. *Mathematische Annalen*, pages 395–407.

Hobson, M. P., Efstathiou, G. P., and Lasenby, A. N. (2006). *General Relativity.* Cambridge University Press, Cambridge.

Hubble, E. (1929). A relation between distance and radial velocity among extra-galactic nebulae. *Proc. of National Acad. of Sciences*, 15:168–173.

Huterer, D. and Turner, Michael, S. (1999). Prospects for probing the dark energy via supernova distance measurements. *Phys. Rev. D*, 60:081301–081306.

Jackson, J. D. (1975). *Classical Electrodynamics.* John Wiley, New York.

Jaffe, R. L. (2005). Casimir effect and the quantum vacuum. *Phys. Rev. D*, 72:021301.

Jahnke, T. et al. (2005). *Mathematik - Stochastik.* Cornelsen, Berlin.

Jaques, V. et al. (2008). Wheeler's delaed-choice thought experiment: Experimental realization and theoretical analysis. *Ann. Phys. Fr.*, 1:1–4.

Karttunen, H., Kröger, P., Oja, H., Poutanen, M., and Donner, K. (1996). *Fundamental Astronomy.* Springer, Berlin, 3 edition.

Karttunen, H., Kröger, P., Oja, H., Poutanen, M., and Donner, K. (2007). *Fundamental Astronomy.* Springer, Berlin, 5 edition.

Kasunic, K. J. (2019). Magnetic aharonov - bohm effects and the quantum phase shift: A heuristiv interpretation. *American Journal of Physics*, 87:745–751.

Kircher, E., Girwidz, R., and Häußler, P. (2001). *Physikdidaktik.* Springer, Berlin, 2 edition.

Klimchitskaya, G. L., Mohideen, U., and Mostepanenko, V. M. (2009). Casimir force between real materials: experiment and theory. *Rev. Mod. Phys.*, 81:1827–1885.

Klimchitskaya, G. L. and Mostepanenko, V. (2020). Recent measurements of the casimir force: Comparison between experiment and theory. *Modern Physics Letters*, 35:1.

Kolmogorov, A. N. (1956). *Foundations of the Theory of Probability - translation edited by Nathan Morrison.* Chelsea Publishing Company, New York, 2 edition.

Kravtsov, A. V. and Borgani, S. (2012). Formation of galaxy clusters. *Annual Rev. of Astr. and Astrophys.*, 50:353–409.

Kumar, A. (2018). *Fundamentals of Quantum Mechanics.* Cambridge University Press, Cambridge.

Lamb, W. E. and Retherford, R. C. (1947). Fine structure of the hydrogen atom by a microwave method. *Phys. Rev.*, 72:241–243.

Landau, L. and Lifschitz, J. (1963). *Course of Theoretical Physics VIII - Electrodynamics of Continuous Media.* Pergamon Press, Oxford, 2. edition.

Landau, L. and Lifschitz, J. (1965). *Course of Theoretical Physics III - Quantum Mechanics*. Pergamon Press, Oxford, 2. revised edition.

Landau, L. and Lifschitz, J. (1971). *Course of Theoretical Physics II - The Classical Theory of Fields*. Pergamon Press, Oxford, 3. edition.

Landau, L. and Lifschitz, J. (1975). *Course of Theoretical Physics VII - Theory of Elasticity*. Pergamon Press, Oxford, 2. edition.

Landau, L. and Lifschitz, J. (1980). *Course of Theoretical Physics V - Statistical Physics I*. Pergamon Press, Oxford, 3. edition.

Landau, L. and Lifschitz, J. (1981). *Course of Theoretical Physics IX - Statistical Physics II*. Pergamon Press, Oxford, 2. edition.

Landau, L. and Lifschitz, J. (1982). *Course of Theoretical Physics IV - Quantum Electrodynamics*. Pergamon Press, Oxford, 2. edition.

Landau, L. and Lifschitz, J. (1987). *Fluid Mechanics*. Pergamon Press, Oxford, 2. edition.

Lay, D. C., Lay, S. R., and McDonald, J. J. (2016). *Linear Algebra*. Pearson Education, Boston - New York - Amsterdam - Singapore.

Lee, J. M. (1997). *Riemannian Manifolds: An Introduction to Curvature*. Springer Verlag, New York.

Lemaître, G. (1927). Un univers homogene de masse constante et de rayon croissant rendant compte de la vitesse radiale des nebuleuses extra-galactiques. *Annales de la Societe Scientifique de Bruxelles*, A47:49–59.

Levi, F. (2015). Nonlocal legacy . *Nature Physics*, page 384.

Lieber, J. and Carmesin, H.-O. (2021). Dynamics in the Early Universe. *PhyDid B, FU Berlin*, pages 49–52.

Manning, A. G. et al. (2015). Wheeler's delayed-choice gedanken experiment with a singel atom. *Nature physics*, 11:339–542.

Maxwell, J. C. (1865). A dynamical theory of the electromagnetic field. *Phil. Trans. of the R.l Soc. London*, 155:459–512.

Mayer, J. R. (1842). Remarks on the forces of inorganic nature. *Annalen der Chemie und Pharmacie*, 43:233.

Michelson, A. A. and Morley, E. (1887). On the relative motion of the earth and the luminiferous ether. *American Journal of Science*, 34:333–345.

Milgrom, M. (1983). A modification of the newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophysical Journal*, 270:365–370.

Moore, T. A. (2013). *A General Relativity Workbook.* University Science Books, Mill Valley, CA.

Müller, P. H. (1975). *Lexikon der Stochastik.* Akademie - Verlag, Berlin.

Newell, D. B. et al. (2018). The CODATA 2017 values of h, e, k, and N_A for the revision of the SI. *Metrologia*, 55:L13–L16.

Newton, I. (1687). *Philosophiae Naturalis Principia Mathematica.* Jussu Societatis Regiae ac typis Josephi Streater, London.

Niiniluoto, I., Sintonen, M., and Wolenski, J. (2004). *Handbook of Epistemology.* Springer, Dordrecht.

Nobbenius, S. (2006). *The Cosmological Constant Problem, an Inspiration for New Physics.* PhD thesis, University Utrecht, Utrecht.

Noether, E. (1918). Invariante Variationsprobleme. *Nachrichten der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Math-phys. Klasse*, pages 235–257.

Olofsson, P. and Andersson, M. (2012). *Probability, Statistics, and Stochastic Processes.* Wiley, New York.

Peebles, P. J. E. (1973). Statistical analysis of catalogs of extragalactic objects. i. theory. *ApJ*, 185:413–440.

Penzias, A. and Wilson, R. W. (1965). A measurement of excess antenna temperature at 4080 mc/s. *Astrophysical Journal Letters*, 142:419–421.

Perlmutter, S. et al. (1998). Discovery of a supernova explosion at half the age of the universe. *Nature*, 391:51–54.

Pesce, D. W. et al. (2020). The Megamaser Cosmology Project: XIII. Combined Hubble Constant Constraints. *Astrophysical Journal Letters*, 891:L1.

Peskin, Michael, E. and Schroeder, D. V. (1995). *An Introduction to Quantum Field Theory.* CRC Press, Boca Raton - London - New York.

Philcox, O. H. E., Ivanov, Mikhail, M., Simonovic, M., and Zaldarriaga, M. (2020). Combining Full-Shape and BAO Analyses of Galaxy Power Spectra: A 1.6% CMB-Independent Constraint on H_0. *JCAP*, 2020:1–42.

Planck, M. (1899). Über irreversible Strahlungsvorgänge. *Verlag der Königlich Preußischen Akademie der Wissenschaften*, pages 440–480.

Planck, M. (1900). On the theory of the energy distribution law of the normal spectrum. *Verhandl. Dtsch. Phys. Ges. at December 14th*, 2:237–245.

Planck-Collaboration (2020). Planck 2018 results. VI. Cosmological parameters. *Astronomy and Astrophysics*, pages 1–73.

Pollard, H. and Tenenbaum, M. (1963). *Ordinary Differential Equations*. Dover, New York.

Popper, K. (1935). *Logik der Forschung*. Julius Springer, Wien, 1 edition.

Popper, K. (1974). *Objektive Erkenntnis*. Hoffmann und Campe, Hamburg, 2. edition.

Popper, K. (2002). *The Logic of Scientific Discovery*. Routledge, london - New York, 1 edition.

Pound, R. V. and Rebka, G. A. (1960). Apparent weight of photons. *PRL*, 4:337–341.

Riemann, B. (1868). Über die Hypothesen, welche der Geometrie zu Grunde liegen. *Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen*, 13:133–150.

Riess, A. G. et al. (2000). Tests of the Accelerating Universe with Near-Infrared Observations of a High-Redshift Type Ia Supernova. *The Astrophysical Journal*, 536:62–67.

Riess, A. G. et al. (2022). A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km s^{-1} Mpc^{-1} Uncertainty from the Hubble Space Telescope and the SHOES Team. *The Astrophys. J. Lett.*, 934:L7:1 – 52.

Sakurai, J. J. and Napolitano, J. (1994). *Modern Quantum Mechanics*. Addison-Wesley, New York - London - Delhi.

Sawitzki, P. and Carmesin, H.-O. (2021). Dimensional transitions in a Bose gas. *PhyDid B, FU Berlin*, pages 53–59.

Scheck, F. (2013). *Quantum Physics*. Springer, Heidelberg - New York - Dordrecht - London, 2 edition.

Schiff, J. L. (1991). *The Laplace Transform: Theory and Applications*. Springer, New York.

Schmidt, F. et al. (2022). Tunable critical casimir forces counteract casimir-lifschitz attraction. *nature physics*, 19:271–278.

Schöneberg, P. and Carmesin, H.-O. (2020). Solution of a Density Problem in the Early Universe. *PhyDid B, FU Berlin*, pages 43–46.

Schöneberg, P. and Carmesin, H.-O. (2021). Solution of the horizon problem. *PhyDid B, FU Berlin*, pages 61–64.

Schrödinger, E. (1926a). Quantisierung als Eigenwertproblem I. *Annalen der Physik*, 79:361–376.

Schrödinger, E. (1926b). Quantisierung als Eigenwertproblem II. *Annalen der Physik*, 79:489–527.

Schrödinger, E. (1926c). Quantisierung als Eigenwertproblem III. *Annalen der Physik*, 80:437.

Schrödinger, E. (1926d). Quantisierung als Eigenwertproblem IV. *Annalen der Physik*, 81:109–139.

Schwartz, M. D. (2014). *Quantum Field Theory and the Standard Model*. Cambridge University Press, Cambridge.

Schwarzschild, K. (1916). Über das Gravitationsfeld eines Massenpunktes nach der Einstein'schen Theorie. *Sitzungsberichte der Deutschen Akad. d. Wiss.*, pages 186–196.

Slipher, V. (1917). Nebulae. *Proceedings of the American Phil. Soc.*, 566:403–409.

Smoot, G. F. (2007). Nobel Lecture: Cosmic microwave background radiation anisotropies: Their discovery and utilization. *Review of Modern Physics*, 79:1347–1379.

Sommerfeld, A. (1978). *Mechanik der deformierbaren Medien*. Verlag Harri Deutsch, Frankfurt, 6. edition.

Sprenger, L. and Carmesin, H.-O. (2018). A Computer Simulation of Cosmic Inflation. *PhyDid B*, pages 61–64.

Stephani, H. (1980). *Allgemeine Relativitätstheorie*. VEB Deutscher Verlag der Wissenschaften, Berlin, 2. edition.

Teschl, G. (2014). *Mathematical Methods in Quantum Mechanics*. American Mathematical Society, Providence, 2. edition.

Thirring, W. (1978). *Lehrbuch der Mathematischen Physik*. Springer, Wien.

Tipler, P. A. and Llewellyn, R. A. (2008). *Modern Physics*. Freeman, New York, 5 edition.

Uddin, S. A. et al. (2024). Carnegie Supernova Project I and II: Measurement of H_0 using Cepheid, TRGB and SBF Distance calibration to type Ia Supernovae. *Astrophys. J.*, accepted:1 – 39.

van der Waals, J. D. (1873). *Over de Continuiteit van den gas- en vloeistoftoestand*. Sijthoff, Leiden.

Walsh, D., Carswell, R. F., and Weyman, R. J. (1979). 0957 + 561 a, b: Twin quasars or grav. lens? *Nature*, 279:381–384.

Weinberg, S. (1967). A Model of Leptons. *Phys. Rev. Lett.*, 19:1264–1266.

Weinberg, S. (1989). The cosmological constant problem. *Reviews of Modern Physics*, 81:1–23.

Weinberg, S. (1996). *The Quantum Theory of Fields*. John Wiley and Sons, New York - London - Sydney - Toronto.

Weinberg, S. (2017). The Trouble with Quantum Mechanics. *The New York Review of Books*.

White, S. D. M., Efstathiou, G., and Frenk, C. S. (1993). The amplitude of mass fluctuations in the universe. *MNRAS*, 262:1023–1028.

Will, C. M. (2014). The Confrontation between General Relativity and Experiment. *Living Revies in Relativity*, 17/4:1–117.

Wirtz, C. (1922). Radialbewegung der Gasnebel. *Astronomische Nachrichten*, 215:281–286.

Workman, R. L. et al. (2022). Review of Particle Physics (by Particle Data Group). *Progr. Theor. Exp. Phys.*, 083C01:1–2270.

Ye, D. et al. (2010). Superluminal Phase Velocity in Dispersive Media. *PIERS Proceedings*, pages 1064–1067.

Ye, D. et al. (2015). Observsation of Wave Packet Distortion during a Negative-Group-Velocity Transmission. *Scientific Reports*, pages 1–5.

Zeldovich, Y. B. (1968). The cosmological constant and the theory of elementary particles. *Sov. Astron. A. J.*, 11:381–393.